Math 7

Unit 6 Lesson 2 Cumulative Practice Problems

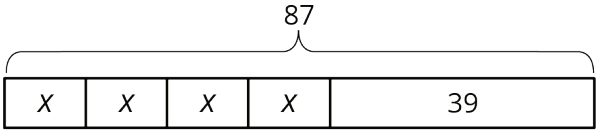
1. The table shows the number of apples and the total weight of the apples.

number of apples	weight of apples (grams)
2	511
5	1200
8	2016

Estimate the weight of 6 apples.

(From Unit 3, Lesson 1.)

2. Select **all** stories that the tape diagram can represent.



A. There are 87 children and 39 adults at a show. The seating in the theater is split into 4 equal sections.

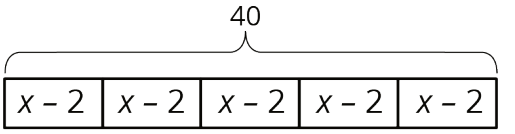
B. There are 87 first graders in after-care. After 39 students are picked up, the teacher put the remaining students into 4 groups for an activity.

C. Lin buys a pack of 87 pencils. She gives 39 to her teacher and shared the remaining pencils between herself and 3 friends.

D. Andre buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.

E. Diego's family spends $87 on 4 tickets to the fair and a $39 dinner.

3. Andre wants to save $40 to buy a gift for his dad. Andre's neighbor will pay him weekly to mow the lawn, but Andre always gives a $2 donation to the food bank in weeks when he earns money. Andre calculates that it will take him 5 weeks to earn the money for his dad's gift. He draws a tape diagram to represent the situation.



a. Explain how the parts of the tape diagram represent the story.

b. How much does Andre's neighbor pay him each week to mow the lawn?

4. Without evaluating each expression, determine which value is the greatest. Explain how you know.

a. $7\frac{5}{6} - 9\frac{3}{4}$

b. $(-7\frac{5}{6}) + (-9\frac{3}{4})$

c. $(-7\frac{5}{6}) \cdot 9\frac{3}{4}$

d. $(-7\frac{5}{6}) \div (-9\frac{3}{4})$

(From Unit 5, Lesson 13.)

5. Solve each equation.

a. $(8.5) \cdot (-3) = a$

b. $(-7) + b = (-11)$

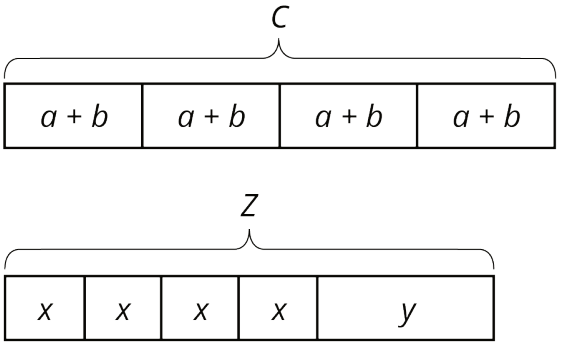
c. $c - (-3) = 15$

d. $d \cdot (-4) = 32$

(From Unit 5, Lesson 15.)

Lesson 2: Reasoning about Contexts with Tape Diagrams (Part 1)

2.1: Notice and Wonder: Remembering Tape Diagrams



1. What do you notice? What do you wonder?

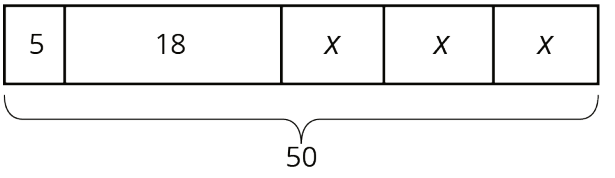
2. What are some possible values for a, b, and c in the first diagram?

   For x, y, and z in the second diagram? How did you decide on those values?

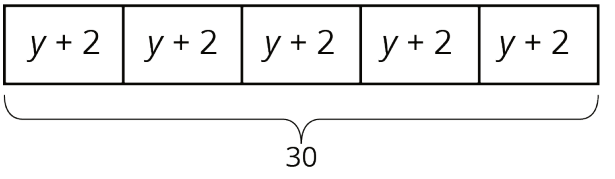
2.2: Every Picture Tells a Story

Here are three stories with a diagram that represents it. With your group, decide who will go first. That person explains why the diagram represents the story. Work together to find any unknown amounts in the story. Then, switch roles for the second diagram and switch again for the third.

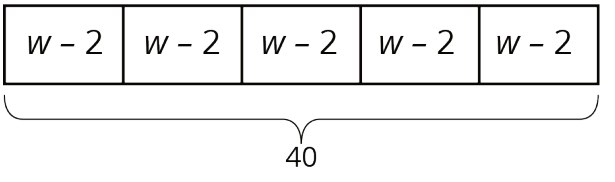
1. Mai made 50 flyers for five volunteers in her club to hang up around school. She gave 5 flyers to the first volunteer, 18 flyers to the second volunteer, and divided the remaining flyers equally among the three remaining volunteers.



2. To thank her five volunteers, Mai gave each of them the same number of stickers. Then she gave them each two more stickers. Altogether, she gave them a total of 30 stickers.



3. Mai distributed another group of flyers equally among the five volunteers. Then she remembered that she needed some flyers to give to teachers, so she took 2 flyers from each volunteer. Then, the volunteers had a total of 40 flyers to hang up.



2.3: Every Story Needs a Picture

Here are three more stories. Draw a tape diagram to represent each story. Then describe how you would find any unknown amounts in the stories.

1. Noah and his sister are making gift bags for a birthday party. Noah puts 3 pencil erasers in each bag. His sister puts x stickers in each bag. After filling 4 bags, they have used a total of 44 items.

2. Noah's family also wants to blow up a total of 60 balloons for the party. Yesterday they blew up 24 balloons. Today they want to split the remaining balloons equally between four family members.

3. Noah's family bought some fruit bars to put in the gift bags. They bought one box each of four flavors: apple, strawberry, blueberry, and peach. The boxes all had the same number of bars. Noah wanted to taste the flavors and ate one bar from each box. There were 28 bars left for the gift bags.

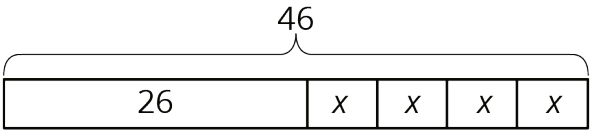
Are you ready for more?

Design a tiling that uses a repeating pattern consisting of 2 kinds of shapes (e.g., 1 hexagon with 3 triangles forming a triangle). How many times did you repeat the pattern in your picture? How many individual shapes did you use?

Lesson 2 Summary

Tape diagrams are useful for representing how quantities are related and can help us answer questions about a situation.

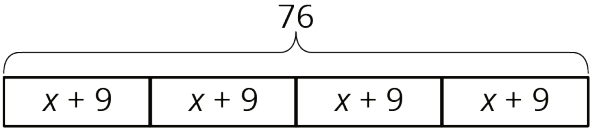
Suppose a school receives 46 copies of a popular book. The library takes 26 copies and the remainder are split evenly among 4 teachers. How many books does each teacher receive? This situation involves 4 equal parts and one other part. We can represent the situation with a rectangle labeled 26 (books given to the library) along with 4 equal-sized parts (books split among 4 teachers). We label the total, 46, to show how many the rectangle represents in all. We use a letter to show the unknown amount, which represents the number of books each teacher receives. Using the same letter, x, means that the same number is represented four times.



Some situations have parts that are all equal, but each part has been increased from an original amount:

A company manufactures a special type of sensor, and packs them in boxes of 4 for shipment. Then a new design increases the weight of each sensor by 9 grams. The new package of 4 sensors weighs 76 grams. How much did each sensor weigh originally?

We can describe this situation with a rectangle representing a total of 76 split into 4 equal parts. Each part shows that the new weight, $x + 9$, is 9 more than the original weight, x.



Unit 6 Lesson 3 Cumulative Practice Problems

1. Solve each equation mentally.

   a. $2x = 10$

   b. $-3x = 21$

   c. $\frac{1}{3}x = 6$

   d. $-\frac{1}{2}x = -7$

   (From Unit 5, Lesson 15.)

