Unit 1 Topic 3: Exponential Equations in One Variable

Algebra 1 Summit

AUTHOR
Algebra 1 Summit

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook®**, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform®**.

Printed: August 11, 2014

flexbook
next generation textbooks

CHAPTER 1 Unit 1 Topic 3: Exponential Equations in One Variable

Dear Parents/Guardians,

Unit 1 Topic 3 focuses on exponential equations in one variable. This topic includes:

- Using the Properties of Exponents to simplify exponential expressions
- Using the Exponential Property of Equality to solve exponential equations

Properties of Exponents:

Consider the following exponential expressions with the same base and what happens through the algebraic operations. Exponential expressions have several components, for example:

ax^b

a is the coefficient

x is the base

b is the exponent or power

Product Property (multiplication):

$\boxed{a^m \times a^n = a^{m+n}}$

When the bases are the same then exponents can be added.

a)

$3^2 \times 3^3$	The base is 3.
3^{2+3}	Keep the base of 3 and add the exponents.
3^5	This answer is in exponential form.

The answer can be taken one step further. The base is numerical so the term can be evaluated.

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3$$
$$3^5 = 243$$
$$\boxed{3^2 \times 3^3 = 3^5 = 243}$$

Note: Rules of exponents are shortcut methods for simplifying. When in doubt expand out the values as shown below to simplify $(x^3)(x^2)$

$(x \bullet x \bullet x)(x \bullet x) = x^5$

expanding expressions works for all rules.

b)

$(x^3)(x^6)$	The base is x.
x^{3+6}	Keep the base of x and add the exponents.
x^9	The answer is in exponential form.
$\boxed{(x^3)(x^6) = x^9}$	

c)

$5x^2y^3 \cdot 3xy^2$	The bases are x and y.
$15(x^2y^3)(xy^2)$	Multiply the coefficients - $5 \times 3 = 15$. Keep the bases of x and y and add the exponents of the same base. If a base does not have a written exponent, it is understood as 1.
$15x^{2+1}y^{3+2}$	
$15x^3y^5$	The answer is in exponential form.
$\boxed{5x^2y^3 \cdot 3xy^2 = 15x^3y^5}$	

e) Simplify the expression: x^3y^2

this expression cannot be simplified further since the bases are not the same

Quotient Property (division):

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

The quotient property is an extension of the product property, but may end with the possibility of a negative in the exponent. When dividing, if the bases are the same the exponents can be subtracted. If a negative exponent results from subtraction, see the negative exponent rules below.

a)

$2^7 \div 2^3$	The base is 2.
2^{7-3}	Keep the base of 2 and subtract the exponents.
2^4	The answer is in exponential form.

The answer can be taken one step further. The base is numerical so the term can be evaluated.

$$2^4 = 2 \times 2 \times 2 \times 2$$
$$2^4 = 16$$
$$\boxed{2^7 \div 2^3 = 2^4 = 16}$$

b)

$\frac{x^8}{x^2}$	The base is x.
x^{8-2}	Keep the base of x and subtract the exponents.
x^6	The answer is in exponential form.
$\boxed{\frac{x^8}{x^2} = x^6}$	

c)

$\frac{y^3}{y^{-5}}$	The base is y.
$y^{3\overset{+}{-}-5}$	Keep the base of y and subtract the exponents.
y^8	The answer is in exponential form.
$\boxed{\frac{y^3}{y^{-5}} = y^8}$	

d) Note: when simplifying all answers must be written with positive exponents, (see next section for explanation)

$\frac{16x^2y^5}{4x^5y^3}$	The bases are x and y.
$4\left(\frac{x^2y^5}{x^5y^3}\right)$	Divide the coefficients - $16 \div 4 = 4$. Keep the bases of x and y and subtract the exponents of the same base.
$4x^{2-5}y^{5-3}$	
$4x^{-3}y^2$	All answers must be written with positive exponents. If the base is in the numerator with a negative exponent, write it in the denominator with a positive exponent.
$\frac{4y^2}{x^3}$	The answer is in exponential form.
$\boxed{\frac{16x^2y^5}{4x^5y^3} = \frac{4y^2}{x^3}}$	

Negative exponent:

$$\boxed{a^{-m} = \frac{1}{a^m}}$$

A negative exponent means reciprocal. This is because of multiplying the base by "a" m times, you are dividing m times.

a) $x^{-3} = \frac{1}{x^3}$

b) $4r^{-2}$ /t would simplify to 4/ r^2t

note: variable should be ordered alphabetically when simplifying

Power to a Power:

$\boxed{(a^m)^n = a^{mn}}$

a)

$(2^3)^2$	The base is 2^3.
$2^{3\times 2}$	Keep the base of 2^3 and multiply the exponents.
2^6	The answer is in exponential form.

