Math 8

Unit 4 Lesson 1 Cumulative Practice Problems

1. Tyler reads $\frac{2}{15}$ of a book on Monday, $\frac{1}{3}$ of it on Tuesday, $\frac{2}{9}$ of it on Wednesday, and $\frac{3}{4}$ of the remainder on Thursday. If he still has 14 pages left to read on Friday, how many pages are there in the book?

2. Clare asks Andre to play the following number puzzle:

   - Pick a number
   - Add 2
   - Multiply by 3
   - Subtract 7
   - Add your original number

   Andre's final result is 27.
   Which number did he start with?

3. In a basketball game, Elena scores twice as many points as Tyler. Tyler scores four points fewer than Noah, and Noah scores three times as many points as Mai. If Mai scores 5 points, how many points did Elena score? Explain your reasoning.

4. Select **all** of the given points in the coordinate plane that lie on the graph of the linear equation $4x - y = 3$.

   A. $(-1, -7)$

   B. $(0, 3)$

   C. $(\frac{3}{4}, 0)$

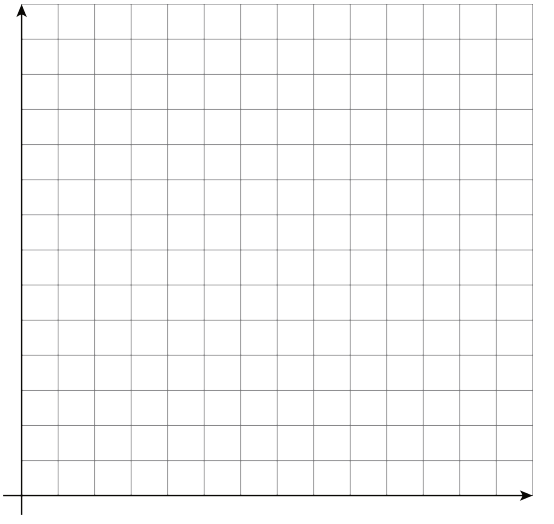
   D. $(1, 1)$

   E. $(2, 5)$

   F. $(4, -1)$

   (From Unit 3, Lesson 12.)

5. A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 feet long, followed by the nested carts (so 0 nested carts means there's just the starting cart). The store measured a row of 13 nested carts to be 23.5 feet long, and a row of 18 nested carts to be 31 feet long.



   a. Create a graph of the situation.

   b. How much does each nested cart add to the length of the row? Explain your reasoning.

   c. If the store design allows for 43 feet for each row, how many total carts fit in a row?

   (From Unit 3, Lesson 5.)

6. Triangle A is an isosceles triangle with two angles of measure x degrees and one angle of measure y degrees.

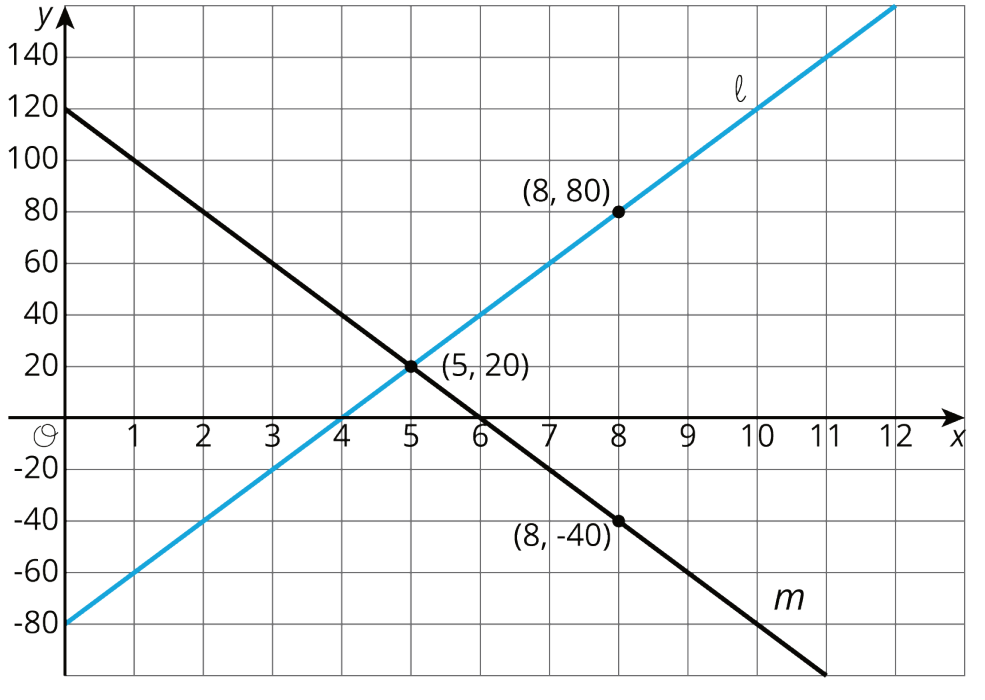
   a. Find three combinations of x and y that make this sentence true.

