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Unit 1 Topic 1: Linear Equation in one variable

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CONCEPT 1 Unit 1 Topic 1: Linear Equations in One Variable

Dear Parents and/or Guardians,

Unit 1 builds on prior knowledge of solving linear equations. Students develop fluency and mastery in writing, interpreting, and translating between various forms of linear equations and inequalities in one variable, and using them to solve problems. This topic includes:

- Creating and interpreting expressions that model situations
- Identifying, adding and subtracting like terms
- Creating and interpreting equations that model situations
- Justifying the steps to solve an equation
- Explaining the meaning of a solution
- Solving literal equations for a specific variable
- Examining Units.

Creating and Interpreting Expressions that Model Situations

A **variable** is a letter, or symbol that represents an unknown value.

A **term** in mathematics is a number, a variable or the combination of both through multiplication.

The **constant** in an equation is an isolated number not combined with a variable.

An **expression** is two or more terms connected by a mathematical symbol.

Example 1:

Write an expression to model the sum of Sally and Jason's marbles if Sally has 14 marbles.

The expression that would model this situation is $j+14$.

Example 2:

This picture represents a quarter of a bigger rectangle. Write an expression to represent the sum of the number of unshaded pieces within the bigger rectangle.
Then write an expression to represent the sum of the number of unshaded pieces in three bigger rectangles.

Let u = the number of unshaded pieces in this picture

Since the picture represents one quarter or one fourth (1/4) of the bigger rectangle, we will need to multiply the u-value by 4 to find the number of unshaded pieces in the bigger rectangle. The first expression would be written as $4u$ to represent the number of unshaded pieces within the bigger rectangle.

To represent the sum of the number of unshaded pieces if there were three bigger rectangles, multiply the $4u$ (which stands for one bigger rectangle) by 3 . The expression would be written as $12u$.

Example 3:

A plumber charges $30 per hour along with a $50 charge to come to a house. Write an expression to represent the cost of hiring the plumber.

Let h = the number of hours the plumber works

"$30 per hour" means that $30 will be charged for each hour the plumber works, this tells us number of hours would be multiplied by 30

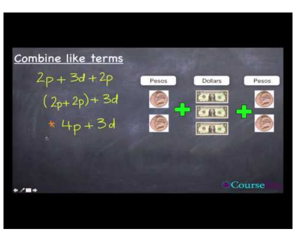
the $50 charge for the plumber doesn't change by the number of hours worked, so it is the constant

The expression that models how many hours the plumber works is $30h+50$

Identifying, Adding and Subtracting Like Terms:

A **coefficient** is the number that is in front of, or multiplied by, the variable.

Like Terms are terms that have the same variable and the same exponent.



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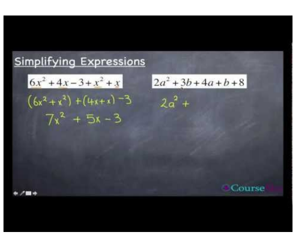

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Example 1:

Simplify the expression: $8f+3k-2f$

Group like terms: $8f-2f+3k$

The simplified expression is $6f+3k$.

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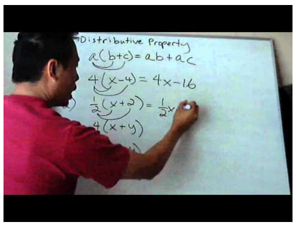

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Example 2:

Simplify the expression: $3(x+5)-2x$

This expression requires the distributive property first to simplify the like terms.

Distribute the "3" to the "x" and the "5" by multiplication.

$3x+15-2x$

Group the like terms.

$3x-2x+15$

Combine the like terms to produce the simplified expression $x+15$.

Example 3:

Simplify the expression: $15-2(6-a)+a$

First distribute the "-2" to the "6" and "-a" through multiplication

$15-12+2a+a$

Group the like terms.

$2a+a+15-12$

Combine the like terms to produce the simplified expression $3a+3$.

Creating and Interpreting Equations that Model Situations

An **equation** is a number sentence in which two expressions are set equal to each other.