2. Complete the magic squares so that the sum of each row, each column, and each diagonal in a grid are all equal.

0	7	2
	3	

1		
	3	-2
		5

4	2	0
-1		

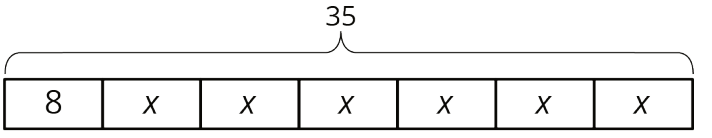
   (From Unit 5, Lesson 3.)

3. Draw a tape diagram to match each equation.

   a. $5(x + 1) = 20$

   b. $5x + 1 = 20$

4. Select **all** the equations that match the tape diagram.



A. $35 = 8 + x + x + x + x + x + x$

B. $35 = 8 + 6x$

C. $6 + 8x = 35$

D. $6x + 8 = 35$

E. $6x + 8x = 35x$

F. $35 - 8 = 6x$

5. Each car is traveling at a constant speed. Find the number of miles each car travels in 1 hour at the given rate.

a. 135 miles in 3 hours

b. 22 miles in $\frac{1}{2}$ hour

c. 7.5 miles in $\frac{1}{4}$ hour

d. $\frac{100}{3}$ miles in $\frac{2}{3}$ hour

e. $97\frac{1}{2}$ miles in $\frac{3}{2}$ hour

(From Unit 4, Lesson 2.)

Lesson 3: Reasoning about Equations with Tape Diagrams

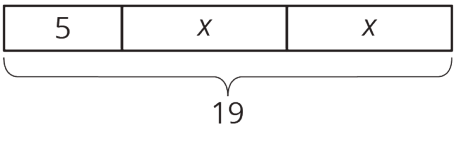
3.1: Find Equivalent Expressions

Select **all** the expressions that are equivalent to $7(2 - 3n)$. Explain how you know each expression you select is equivalent.

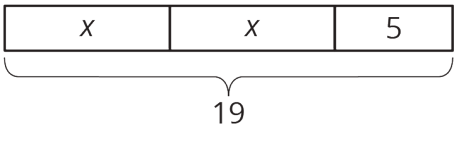
1. $9 - 10n$
2. $14 - 3n$
3. $14 - 21n$
4. $(2 - 3n) \cdot 7$
5. $7 \cdot 2 \cdot (-3n)$

3.2: Matching Equations to Tape Diagrams

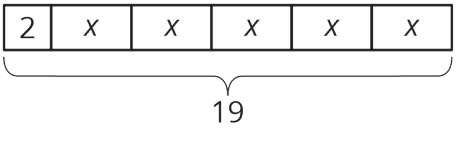
A



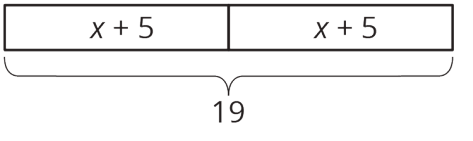
B



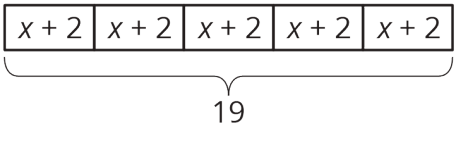
C



D



E



1. Match each equation to one of the tape diagrams. Be prepared to explain how the equation matches the diagram.

2. Sort the equations into categories of your choosing. Explain the criteria for each category.

- $2x + 5 = 19$
- $2 + 5x = 19$
- $2(x + 5) = 19$
- $5(x + 2) = 19$
- $19 = 5 + 2x$
- $(x + 5) \cdot 2 = 19$
- $19 = (x + 2) \cdot 5$
- $19 \div 2 = x + 5$
- $19 - 2 = 5x$

3.3: Drawing Tape Diagrams to Represent Equations

- $114 = 3x + 18$
- $114 = 3(y + 18)$

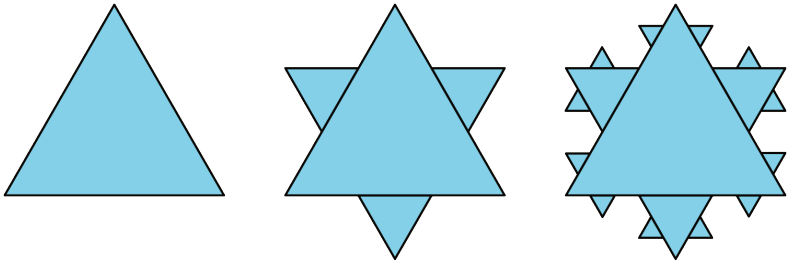
1. Draw a tape diagram to match each equation.

2. Use any method to find values for x and y that make the equations true.

Are you ready for more?

To make a Koch snowflake:

- Start with an equilateral triangle that has side lengths of 1. This is step 1.
- Replace the middle third of each line segment with a small equilateral triangle with the middle third of the segment forming the base. This is step 2.
- Do the same to each of the line segments. This is step 3.
- Keep repeating this process.



step 1 step 2 step 3

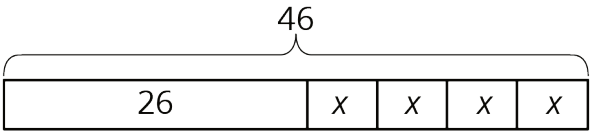
1. What is the perimeter after step 2? Step 3?

2. What happens to the perimeter, or the length of line traced along the outside of the figure, as the process continues?

Lesson 3 Summary

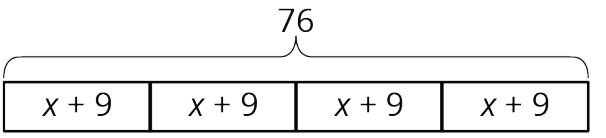
We have seen how tape diagrams represent relationships between quantities. Because of the meaning and properties of addition and multiplication, more than one equation can often be used to represent a single tape diagram.

Let's take a look at two tape diagrams.



We can describe this diagram with several different equations. Here are some of them:

- $26 + 4x = 46$, because the parts add up to the whole.
- $4x + 26 = 46$, because addition is commutative.
- $46 = 4x + 26$, because if two quantities are equal, it doesn't matter how we arrange them around the equal sign.
- $4x = 46 - 26$, because one part (the part made up of 4 x's) is the difference between the whole and the other part.



For this diagram:

- $4(x + 9) = 76$, because multiplication means having multiple groups of the same size.
- $(x + 9) \cdot 4 = 76$, because multiplication is commutative.
- $76 \div 4 = x + 9$, because division tells us the size of each equal part.

Unit 6 Lesson 4 Cumulative Practice Problems

1. Draw a square with side length 7 cm.

   a. Predict the perimeter and the length of the diagonal of the square.

   b. Measure the perimeter and the length of the diagonal of the square.

   c. Describe how close the predictions and measurements are.

(From Unit 3, Lesson 1.)

2. Find the products.

   a. $(100) \cdot (-0.09)$

   b. $(-7) \cdot (-1.1)$

   c. $(-7.3) \cdot (5)$

   d. $(-0.2) \cdot (-0.3)$

(From Unit 5, Lesson 9.)

3. Here are three stories:

   - A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 to see the show.

   - Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.

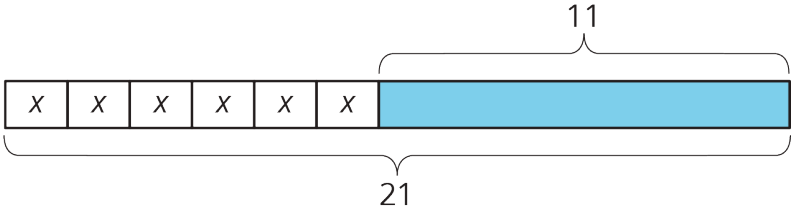
   - Jada works for 6 hours preparing for the art fair. She spends 3 hours on a sculpture and then paints 27 picture frames.