   b. Write an equation relating x and y.

   c. If you were to sketch the graph of this linear equation, what would its slope be? How can you interpret the slope in the context of the triangle?

(From Unit 3, Lesson 13.)

7. Consider the following graphs of linear equations. Decide which line has a positive slope, and which has a negative slope. Then calculate each line's exact slope.

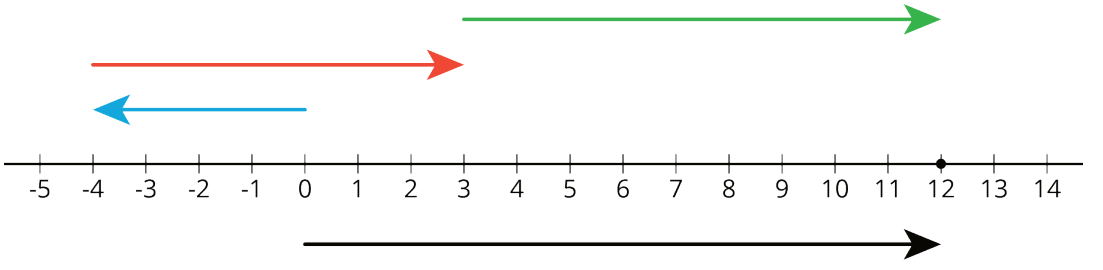


(From Unit 3, Lesson 10.)

Lesson 1: Number Puzzles

1.1: Notice and Wonder: A Number Line

What do you notice? What do you wonder?



1.2: Telling Temperatures

Solve each puzzle. Show your thinking. Organize it so it can be followed by others.

1. The temperature was very cold. Then the temperature doubled.
   Then the temperature dropped by 10 degrees. Then the temperature increased by 40 degrees. The temperature is now 16 degrees. What was the starting temperature?

2. Lin ran twice as far as Diego. Diego ran 300 m farther than Jada. Jada ran $\frac{1}{3}$ the distance that Noah ran. Noah ran 1200 m. How far did Lin run?

1.3: Making a Puzzle

Write another number puzzle with at least three steps. On a different piece of paper, write a solution to your puzzle.

Trade puzzles with your partner and solve theirs. Make sure to show your thinking.

With your partner, compare your solutions to each puzzle. Did they solve them the same way you did? Be prepared to share with the class which solution strategy you like best.

Are you ready for more?

Here is a number puzzle that uses math. Some might call it a magic trick!

1. Think of a number.
2. Double the number.
3. Add 9.
4. Subtract 3.
5. Divide by 2.
6. Subtract the number you started with.
7. The answer should be 3.

Why does this always work? Can you think of a different number puzzle that uses math (like this one) that will always result in 5?

Lesson 1 Summary

Here is an example of a puzzle problem:

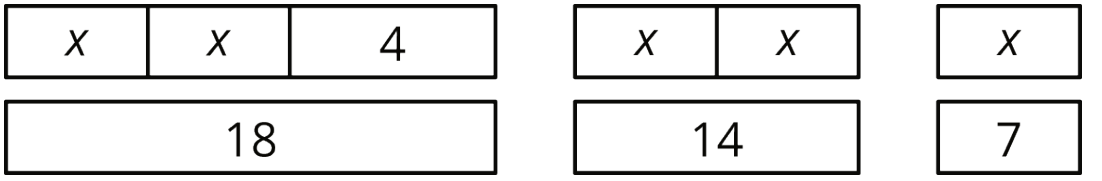
Twice a number plus 4 is 18. What is the number?