Example 1:

Eighteen less than three times a number is 70. Model this situation by creating an equation.

The equation that would model this situation is $3n - 18 = 70$.

Example 2:

The perimeter of a rectangle is equal to 144 inches with a length of 5 inches. Create an equation to model the width of the rectangle.

Perimeter of a rectangle formula is $P = 2L + 2W$, $P = Perimeter, L = Length, W = Width$

What do we know? Perimeter is 144 inches, so $P = 144$

Length is 5 inches, so $L = 5$

What is the unknown? The measurement of the width.

Substitute the length measurement and the perimeter into the formula

$144 = 2(5) + 2W$

Simplify.

$144 = 10 + 2W$

The equation that would model this situation is $2W + 10 = 144$.

Example 3:

Sally spent \$35 at the grocery store buying food for her family BBQ. She bought 6 bags of chips, 4 packages of hotdogs, and 2 pounds of hamburgers. In the weekly advertisement hamburgers were displayed as \$1.99 per pound. There was also a special on chips that was displayed as 2 for \$5. Write an equation to model the cost of each package of hotdogs.

What do we know? Sally spent \$35, so: *chips* + *hotdogs* + *hamburgers* = *\$35*.

The special on chips is 2 bags for \$5 which means each bag is \$2.50.

Sally bought 6 bags of chips.

The cost of the chips is: $6 \cdot 2.50 = 15$ dollars

She bought 2 pounds of hamburger meat at \$1.99 per pound

The cost of the hamburger meat is: $2 \cdot 1.99 = 3.98$ dollars

What is the unknown? How much each hotdog package costs, let h = cost per package of hotdogs

To find the total cost for all the hotdog packages, is multiply the number of packages by the price. $4h$ is the cost of all the hotdog packages because she bought 4 packages.

Create the equation to show the information listed above.

$35 = 15 + 3.98 + 4h$

Simplify by combining like terms.

$35 = 18.98 + 4h$

The equation that would model this situation is $4h + 18.98 = 35$.

Justify the Steps to Solve an Equation

The **reciprocal** of a fraction is when the fraction is "flipped" by making numerator and denominator switch spots in a fraction. For example, the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$.

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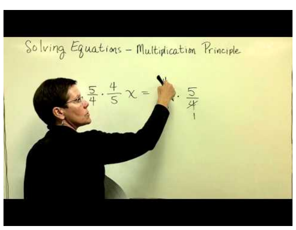

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TABLE 1.1:

Property	Definition	Numerical Example	Algebraic Example
Commutative Property of Addition or Multiplication	**Order doesn't matter...** *Add or multiply numbers in any order and you will get the same SUM or PRODUCT*	$5+3=3+5$ $2(6)=6(2)$	$a+b=b+a$ $2*a=a*2$

TABLE 1.1: (continued)