Here are three equations:

- $3x + 6 = 27$
- $6x + 3 = 27$
- $27x + 3 = 6$

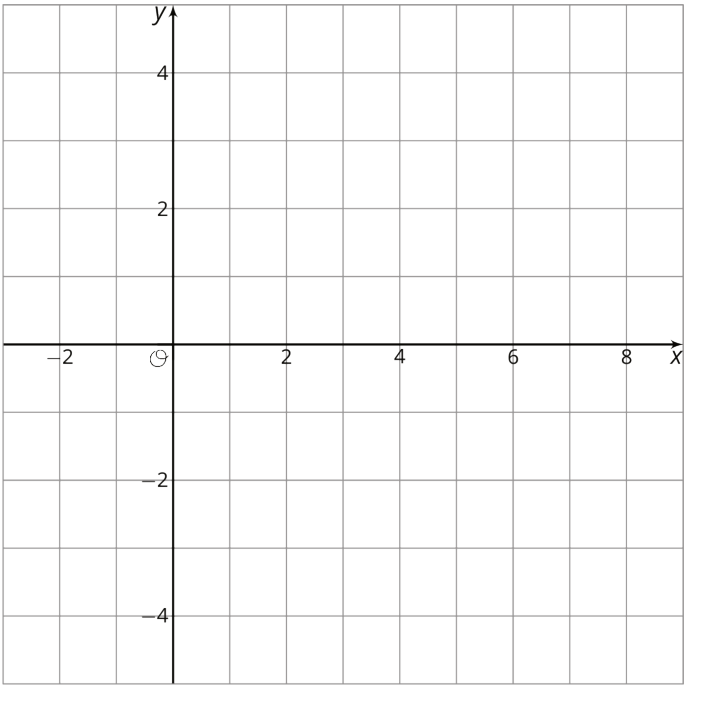
a. Decide which equation represents each story. What does x represent in each equation?
b. Find the solution to each equation. Explain or show your reasoning.
c. What does each solution tell you about its situation?

4. Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$6x + 11 = 21$

5. a. Plot these points on the coordinate plane:
$A = (3, 2), B = (7.5, 2), C = (7.5, -2.5), D = (3, -2)$



b. What is the vertical difference between D and A?

c. Write an expression that represents the vertical distance between B and C.

(From Unit 5, Lesson 7.)

Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)

4.1: Algebra Talk: Seeing Structure

Find a solution to each equation without writing anything down.

$x + 1 = 5$

$2(x + 1) = 10$

$3(x + 1) = 15$

$500 = 100(x + 1)$

4.2: Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Diego has 7 packs of markers. Each pack has x markers in it. After Lin gives him 9 more markers, he has a total of 30 markers.

2. Elena is cutting a 30-foot piece of ribbon for a craft project. She cuts off 7 feet, and then cuts the remaining piece into 9 equal lengths of x feet each.

3. A construction manager weighs a bundle of 9 identical bricks and a 7-pound concrete block. The bundle weighs 30 pounds.

4. A skating rink charges a group rate of $9 plus a fee to rent each pair of skates. A family rents 7 pairs of skates and pays a total of $30.

5. Andre bakes 9 pans of brownies. He donates 7 pans to the school bake sale and keeps the rest to divide equally among his class of 30 students.

4.3: Situations, Diagrams, and Equations

Each situation in the previous activity is represented by one of the equations.

- $7x + 9 = 30$
- $30 = 9x + 7$
- $30x + 7 = 9$

1. Match each situation to an equation.
2. Find the solution to each equation. Use your diagrams to help you reason.
3. What does each solution tell you about its situation?

Are you ready for more?

While in New York City, is it a better deal for a group of friends to take a taxi or the subway to get from the Empire State Building to the Metropolitan Museum of Art? Explain your reasoning.

Lesson 4 Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are three situations:

1. An architect is drafting plans for a new supermarket. There will be a space 144 inches long for rows of nested shopping carts. The first cart is 34 inches long and each nested cart adds another 10 inches. The architect want to know how many shopping carts will fit in each row.

2. A bakery buys a large bag of sugar that has 34 cups. They use 10 cups to make some cookies. Then they use the rest of the bag to make 144 giant muffins. Their customers want to know how much sugar is in each muffin.

3. Kiran is trying to save $144 to buy a new guitar. He has $34 and is going to save $10 a week from money he earns mowing lawns. He wants to know how many weeks it will take him to have enough money to buy the guitar.

We see the same three numbers in the situations: 10, 34, and 144. How could we represent each situation with an equation?

In the first situation, there is one shopping cart with length 34 and then an unknown number of carts with length 10. Similarly, Kiran has 34 dollars saved and then will save 10 each week for an unknown number of weeks. Both situations have one part of 34 and then equal parts of size 10 that all add together to 144. Their equation is $34 + 10x = 144$.

Since it takes 11 groups of 10 to get from 34 to 144, the value of x in these two situations is $(144 - 34) \div 10$ or 11. There will be 11 shopping carts in each row, and it will take Kiran 11 weeks to raise the money for the guitar.

In the bakery situation, there is one part of 10 and then 144 equal parts of unknown size that all add together to 34. The equation is $10 + 144x = 34$. Since 24 is needed to get from 10 to 34, the value of x is $(34 - 10) \div 144$ or $\frac{1}{6}$. There is $\frac{1}{6}$ cup of sugar in each giant muffin.

Unit 6 Lesson 5 Cumulative Practice Problems

1. Here are some prices customers paid for different items at a farmer's market. Find the cost for 1 pound of each item.

   a. \$5 for 4 pounds of apples

   b. \$3.50 for $\frac{1}{2}$ pound of cheese

   c. \$8.25 for $1\frac{1}{2}$ pounds of coffee beans

   d. \$6.75 for $\frac{3}{4}$ pounds of fudge

   e. \$5.50 for a $6\frac{1}{4}$ pound pumpkin

   (From Unit 4, Lesson 2.)

2. Find the products.

   a. $\frac{2}{3} \cdot \left(\frac{-4}{5}\right)$

   b. $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right)$

   c. $\left(\frac{-2}{39}\right) \cdot 39$

   d. $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right)$

   (From Unit 5, Lesson 9.)

3. Here are two stories:

   - A family buys 6 tickets to a show. They also *each* spend $3 on a snack. They spend $24 on the show.

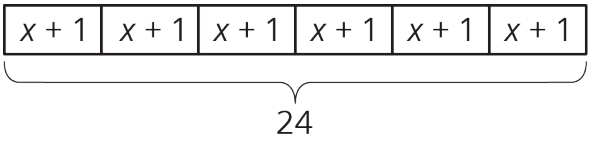
   - Diego has 24 ounces of juice. He pours equal amounts for each of his 3 friends, and then adds 6 more ounces for each.

   Here are two equations:

   - $3(x + 6) = 24$
   - $6(x + 3) = 24$

   a. Which equation represents which story?
   b. What does x represent in each equation?
   c. Find the solution to each equation. Explain or show your reasoning.
   d. What does each solution tell you about its situation?

4. Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$6(x + 1) = 24$$

5. Below is a set of data about temperatures. The *range* of a set of data is the distance between the lowest and highest value in the set. What is the range of these temperatures?

   9°C, -3°C, 22°C, -5°C, 11°C, 15°C

   (From Unit 5, Lesson 7.)

6. A store is having a 25% off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs $24.

   (From Unit 4, Lesson 11.)

Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)

5.1: Algebra Talk: Seeing Structure

Solve each equation mentally.

$x - 1 = 5$

$2(x - 1) = 10$

$3(x - 1) = 15$

$500 = 100(x - 1)$

5.2: More Situations and Diagrams

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Each of 5 gift bags contains x pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.

2. Noah drew an equilateral triangle with sides of length 5 inches. He wants to increase the length of each side by x inches so the triangle is still equilateral and has a perimeter of 20 inches.

3. An art class charges each student $3 to attend plus a fee for supplies. Today, $20 was collected for the 5 students attending the class.

4. Elena ran 20 miles this week, which was three times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

5.3: More Situations, Diagrams, and Equations

Each situation in the previous activity is represented by one of the equations.

- $(x + 3) \cdot 5 = 20$

- $3(x + 5) = 20$

1. Match each situation to an equation.
2. Find the solution to each equation. Use your diagrams to help you reason.
3. What does each solution tell you about its situation?

Are you ready for more?

Han, his sister, his dad, and his grandmother step onto a crowded bus with only 3 open seats for a 42-minute ride. They decide Han's grandmother should sit for the entire ride. Han, his sister, and his dad take turns sitting in the remaining two seats, and Han's dad sits 1.5 times as long as both Han and his sister. How many minutes did each one spend sitting?

Lesson 5 Summary

Equations with parentheses can represent a variety of situations.

1. Lin volunteers at a hospital and is preparing toy baskets for children who are patients. She adds 2 items to each basket, after which the supervisor's list shows that 140 toys have been packed into a group of 10 baskets. Lin wants to know how many toys were in each basket before she added the items.