There are many different ways to represent and solve puzzle problems.

- We can reason through it.

  Twice a number plus 4 is 18.
  Then twice the number is $18 - 4 = 14$.
  That means the number is 7.

- We can draw a diagram.



- We can write and solve an equation.

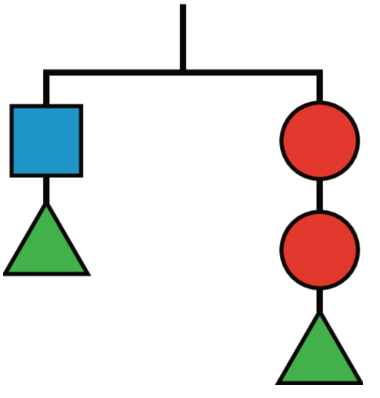
$$2x + 4 = 18$$
$$2x = 14$$
$$x = 7$$

Reasoning and diagrams help us see what is going on and why the answer is what it is. But as number puzzles and story problems get more complex, those methods get harder, and equations get more and more helpful. We will use different kinds of diagrams to help us understand problems and strategies in future lessons, but we will also see the power of writing and solving equations to answer increasingly more complex mathematical problems.

Unit 4 Lesson 2 Cumulative Practice Problems

1. Which of the changes would keep the hanger in balance?

   Select all that apply.



A. Adding two circles on the left and a square on the right

B. Adding 2 triangles to each side

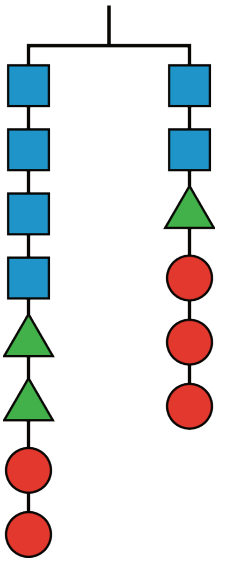
C. Adding two circles on the right and a square on the left

D. Adding a circle on the left and a square on the right

E. Adding a triangle on the left and a square on the right

2. Here is a balanced hanger diagram.

   Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square. Select *all* equations that represent the hanger.



   A. $x + x + x + x + 11 = x + 11.5$

   B. $2x = 0.5$

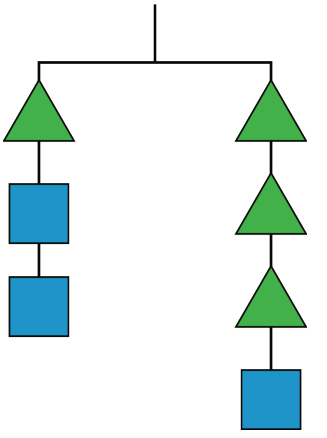
   C. $4x + 5 + 6 = 2x + 2.5 + 6$

   D. $2x + 2.5 = 3$

   E. $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

3. What is the weight of a square if a triangle weighs 4 grams?

   Explain your reasoning.



4. Andre came up with the following puzzle. “I am three years younger than my brother, and I am 2 years older than my sister. My mom's age is one less than three times my brother's age. When you add all our ages, you get 87. What are our ages?”

   a. Try to solve the puzzle.

   b. Jada writes this equation for the sum of the ages:
   $(x) + (x + 3) + (x - 2) + 3(x + 3) - 1 = 87.$