The answer can be taken one step further. The base is numerical so the term can be evaluated.

$$2^6 = 2\times 2\times 2\times 2\times 2\times 2$$
$$2^6 = 64$$
$$\boxed{(2^3)^2 = 2^6 = 64}$$

b) $(a^3)^4$ would be simplified using the power to power rule (a ^{3x4}) = a^{12}

Justification: $((a\cdot a\cdot a)(a\cdot a\cdot a)(a\cdot a\cdot a)(a\cdot a\cdot a) = a^{12}$

The answer can be taken one step further. The base is numerical so the term can be evaluated.

c) $(3y^4)^2$

Justification: $3^2\cdot(y^4)^2$

the two distributes to both parts of the inside of the parenthesis:

Answer: $9y^8$

Zero Property:

The zero property of exponents states that anything raised to the zero property becomes 1.

a) $3^0 = 1$

b) $c^0 = 1$

Two videos by James Sousa:

First on Properties of Exponents: http://www.youtube.com/watch?v=0GAMbuPJGOY

Second on Negative Exponents: https://www.youtube.com/watch?v=R7Yp5TW1NTs

More complex examples applying the properties of exponents:

Example A

Simplify the following expression. All exponents should be positive in the final answer.

$$\frac{(a^{-2}b^3)^{-3}}{ab^2c^0}$$

Solution:

$$\frac{(a^{-2}b^3)^{-3}}{ab^2c^0} = \frac{a^6b^{-9}}{ab^2 \cdot 1} = \frac{a^5}{b^{11}}$$

Example B

Simplify the following expression until all exponents are positive.

$$(2x)^5 \cdot \frac{4^2}{2^{-3}} =$$

Solution:
note: the 4^2 was rewritten as
2^4 because they are equvalent
and then could be combined with the
other base 2's.

$$(2x)^5 \cdot \frac{4^2}{2^{-3}} = \frac{2^5x^52^4}{2^{-3}} = \frac{2^9x^5}{2^{-3}} = 2^{12}x^5$$

Example C

Simplify the following expression until all exponents are positive.

$$\frac{(a^{-3}b^2c^4)^{-1}}{(a^2b^{-4}c^0)^3}$$

Solution:

$$\frac{(a^{-3}b^2c^4)^{-1}}{(a^2b^{-4}c^0)^3} = \frac{a^3b^{-2}c^{-4}}{a^6b^{-12}} = \frac{b^{10}}{a^3c^4}$$

Using the Exponential Property of Equality to solve exponential equations:

In this section, students will be able to construct arguments to justify their reasoning in problems involving exponential expressions and equations. In addition, students will be able to articulate the differences and similarities in solving linear and exponential equations. In order to do this, they need to have a clear understanding of the Exponential Property of Equality and be able to use it to find equivalent exponential expressions and solve equations involving exponents.

The Exponential Property of Equality states that two exponential expressions with the same base will have equal exponents.

If $b^x = b^y$, then $x = y$.

Example A

If $2^x = 2^5$, then $x = 5$.

Example B

Solve for x.

	$5^{x+1} = 5^{3x}$
Apply the exponential property of equality, when the bases are the same, the exponent can be set equal.	$x+1 = 3x$
Apply Subtraction Property of Equality.	$x+1-x = 3x-x$
	$1 = 2x$
Apply Division Property of Equality.	$1 \div 2 = 2x \div 2$
Solution.	$\frac{1}{2} = x$

Students will learn to apply this property to solve more complex exponential equations by finding common bases, then setting the exponents equal and solving.

Video about solving equations using the exponential property of equality:

https://www.youtube.com/watch?v=aPyE9SKtczs

Example C

Solve for n.

	$3^n = 27^2$
Since $27 = 3^3$, then	$3^n = (3^3)^2$
Simplify.	$3^n = 3^6$
	$n = 6$

Example D

Solve for x.

	$2^{2x-1} = 8^x$
Rewrite in a common base.	$2^{2x-1} = (2^3)^x$
Simplify.	$2^{2x-1} = 2^{3x}$
Set exponents equal.	$2x - 1 = 3x$
Apply Subtraction Property of Equality.	$2x - 1 - 2x = 3x - 2x$
Solution: $-1 = x$	

Example E

Solve for m.

	$\frac{1}{25} = 125^{2m}$
Rewrite in a common base.	$5^{-2} = (5^3)^{2m}$
Simplify.	$5^{-2} = 5^{6m}$
Set exponents equal.	$-2 = 6m$
Apply Division Property of Equality.	$-2 \div 6 = 6m \div 6$
Simplify.	$-\frac{1}{3} = m$

Example F

Solve for x.

$$100^{3x+1} = \left(\frac{1}{1000}\right)^x$$

Rewrite in a common base. $10^{2(3x+1)} = (10^{-3})^x$

Simplify. $10^{6x+2} = 10^{-3x}$

Set exponents equal. $6x+2 = -3x$

Apply Subtraction Property of Equality.

$$6x+2-6x = -3x-6x$$
$$2 = -9x$$

Apply Division Property of Equality.

$$2 \div -9 = -9x \div -9$$
$$-\frac{2}{9} = x$$

Example G

Janet and Isaac make cupcakes to sell at the school bake sale. They each make 1 cupcake to sample. The next day Janet doubles her production. She continues to double it each day for the next few weeks. Six days after he make the sample Isaac has gathered some friends to help. He makes eight times as many cupcakes than the day before and continues in this pattern.

a) On what day will they have the same number of cupcakes?

b) How many cupcakes will they have on that day?

Solution:

a) Set up and solve an equation

$2^x = 8^{x-6}$

$2^x = 2^{3(x-6)}$

$x = 3x - 18$

$x = 9$

On day 9, Janet and Isaac will have the same number of cupcakes.

b) $2^9 = 512$

On day 9, Janet and Isaac each have 512 cupcakes.