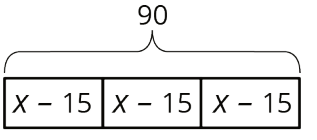
2. A large store has the same number of workers on each of 2 teams to handle different shifts. They decide to add 10 workers to each team, bringing the total number of workers to 140. An executive at the company that runs this chain of stores wants to know how many employees were in each team before the increase.

Each bag in the first story has an unknown number of toys, x, that is increased by 2. Then ten groups of $x + 2$ give a total of 140 toys. An equation representing this situation is $10(x + 2) = 140$. Since 10 times a number is 140, that number is 14, which is the total number of items in each bag. Before Lin added the 2 items there were $14 - 2$ or 12 toys in each bag.

The executive in the second story knows that the size of each team of y employees has been increased by 10. There are now 2 teams of $y + 10$ each. An equation representing this situation is $2(y + 10) = 140$. Since 2 times an amount is 140, that amount is 70, which is the new size of each team. The value of y is $70 - 10$ or 60. There were 60 employees on each team before the increase.

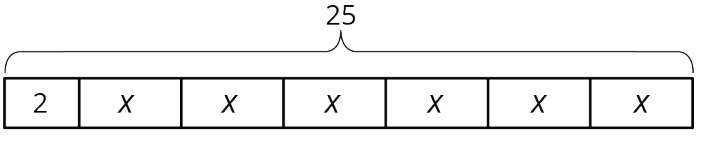
Unit 6 Lesson 6 Cumulative Practice Problems

1. A school ordered 3 large boxes of board markers. After giving 15 markers to each of 3 teachers, there were 90 markers left. The diagram represents the situation. How many markers were originally in each box?



(From Unit 6, Lesson 2.)

2. The diagram can be represented by the equation $25 = 2 + 6x$. Explain where you can see the 6 in the diagram.



(From Unit 6, Lesson 3.)

3. Match each equation to a story. (Two of the stories match the same equation.)

a. $3(x + 5) = 17$

b. $3x + 5 = 17$

c. $5(x + 3) = 17$

d. $5x + 3 = 17$

a. Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?

b. A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?

c. Elena spends $17 and buys a $3 book and a bookmark for each of her 5 cousins. How much does each bookmark cost?

d. Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?

e. Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

4. Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation $2(x + 20) = 90$ describes this situation. Match each expression with the statement in the story with the expression it represents.

A. x

B. $x + 20$

C. $2(x + 20)$

D. 90

1. The number of minutes that Jada walked

2. The number of minutes that Elena walked

3. The number of minutes that Lin walked

Lesson 6: Distinguishing between Two Types of Situations

6.1: Which One Doesn't Belong: Seeing Structure

Which equation doesn't belong?

$4(x + 3) = 9$

$4 + 3x = 9$

$4 \cdot x + 12 = 9$

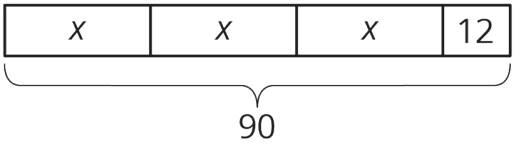
$9 = 12 + 4x$

6.2: Card Sort: Categories of Equations

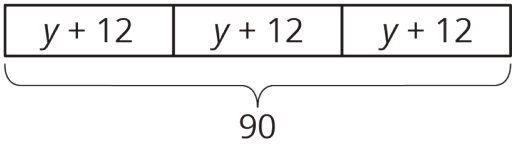
Your teacher will give you a set of cards that show equations. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories. Then, sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

6.3: Even More Situations, Diagrams, and Equations

A



B



Story 1: Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Story 2: Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

1. Which diagram goes with which story? Be prepared to explain your reasoning.

2. In each diagram, what part of the story does the variable represent?

3. Write an equation corresponding to each story. If you get stuck, use the diagram.

4. Find the value of the variable in the story.

Are you ready for more?

A tutor is starting a business. In the first year, they start with 5 clients and charge $10 per week for an hour of tutoring with each client. For each year following, they double the number of clients and the number of hours each week. Each new client will be charged 150% of the charges of the clients from the previous year.

1. Organize the weekly earnings for each year in a table.

2. Assuming a full-time week is 40 hours per week, how many years will it take to reach full time and how many new clients will be taken on that year?

3. After reaching full time, what is the tutor's annual salary if they take 2 weeks of vacation?

4. Is there another business model you'd recommend for the tutor? Explain your reasoning.

Lesson 6 Summary

In this unit, we encounter two main types of situations that can be represented with an equation. Here is an example of each type:

1. After adding 8 students to each of 6 same-sized teams, there were 72 students altogether.

2. After adding an 8-pound box of tennis rackets to a crate with 6 identical boxes of ping pong paddles, the crate weighed 72 pounds.

The first situation has all equal parts, since additions are made to *each* team. An equation that represents this situation is $6(x + 8) = 72$, where x represents the original number of students on each team. Eight students were added to each group, there are 6 groups, and there are a total of 72 students.

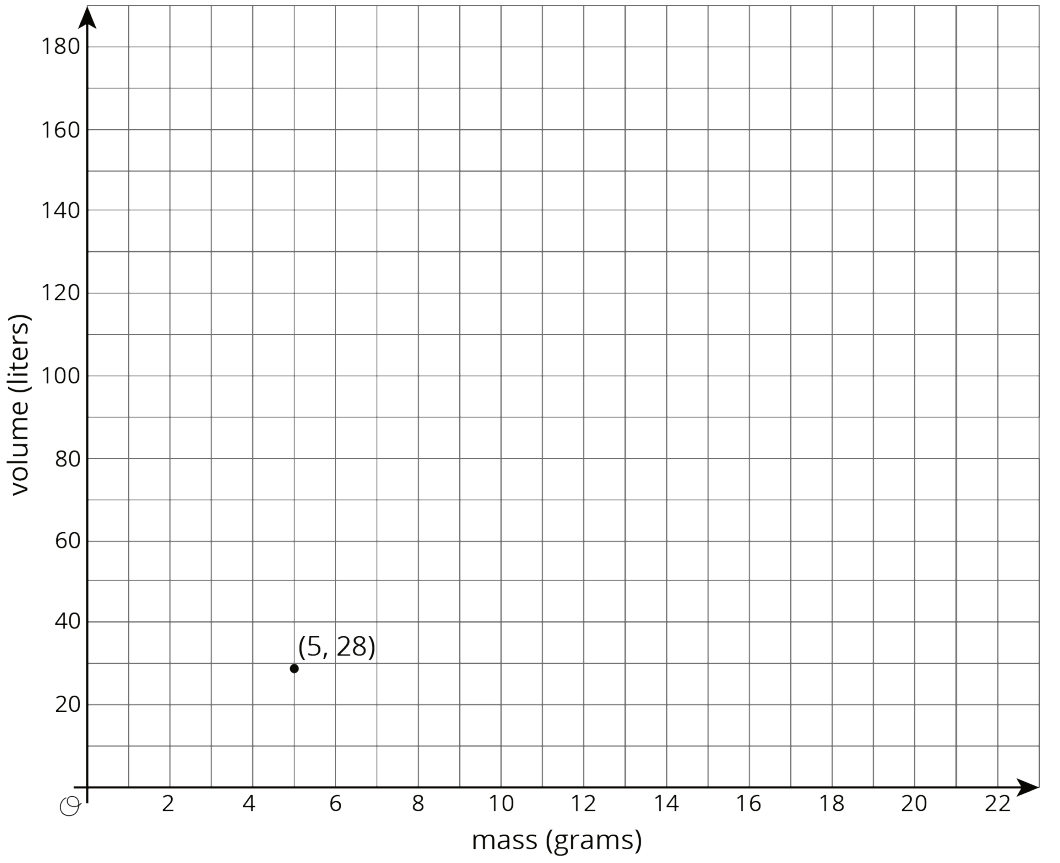
In the second situation, there are 6 equal parts added to one other part. An equation that represents this situation is $6x + 8 = 72$, where x represents the weight of a box of ping pong paddles, there are 6 boxes of ping pong paddles, there is an additional box that weighs 8 pounds, and the crate weighs 72 pounds altogether.

In the first situation, there were 6 equal groups, and 8 students added to each group. $6(x + 8) = 72$.

In the second situation, there were 6 equal groups, but 8 more pounds in addition to that. $6x + 8 = 72$.