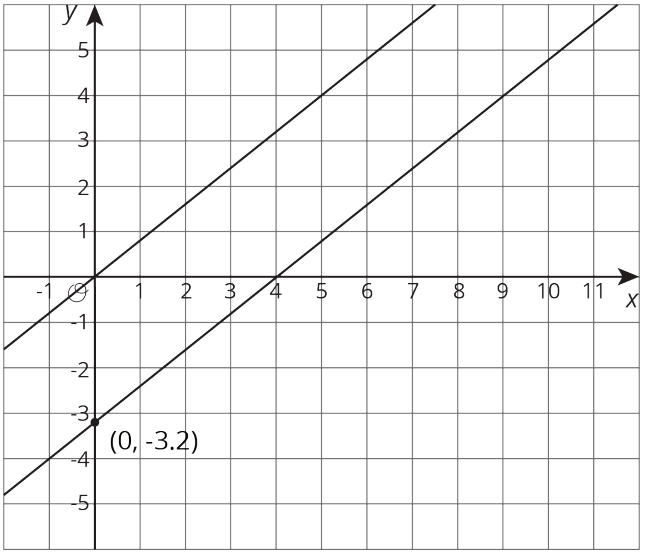
   Explain the meaning of the variable and each term of the equation.

   c. Write the equation with fewer terms.

   d. Solve the puzzle if you haven’t already.

(From Unit 4, Lesson 1.)

5. These two lines are parallel. Write an equation for each.



(From Unit 3, Lesson 8.)

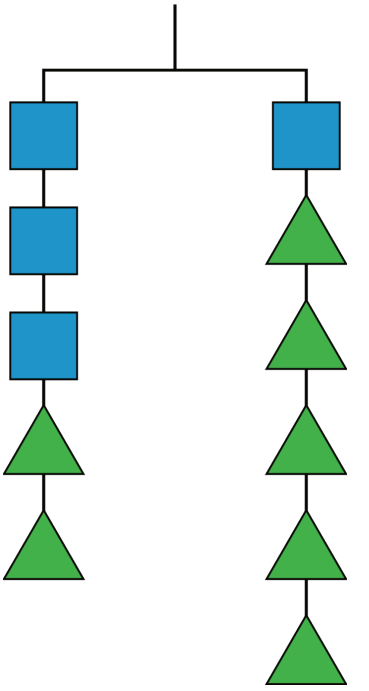
Lesson 2: Keeping the Equation Balanced

2.1: Notice and Wonder: Hanging Socks

What do you notice? What do you wonder?

2.2: Hanging Blocks

This picture represents a hanger that is balanced because the weight on each side is the same.



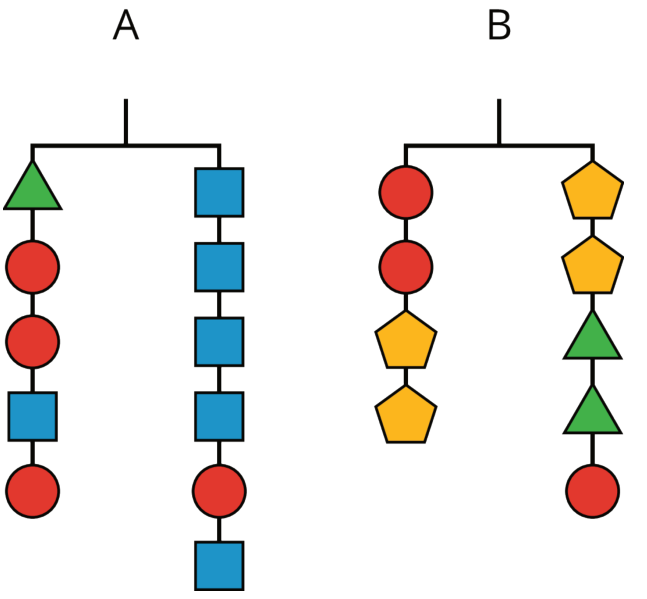
1. Elena takes two triangles off of the left side and three triangles off of the right side. Will the hanger still be in balance, or will it tip to one side? Which side? Explain how you know.

2. If a triangle weighs 1 gram, how much does a square weigh?

2.3: More Hanging Blocks

A triangle weighs 3 grams and a circle weighs 6 grams.