Associative Property of Addition or Multiplication	**Grouping** *When three or more numbers are added or multiplied, the sum or product is the same regardless of the way in which the numbers are grouped.*	$6+(4+3)=(6+4)+3$ $4\cdot(1\cdot 2)=(2\cdot 4)\cdot 1$	$a+(b+c)=(a+b)+c$ $a*(b*c)=(a*b)*c$
Distributive Property	**Giving** *Use multiplication to "give" the number multiplied to the parenthesis to each of the numbers inside of the parenthesis*	$5(3+2)=(5\cdot 3)+(5\cdot 2)$	$a(b+c)=ab+ac$
Additive or Multiplicitive Identity Property of Equality	*The sum of any number and 0 is that same number.* *The product of any number and 1 is that same number.*	$12+0=12$ $4\cdot 1=4$	$x+0=x$ $h\cdot 1=h$
Additive and Multiplicitive Inverse Property	**Opposite and reciprocal** *Add any number with its OPPOSITE number and the sum is 0.* *Multiply any number by its RECIPROCAL and the product is 1.*	$3+(-3)=0$ $\frac{2}{1}\cdot\frac{1}{2}=1$	$f+(-f)=0$ $\frac{a}{b}\cdot\frac{b}{a}=1, for\ a,b\neq 0$
Multiplicative Property of Zero	The product of any number and 0 is 0.	$5\cdot 0=0$	$k(0)=0$
Subtraction Property of Equality	When the same number is subtracted from each side of an equation, the sides remain equal. **If *a* = *b*, then *a* – *c* = *b* – *c***	$7=7$ $7-5=7-5$ $2=2$	$x+3=8$ $x+3-3=8-3$ $x=5$
Addition Property of Equality	When the same number is added to each side of an equation, the sides remain the equal. **If *a* = *b*, then *a* + *c* = *b* + *c***	$5=5$ $5+3=5+3$ $8=8$	$b-9=2$ $b-9+9=2+9$ $b=11$
Multiplication Property of Equality	When the same number is multiplied on each side of an equation, the sides remain equal. **If *a* = *b*, then *ac* = *bc***	$2=2$ $2(-3)=2(-3)$ $-6=-6$	$\frac{x}{6}=-3$ $\frac{x}{6}\cdot 6=(-3)\cdot 6$ $x=-18$

TABLE 1.1: (continued)

Division Property of Equality	When each side of an equation is divided by the same number, the sides remain equal. **If *a* = *b* *and* *c*≠ 0, then** $\frac{a}{c} = \frac{b}{c}$	$15 = 15$ $\frac{15}{-3} = \frac{15}{-3}$ $-5 = -5$	$10v = 70$ $\frac{10v}{10} = \frac{70}{10}$ $v = 7$

Example 1: Solve the equation and justify your steps.

TABLE 1.2:

Equation	Justification
-2x = 16	Given
$\frac{-2}{-2}x = \frac{16}{-2}$	Division Property of Equality (Divide by -2)
x = -8	Solution

Example 2: Solve the equation and justify your steps.

TABLE 1.3:

Equation	Justification
$\frac{2}{3}x+8 = 10$	Given
$\frac{2}{3}x+8-8 = 10-8$	Subtraction Property of Equality (subtract 8 from each side)
$\frac{2}{3}x = 2$	Simplify by combining like terms
$\frac{3}{2}\cdot\frac{2}{3}x = 2\cdot\frac{3}{2}$	Multiplicative Inverse Property or Multiplication Property of Equality (multiply by the reciprocal of the coefficient to eliminate the coefficient)
$x = 3$	Solution

Example 3: Solve the equation and justify your steps.

TABLE 1.4:

Equation	Justification
$4n+3(2n-4) = n$	Given
$4n+6n-12 = n$	Distributive Property
$10n-12 = n$	Combine like terms
$10n-10n-12 = n-10n$	Subtraction Property of Equality
$-12 = -9n$	Simplify
$\frac{-12}{-9} = \frac{-9}{-9}n$	Division Property of Equality
$\frac{12}{9} = n$	Simplify
$n = \frac{4}{3}$	Solution

Explaining the Meaning of a Solution

Example 1:

Solve the following equation and classify if the solution is *always true, sometimes true* or *never true*. $3(r+1)-5=3r-2$

$3r+3-5=3r-2$

$\underline{3r-2=3r-2}$

$3r-3r-2=3r-3r-2$

$-2=-2$

$-2+2=-2+2$

$0=0$

This solution is **always true** because r can be replaced with any number and each side of the equation will always be true. Equations that are always true will show the same thing on each side throughout solving each step. In the second line of the equation 3r - 2 = 3r -2 is shown. At this step in solving the equation we can see the same thing is already on both sides. This shows the equation is already balanced and any number could be substituted in for r. The equation could continued to be solved to show at the end 0=0. Since zero will always equal zero in the last step the equation is always true.