Unit 6 Lesson 7 Cumulative Practice Problems

1. There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters. (5, 28) is shown on the graph below.



a. What is the constant of proportionality in this relationship?

b. In this situation, what is the meaning of the number you found in part a?

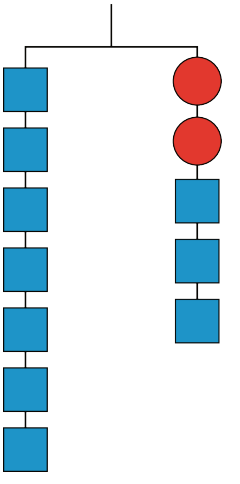
c. Add at least three more points to the graph above, and label with their coordinates.

d. Write an equation that shows the relationship between the mass of a sample of helium and its volume. Use m for mass and v for volume.

(From Unit 2, Lesson 11.)

2. Explain how the parts of the balanced hanger compare to the parts of the equation.

$7 = 2x + 3$

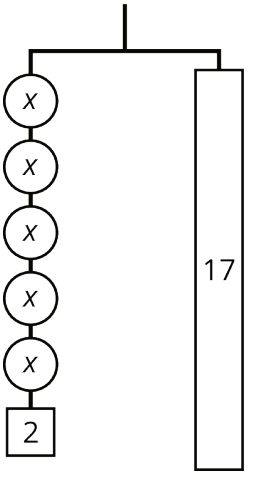


3. For the hanger below:

a. Write an equation to represent the hanger.

b. Draw more hangers to show each step you would take to find x. Explain your reasoning.

c. Write an equation to describe each hanger you drew. Describe how each equation matches its hanger.



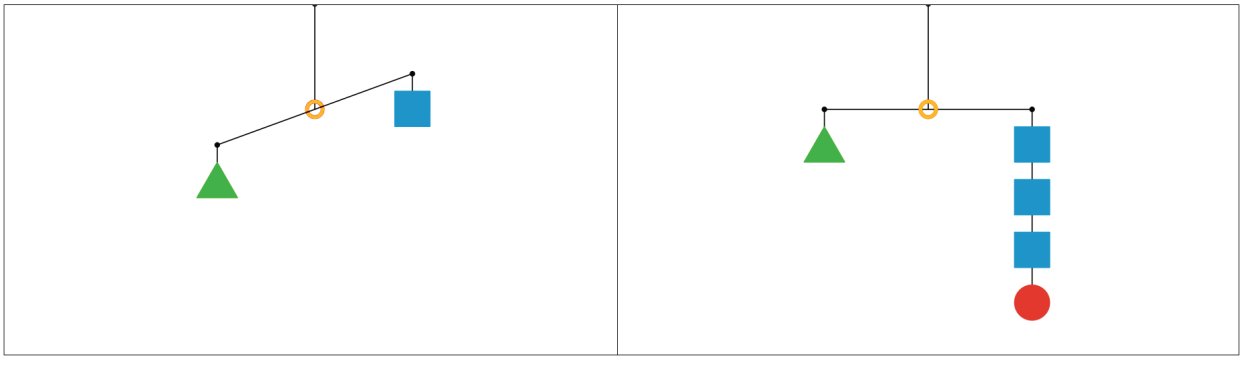
Lesson 7: Reasoning about Solving Equations (Part 1)

7.1: Hanger Diagrams

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true



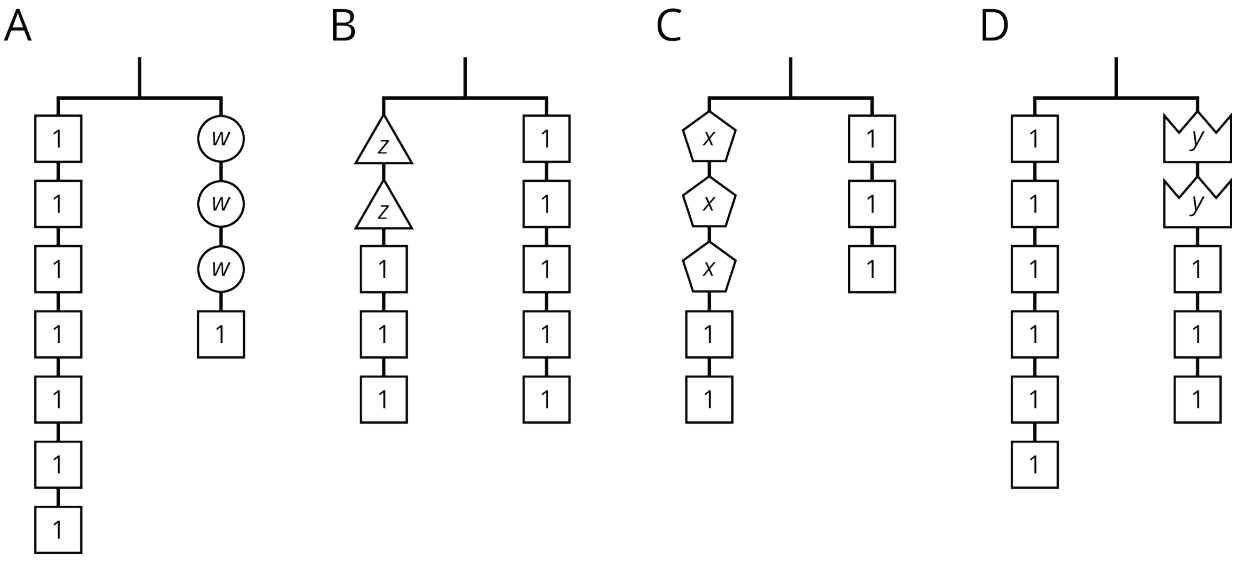
7.2: Hanger and Equation Matching

On each balanced hanger, figures with the same letter have the same weight.

1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   - $2\square + 3 = 5$
   - $3\square + 2 = 3$
   - $6 = 2\square + 3$
   - $7 = 3\square + 1$

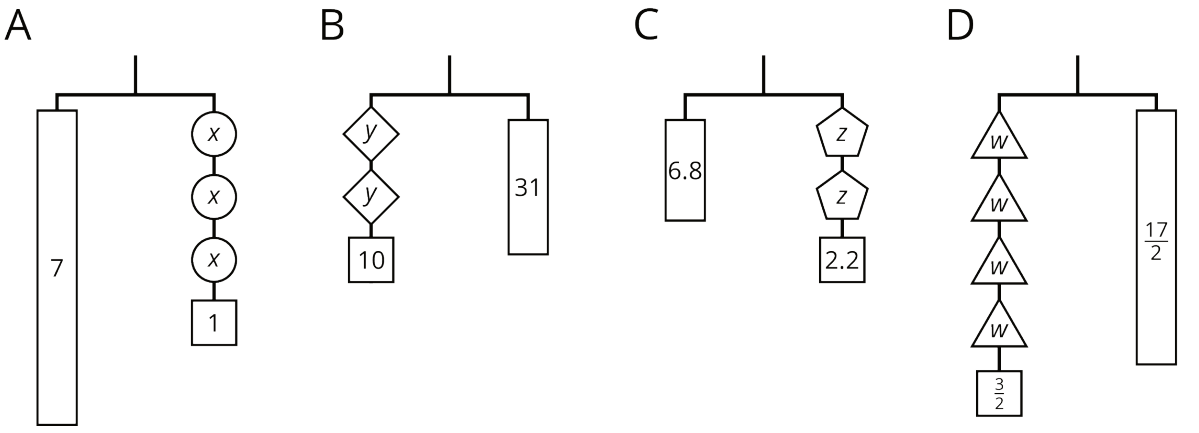
2. Find the solution to each equation. Use the hanger to explain what the solution means.



7.3: Use Hangers to Understand Equation Solving

Here are some balanced hangers where each piece is labeled with its weight. For each diagram:

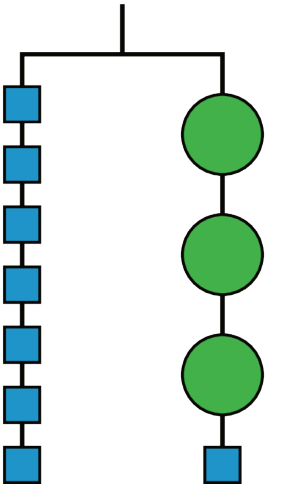
1. Write an equation.
2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.
3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.