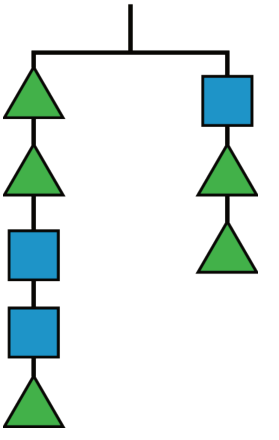
1. Find the weight of a square in Hanger A and the weight of a pentagon in Hanger B.



2. Write an equation to represent each hanger.

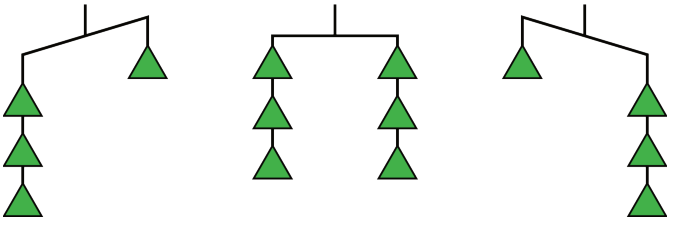
Are you ready for more?

What is the weight of a square on this hanger if a triangle weighs 3 grams?

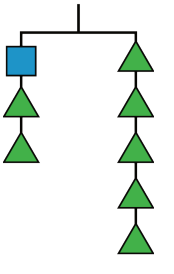


Lesson 2 Summary

If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than the other, the hanger will tilt to the heavier side.

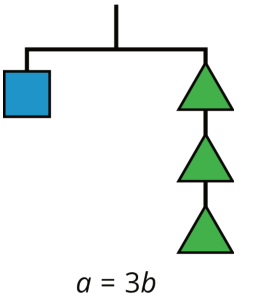


We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on each side have equal value, just like a balanced hanger has equal weights on each side.



$a + 2b = 5b$

If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance.

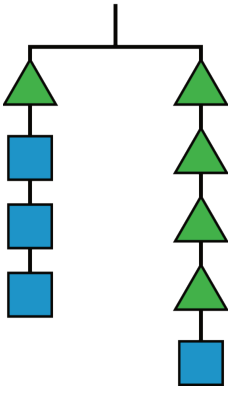


$a = 3b$

We can do these moves with equations as well: adding or subtracting the same amount from each side of an equation maintains the equality.

Unit 4 Lesson 3 Cumulative Practice Problems

1. In this hanger, the weight of the triangle is x and the weight of the square is y.



   a. Write an equation using x and y to represent the hanger.

   b. If x is 6, what is y?

2. Andre and Diego were each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each make to the equation.

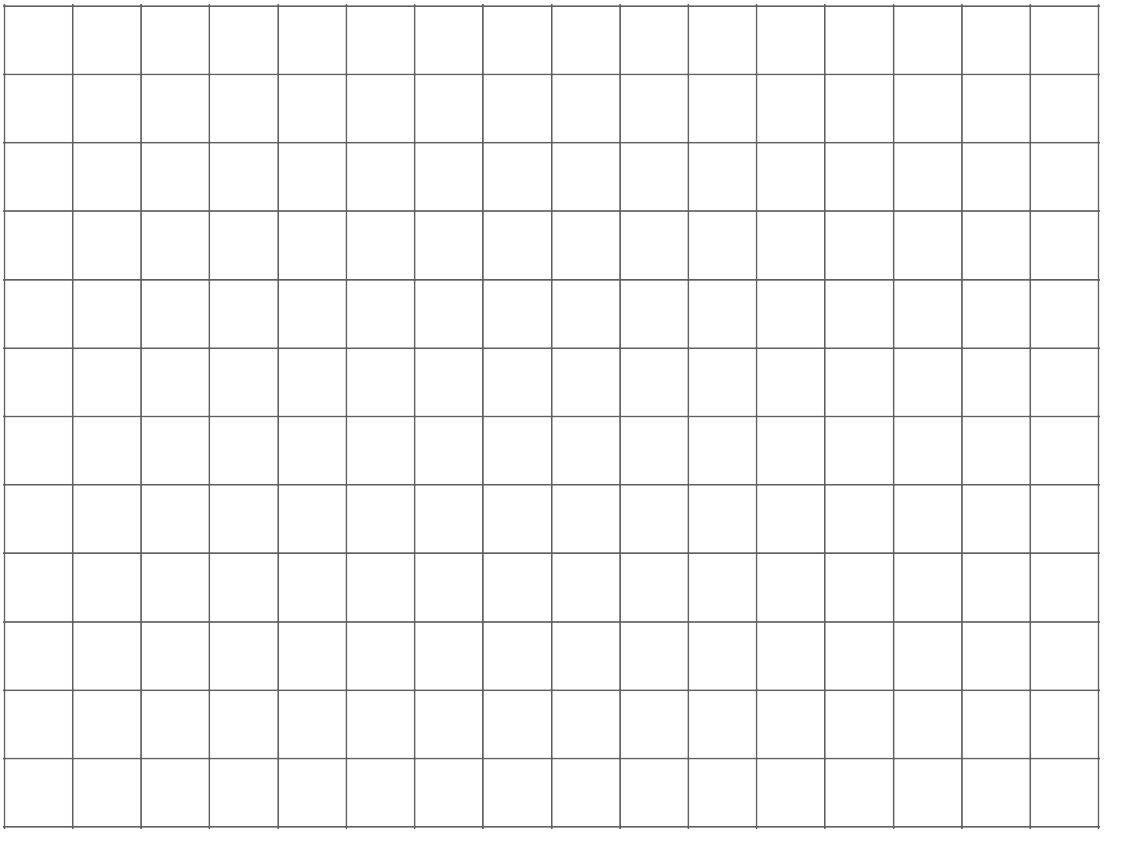
   a. The result of Andre's first step was $-x + 6 = -8$.