Example 2:

Solve the following equation and classify the solution as *always true, sometimes true* or *never true*. $\frac{3}{4}x+16=2-\frac{1}{8}x$

$\frac{3}{4}x+\frac{1}{8}x+16=2-\frac{1}{8}x+\frac{1}{8}x$

$\frac{7}{8}x+16=2$

$\frac{7}{8}x+16-16=2-16$

$\frac{7}{8}x=-14$

$\frac{8}{7}\cdot\frac{7}{8}x=-14\cdot\frac{8}{7}$

$x=-16$

This solution is **sometimes true** because it has one solution shown, $x=-16$. Where there is only one solution to satisfy an equation it will be sometimes true, in this example the equation is true when x = -16.

Example 3:

Solve the following equation and classify the solution as *always true, sometimes true* or *never true*. $2m+5=5(m-7)-3m$

$2m+5=5m-35-3m$

$2m+5=2m-35$

$2m-2m+5=2m-2m-35$

$5=-35$

This solution is **never true** because 5 will never equal to -35 and if you replace m with any number each side of the equation will never have the same value.

Solving Literal Equations for a Specific Variable

A **literal equation** is an equation that contains multiple variables. Generally when solving literal equation, you will be asked to solve for (isolate) a particular variable.

Example 1:

Solve the equation $3x+4y=6$ for the variable y.

$3x+4y=6$

$3x-3x+4y=6-3x$

$4y=6-3x$

$\frac{4}{4}y=\frac{6-3x}{4}$

$y=\frac{3}{2}-\frac{3}{4}x$

The solution for y from the literal equation $3x+4y=6$ is $y=\frac{3}{2}-\frac{3}{4}x$ or $y=\frac{-3x+6}{4}$ is also an acceptable answer

Example 2:

Solve the equation $p=\frac{r-c}{n}$ for the variable r.

$p=\frac{r-c}{n}$

$np=\frac{r-c}{n}n$

$np=r-c$

$np+c=r-c+c$

$np+c=r$

The solution for r from the literal equation $p=\frac{r-c}{n}$ is $r=np+c$.

Example 3:

The formula $A = \frac{1}{2}bh$ describes the area of a triangle. A represents the area, b represents the base, and h represents the height. Solve this formula for the variable b.

$A = \frac{1}{2}bh$

$\frac{2}{1} \cdot A = \frac{2}{1} \cdot \frac{1}{2}bh$

$2A = bh$

$\frac{2A}{h} = \frac{bh}{h}$

$\frac{2A}{h} = b$

The solution for b from the literal equation $A = \frac{1}{2}bh$ is $b = \frac{2A}{h}$.

Examining Units

Example 1:

To convert a temperature from Farenheit to Celsius we use the formula $\frac{9}{5}C + 32 = F$. If it is $47^\circ F$ what is the temperature as a Celsius measure?

We know the formula from the text is $\frac{9}{5}C + 32 = F$. We have two variables, F and C. We already know that $F = 47$. To find C we substitute 47 for F and solve for C.

$\frac{9}{5}C + 32 = F$

$\frac{9}{5}C + 32 = 47$

$\frac{9}{5}C + 32 - 32 = 47 - 32$

$\frac{9}{5}C = 15$

$\frac{5}{9} \cdot \frac{9}{5}C = 15 \cdot \frac{5}{9}$

$C = \frac{75}{9} \quad \frac{\div 3}{\div 3}$

$C = \frac{25}{3} = 8.\bar{3}$

$47^\circ F$ is equivalent to $8.\bar{3}^\circ C$.

Example 2:

Before traveling to Mexico, Jake goes to the bank to exchange his US Dollars into Mexican Pesos. The teller says that the exchange rate right now is 1 US Dollar = 13.0019 Mexican Pesos. If Jake gives the bank teller 100 US Dollars, how many Mexican Pesos can he expect to receive?

Part A:

Using the exchange rate provided above, if $1 = 13.0019 Mexican Pesos, multiply 13.0019 by 100 to find the number of Mexican Pesos.

Jake can expect to receive 1300.19 Mexican Pesos from the bank teller.

Part B: If Jake received 3,250.475 Mexican Pesos, how many US Dollars did he exchange?

Jake exchanged 250 dollars.