Lesson 7 Summary

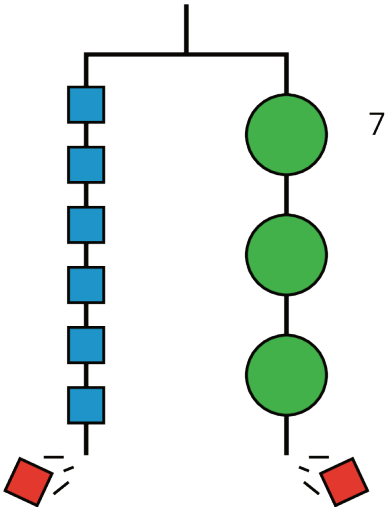
In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use a balanced hanger to think about steps to finding an unknown amount in an associated equation.

The hanger shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.



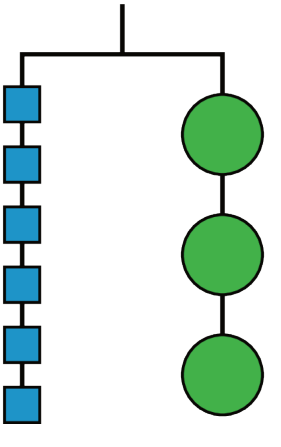
$7 = 3x + 1$

We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.



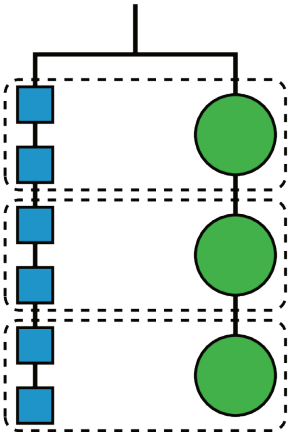
$7 - 1 = 3x + 1 - 1$

An equation for the new balanced hanger is $6 = 3x$.



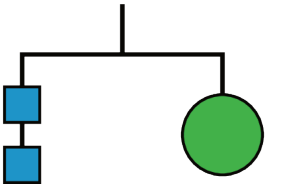
$6 = 3x$

So the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$.



$6 = 3x$

The two sides of the hanger balance with these weights: 6 1-unit weights on one side and 3 weights of unknown size on the other side.



$2 = x$

Here is a concise way to write the steps above:

$7 = 3x + 1$

$6 = 3x$ after subtracting 1 from each side

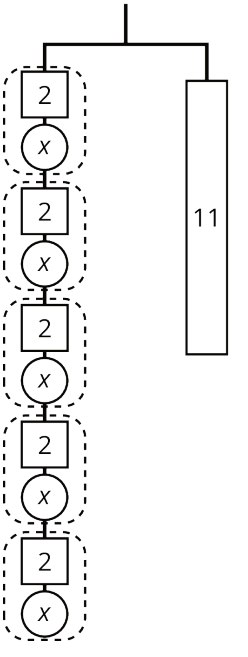
$2 = x$ after multiplying each side by $\frac{1}{3}$

Unit 6 Lesson 8 Cumulative Practice Problems

1. Here is a hanger:

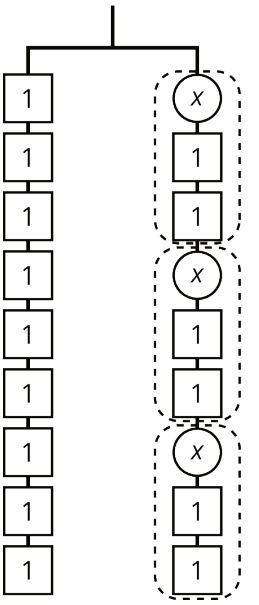
   a. Write an equation to represent the hanger.

   b. Solve the equation by reasoning about the equation or the hanger. Explain your reasoning.



2. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.

- x
- 9
- 3
- $x + 2$
- $3(x + 2)$
- the equal sign



3. Select the word from the following list that best describes each situation.

A. You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.

B. For every car sold, a car salesman is paid 6% of the car's price.

C. Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.

D. An antique furniture store pays $200 for a chair, adds 50% of that amount, and sells the chair for $300.

E. The normal price of a mattress is $600, but it is on sale for 10% off.

F. For any item you purchase in Texas, you pay an additional 6.25% of the item's price to the state government.

1. Tax

2. Commission

3. Discount

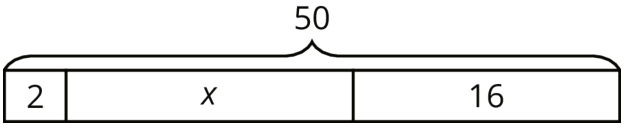
4. Markup

5. Tip or gratuity

6. Interest

(From Unit 4, Lesson 11.)

4. Clare drew this diagram to match the equation $2x + 16 = 50$, but she got the wrong solution as a result of using this diagram.



a. What value for x can be found using the diagram?

b. Show how to fix Clare's diagram to correctly match the equation.

c. Use the new diagram to find a correct value for x.

d. Explain the mistake Clare made when she drew her diagram.

(From Unit 6, Lesson 3.)

Lesson 8: Reasoning about Solving Equations (Part 2)

8.1: Equivalent to $2(x+3)$

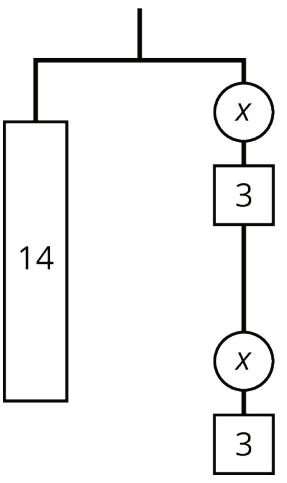
Select **all** the expressions equivalent to $2(x+3)$.

1. $2 \cdot (x+3)$
2. $(x+3)2$
3. $2 \cdot x + 2 \cdot 3$
4. $2 \cdot x + 3$
5. $(2 \cdot x) + 3$
6. $(2+x)3$

8.2: Either Or

1. Explain why either of these equations could represent this hanger:

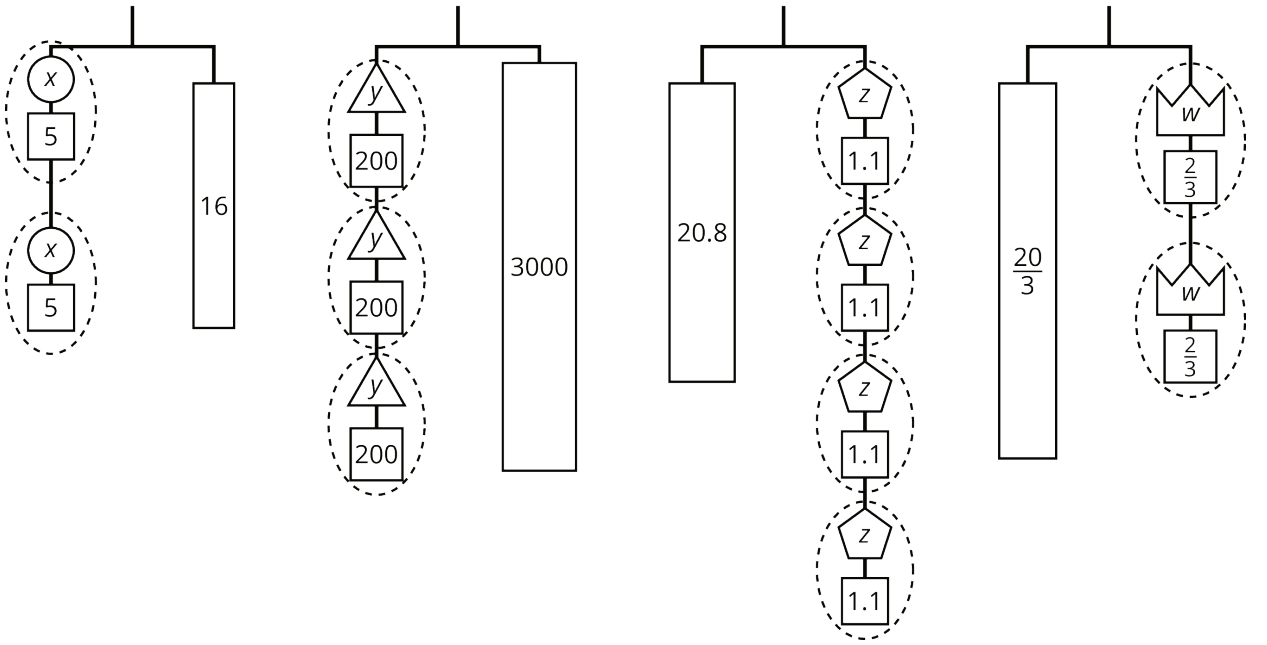
$14 = 2(x+3)$ or $14 = 2x+6$



2. Find the weight of one circle. Be prepared to explain your reasoning.

8.3: Use Hangers to Understand Equation Solving, Again

Here are some balanced hangers. Each piece is labeled with its weight.