   b. The result of Diego's first step was $6 = x - 8$.

3. a. Complete the table with values for x or y that make this equation true: $3x + y = 15$.

x	2		6	0	3		
y		3				0	8

b. Create a graph, plot these points, and find the slope of the line that goes through them.



(From Unit 3, Lesson 11.)

4. Match each set of equations with the move that turned the first equation into the second.

A. $6x + 9 = 4x - 3$
$2x + 9 = -3$

B. $-4(5x - 7) = -18$
$5x - 7 = 4.5$

C. $8 - 10x = 7 + 5x$
$4 - 10x = 3 + 5x$

D. $\frac{-5x}{4} = 4$
$5x = -16$

E. $12x + 4 = 20x + 24$
$3x + 1 = 5x + 6$

1. Multiply both sides by $\frac{-1}{4}$

2. Multiply both sides by -4

3. Multiply both sides by $\frac{1}{4}$

4. Add $-4x$ to both sides

5. Add -4 to both sides

5. Select **all** the situations for which only zero or positive solutions make sense.

A. Measuring temperature in degrees Celsius at an Arctic outpost each day in January.

B. The height of a candle as it burns over an hour.

C. The elevation above sea level of a hiker descending into a canyon.

D. The number of students remaining in school after 6:00 p.m.

E. A bank account balance over a year.

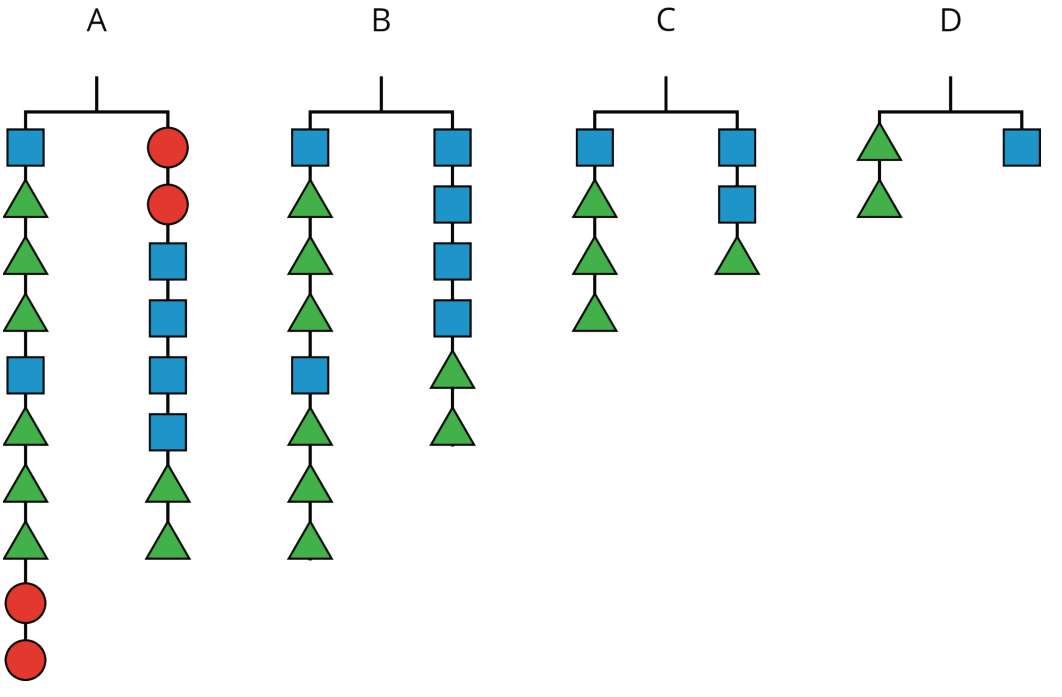
F. The temperature in degrees Fahrenheit of an oven used on a hot summer day.

(From Unit 3, Lesson 14.)

Lesson 3: Balanced Moves

3.1: Matching Hangers

Figures A, B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.



Here are some equations. Each equation represents one of the hanger diagrams.

$$2(x + 3y) = 4x + 2y$$
$$2y = x$$
$$2(x + 3y) + 2z = 2z + 4x + 2y$$
$$x + 3y = 2x + y$$

1. Write the equation that goes with each figure:
   A:
   B:
   C:
   D:

2. Each variable (x, y, and z) represents the weight of one shape. Which goes with which?

3. Explain what was done to each equation to create the next equation. If you get stuck, think about how the hangers changed.

3.2: Matching Equation Moves

Your teacher will give you some cards. Each of the cards 1 through 6 show two equations. Each of the cards A through E describe a move that turns one equation into another.

1. Match each number card with a letter card.

2. One of the letter cards will not have a match. For this card, write two equations showing the described move.

3.3: Keeping Equality

1. Noah and Lin both solved the equation $14a = 2(a - 3)$.
   Do you agree with either of them? Why?

   Noah's solution:

   $$\begin{aligned} 14a &= 2(a-3) \\ 14a &= 2a - 6 \\ 12a &= -6 \\ a &= -\frac{1}{2} \end{aligned}$$

   Lin's solution:

   $$\begin{aligned} 14a &= 2(a-3) \\ 7a &= a - 3 \\ 6a &= -3 \\ a &= -\frac{1}{2} \end{aligned}$$

2. Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?

3. Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

Are you ready for more?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

HANGER + HANGER + HANGER = ALGEBRA

Lesson 3 Summary

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $-2x - 3$ have equal value, we can write the equation

$$4x + 9 = -2x - 3$$

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add *negative numbers*! For example, we can add -9 to each side of the equation.

$$\begin{aligned} 4x + 9 + -9 &= -2x - 3 + -9 && \text{add -9 to each side} \\ 4x &= -2x - 12 && \text{combine like terms} \end{aligned}$$

Since expressions represent numbers, we can also add *expressions* to each side of an equation. For example, we can add $2x$ to each side and still maintain equality.

$$\begin{aligned} 4x + 2x &= -2x - 12 + 2x && \text{add } 2x \text{ to each side} \\ 6x &= -12 && \text{combine like terms} \end{aligned}$$

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (so long as we do not divide by zero).

$$6x \cdot \frac{1}{6} = -12 \cdot \frac{1}{6} \qquad \text{multiply each side by } \frac{1}{6}$$

or

$$6x \div 6 = -12 \div 6 \qquad \text{divide each side by 6}$$

Now we can see that $x = -2$ is the solution to our equation.

We will use these moves in systematic ways to solve equations in future lessons.