For each diagram:

1. Assign one of these equations to each hanger:
   $2(x+5) = 16$ $\qquad$ $3(y+200) = 3{,}000$

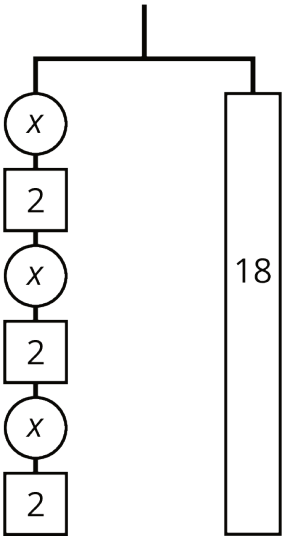
   $20.8 = 4(z+1.1)$ $\qquad$ $\frac{20}{3} = 2\left(w + \frac{2}{3}\right)$

2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.

3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.

Lesson 8 Summary

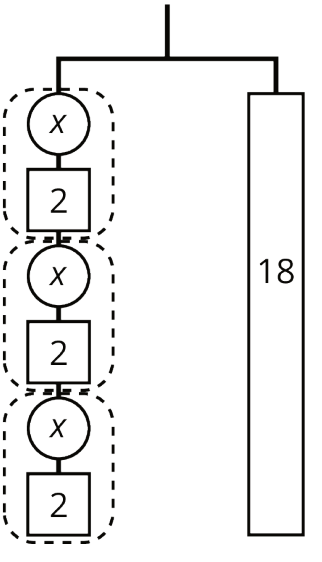
The balanced hanger shows 3 equal, unknown weights and 3 2-unit weights on the left and an 18-unit weight on the right.



There are 3 unknown weights plus 6 units of weight on the left. We could represent this balanced hanger with an equation and solve the equation the same way we did before.

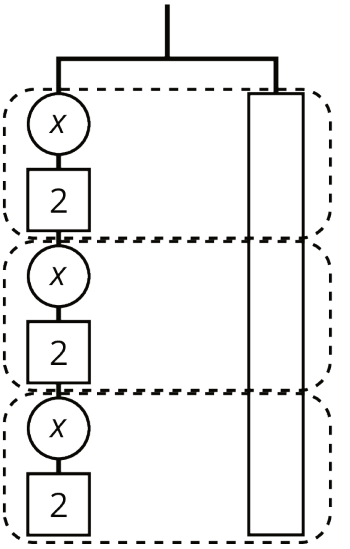
$$\begin{aligned} 3x + 6 &= 18 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

Since there are 3 groups of $x + 2$ on the left, we could represent this hanger with a different equation: $3(x + 2) = 18$.



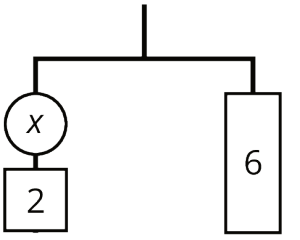
$3(x + 2) = 18$

The two sides of the hanger balance with these weights: 3 groups of $x + 2$ on one side, and 18, or 3 groups of 6, on the other side.



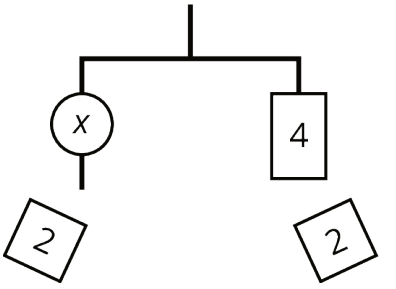
$3(x + 2) = 18$

The two sides of the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 3(x + 2) = \frac{1}{3} \cdot 18$.



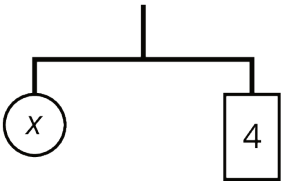
$x + 2 = 6$

We can remove 2 units of weight from each side, and the hanger will stay balanced. This is the same as subtracting 2 from each side of the equation.



$x + 2 = 4 + 2$

An equation for the new balanced hanger is $x = 4$. This gives the solution to the original equation.



$x = 4$

Here is a concise way to write the steps above:

$$\begin{aligned} 3(x+2) &= 18 \\ x+2 &= 6 && \text{after multiplying each side by } \tfrac{1}{3} \\ x &= 4 && \text{after subtracting 2 from each side} \end{aligned}$$

Unit 6 Lesson 11 Cumulative Practice Problems

1. Find the value of each variable.

   a. $a \cdot 3 = -30$

   b. $-9 \cdot b = 45$

   c. $-89 \cdot 12 = c$

   d. $d \cdot 88 = -88{,}000$

   (From Unit 5, Lesson 9.)

2. Match each equation to its solution and to the story it describes.

   Equations:

   a. $5x - 7 = 3$

   b. $7 = 3(5 + x)$

   c. $3x + 5 = -7$

   d. $\frac{1}{3}(x + 7) = 5$

   Solutions:

   a. -4

   b. $\frac{-8}{3}$

   c. 2

   d. 8

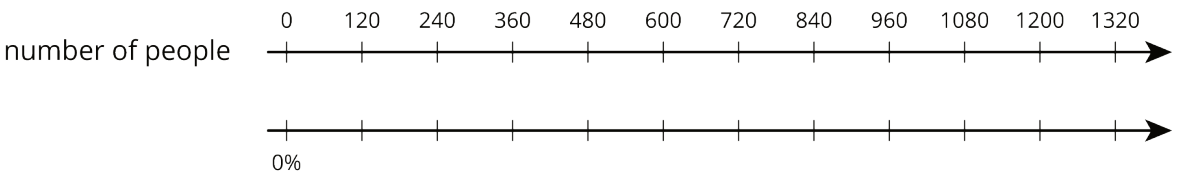
   Stories:

   - The temperature is -7. Since midnight the temperature tripled and then rose 5 degrees. What was temperature at midnight?
   - Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?
   - A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now in 7 hours they can build 3 guitars. By how much did they reduce the time it takes to build each guitar?
   - A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

3. The baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

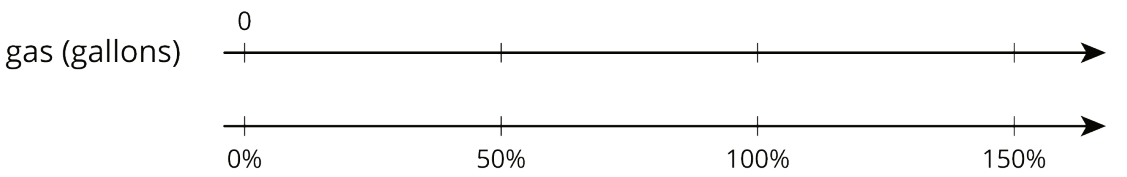
4. Six teams are out on the field playing soccer. The teams all have the same number of players. The head coach asks for 2 players from each team to come help him move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

5. A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?



(From Unit 4, Lesson 7.)

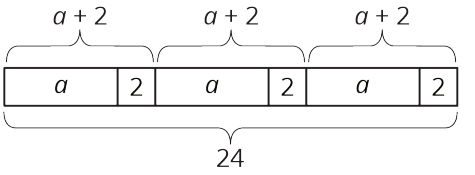
6. The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does it hold?



(From Unit 4, Lesson 7.)

Lesson 11: Using Equations to Solve Problems

11.1: Remember Tape Diagrams

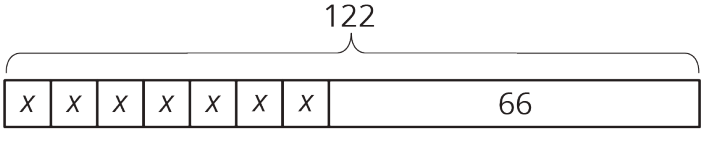


1. Write a story that could be represented by this tape diagram.

2. Write an equation that could be represented by this tape diagram.

11.2: At the Fair

1. Tyler is making invitations to the fair. He has already made some of the invitations, and he wants to finish the rest of them within a week. He is trying to spread out the remaining work, to make the same number of invitations each day. Tyler draws a diagram to represent the situation.



   a. Explain how each part of the situation is represented in Tyler's diagram:

      How many total invitations Tyler is trying to make.

      How many invitations he has made already.

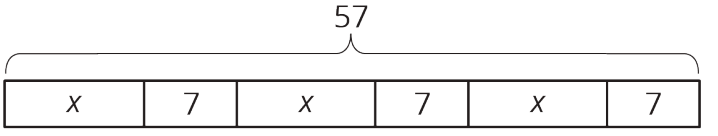
      How many days he has to finish the invitations.

b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.

c. Use Tyler's diagram to write an equation that represents the situation. Explain how each part of the situation is represented in your equation.

d. Show how to solve your equation.

2. Noah and his sister are making prize bags for a game at the fair. Noah is putting 7 pencil erasers in each bag. His sister is putting in some number of stickers. After filling 3 of the bags, they have used a total of 57 items.



a. Explain how the diagram represents the situation.

b. Noah writes the equation $3(x + 7) = 57$ to represent the situation. Do you agree with him? Explain your reasoning.

c. How many stickers is Noah's sister putting in each prize bag? Explain or show your reasoning.

3. A family of 6 is going to the fair. They have a coupon for $1.50 off each ticket. If they pay $46.50 for all their tickets, how much does a ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

11.3: Running Around

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.

   a. How many miles did Priya run last week?

   b. Elena wrote the equation $\frac{1}{9}(x-7)=4$ to describe the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?

2. One day last week, 6 teachers joined $\frac{5}{7}$ of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?

3. Priya and Han plan a fundraiser for the running club. They begin with a balance of -80 because of expenses. In the first hour of the fundraiser they collect equal donations from 9 family members, which brings their balance to -44. How much did each parent give?

4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at a rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

Are you ready for more?

A musician performed at three local fairs. At the first he doubled his money and spent $30. At the second he tripled his money and spent $54. At the third, he quadrupled his money and spent $72. In the end he had $48 left. How much did he have before performing at the fairs?

Lesson 11 Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next six days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where x represents the distance, in miles, she ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

Unit 6 Lesson 12 Cumulative Practice Problems

1. A backpack normally costs $25 but it is on sale for $21. What percentage is the discount?

(From Unit 4, Lesson 12.)

2. Find each product.

   a. $\frac{2}{5} \cdot (-10)$

   b. $-8 \cdot \left(\frac{-3}{2}\right)$

   c. $\frac{10}{6} \cdot 0.6$

   d. $\left(\frac{-100}{37}\right) \cdot (-0.37)$

(From Unit 5, Lesson 9.)

3. Select **all** expressions that show x increased by 35%.

   A. $1.35x$

   B. $\frac{35}{100}x$

   C. $x + \frac{35}{100}x$

   D. $(1 + 0.35)x$

   E. $\frac{100+35}{100}x$

   F. $(100 + 35)x$

4. Complete each sentence with the word *discount*, *deposit*, or *withdrawal*.

   a. Clare took $20 out of her bank account. She made a _____.

   b. Kiran used a coupon when he bought a pair of shoes. He got a _____.

   c. Priya put $20 into her bank account. She made a _____.

   d. Lin paid less than usual for a pack of gum because it was on sale. She got a _____.

(From Unit 4, Lesson 11.)

5. Here are two stories:

   - The initial freshman class at a college is 10% smaller than last year's class. But then during the first week of classes, 20 more students enroll. There are then 830 students in the freshman class.
   - A store reduces the price of a computer by $20. Then during a 10% off sale, a customer pays $830.

   Here are two equations:

   - $0.9x + 20 = 830$
   - $0.9(x - 20) = 830$

   a. Decide which equation represents each story.

   b. Explain why one equation has parentheses and the other doesn't.

   c. Solve each equation, and explain what the solution means in the situation.

Lesson 12: Solving Problems about Percent Increase or Decrease

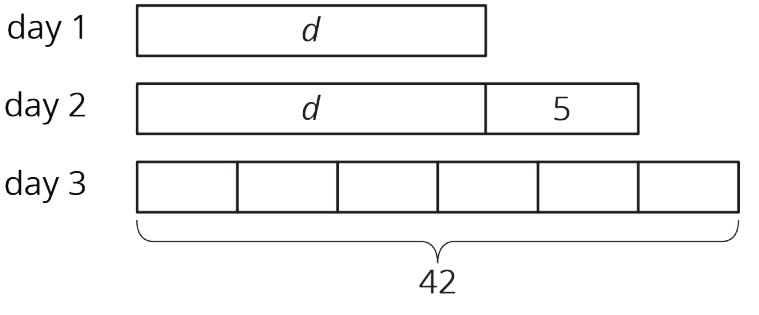
12.1: 20% Off

An item costs x dollars and then a 20% discount is applied. Select **all** the expressions that could represent the price of the item after the discount.

1. $\frac{20}{100}x$
2. $x - \frac{20}{100}x$
3. $(1 - 0.20)\,x$
4. $\frac{100-20}{100}x$
5. $0.80x$
6. $(100 - 20)x$

12.2: Walking More Each Day

1. Mai started a new exercise program. On the second day, she walked 5 minutes more than on the first day. On the third day, she increased her walking time from day 2 by 20% and walked for 42 minutes. Mai drew a diagram to show her progress.



Explain how the diagram represents the situation.

2. Noah said the equation $1.20(d + 5) = 42$ also represents the situation. Do you agree with Noah? Explain your reasoning.

3. Find the number of minutes Mai walked on the first day. Did you use the diagram, the equation, or another strategy? Explain or show your reasoning.

4. Mai has been walking indoors because of cold temperatures. On Day 4 at noon, Mai hears a report that the temperature is only 9 degrees Fahrenheit. She remembers the morning news reporting that the temperature had doubled since midnight and was expected to rise 15 degrees by noon. Mai is pretty sure she can draw a diagram to represent this situation but isn't sure if the equation is $9 = 15 + 2t$ or $2(t + 15) = 9$. What would you tell Mai about the diagram and the equation and how they might be useful to find the temperature, t, at midnight?

12.3: A Sale on Shoes

1. A store is having a sale where all shoes are discounted by 20%. Diego has a coupon for $3 off of the regular price for one pair of shoes. The store first applies the coupon and then takes 20% off of the reduced price. If Diego pays $18.40 for a pair of shoes, what was their original price before the sale and without the coupon?

2. Before the sale, the store had 100 pairs of flip flops in stock. After selling some, they notice that $\frac{3}{5}$ of the flip flops they have left are blue. If the store has 39 pairs of blue flip flops, how many pairs of flip flops (any color) have they sold?

3. When the store had sold $\frac{2}{9}$ of the boots that were on display, they brought out another 34 pairs from the stock room. If that gave them 174 pairs of boots out, how many pairs were on display originally?

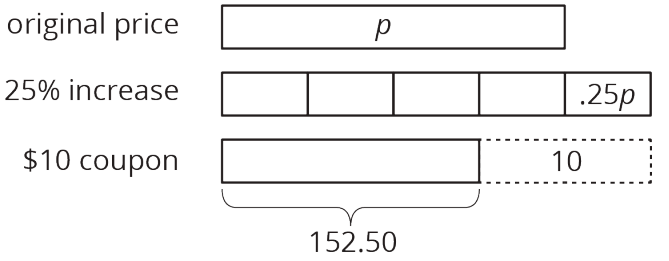
4. On the morning of the sale, the store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during the sale. If the store had 288 pairs after the donation and the sale, how many pairs of shoes did they have at the start?

Are you ready for more?

A coffee shop offers a special: 33% extra free or 33% off the regular price. Which offer is a better deal? Explain your reasoning.

Lesson 12 Summary

We can solve problems where there is a percent increase or decrease by using what we know about equations. For example, a camping store increases the price of a tent by 25%. A customer then uses a $10 coupon for the tent and pays $152.50. We can draw a diagram that shows first the 25% increase and then the $10 coupon.



The price after the 25% increase is $p + .25p$ or $1.25p$. An equation that represents the situation could be $1.25p - 10 = 152.50$. To find the original price before the increase and discount, we can add 10 to each side and divide each side by 1.25, resulting in $p = 130$. The original price of the tent was $130.