Answers to Summer Rev HPC

1a) $(1,-4)$ b) $(-1,4)$ c) $(-1,-4)$ d) $(5,4)$

2a) $(2,3)$ b) $(-2,-3)$ c) $(-2,3)$ d) $(4,-3)$

3) A function where $f(-x)=f(x)$ $f(x)=x^2$

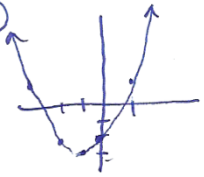
4) A function where $f(-x)=-f(x)$ $f(x)=x^3$

5) $x=1$ $y=3$

6) $y=2x-1$ (7) $y=3$ (8) $x=1$ (9) $y=\frac{5}{3}x+\frac{4}{3}$

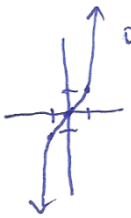
10a) $y=2x-12$ b) $y=-\frac{1}{2}x+3$

(11)



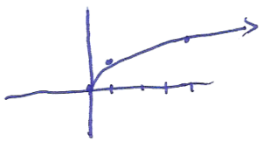
D: $\mathbb{R}$
R: $[-3,\infty)$

(12)



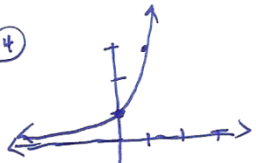
D: $\mathbb{R}$
R: $\mathbb{R}$

(13)



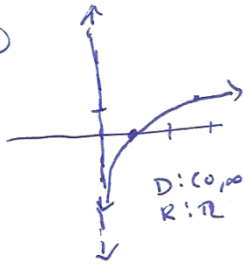
D: $[0,\infty)$ R: $[0,\infty)$

(14)



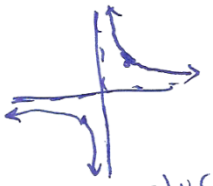
D: $\mathbb{R}$ R: $(0,\infty)$

(15)



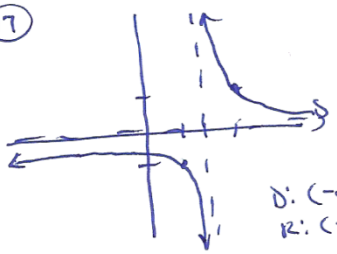
D: $(0,\infty)$
R: $\mathbb{R}$

(16)



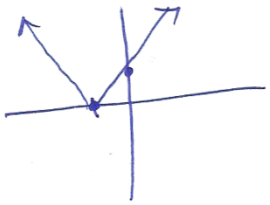
D: $(-\infty,0)\cup(0,\infty)$
R: $(-\infty,0)\cup(0,\infty)$

(17)



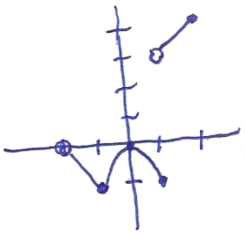
D: $(-\infty,2)\cup(2,\infty)$
R: $(-\infty,0)\cup(0,\infty$

(18)



D: $\mathbb{R}$
R: $[0,\infty)$

(19)



(20) $f(x)=\begin{cases} \frac{5}{2}x & 0\le x<2 \\ -\frac{5}{2}x+10 & 2\le x\le 4 \end{cases}$

(21) a] f: D: $(-\infty,0)\cup(0,\infty)$ R: $(-\infty,0)\cup(0,\infty)$ g: D: $\mathbb{R}$ R: $[-4,\infty)$ b] $f+g=\frac{1}{x}+x^2-4=\frac{x^3-4x+1}{x}$

$f^{-1}=\frac{1}{x}$; $\frac{f}{g}=\frac{\frac{1}{x}}{x^2-4}=\frac{1}{x^3-4x}$; $f\circ g=\frac{1}{x^2-4}$; $g\circ f=\left(\frac{1}{x}\right)^2-4=\frac{1-4x^2}{x^2}$

c] $f\circ g$ D: $\mathbb{R}$ except ± 2; R: $\mathbb{R}\setminus 0$ $g\circ f$: D: $\mathbb{R}$ except 0 R: $[-4,\infty)$

22) $(x-2)^2+(y-1)^2=25$ (23) $(x-1)^2+(y+2)^2=11$ center $(1,-2)$ $r=\sqrt{11}$

(24) $8y=x^2$ (25) f: $(2,\frac{1}{4})$ d: $y=-\frac{1}{4}$ (26) v: $(\pm 6,0)$ f: $(\pm 2\sqrt{5},0)$

(27) hyperbola (28)
$$\begin{aligned} 8x+y&=11\\ x-y&=97\\ 9x&=108\\ x&=12\\ y&=-85\end{aligned}$$
$(12,-85)$

(29)
$$\begin{aligned} 2x+y&=6\\ 4x+2y&=8\end{aligned}\qquad\begin{aligned} -4x-2y&=-12\\ 4x+2y&=8\\ \hline 0&=-6\end{aligned}$$
$\varnothing$

(30) $x+y-z=-1$, $4x-3y+2z=16$, $2x-2y-3z=5$

$$\begin{bmatrix}1&1&-1\\4&-3&2\\2&-2&-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\16\\5\end{bmatrix}\qquad\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1&1&-1\\4&-3&2\\2&-2&-3\end{bmatrix}^{-1}\begin{bmatrix}-1\\16\\5\end{bmatrix}=\begin{bmatrix}2\\-2\\1\end{bmatrix}$$

(31) $\sqrt{x+1}=\sqrt{x+6}-1$
$$\begin{aligned} x+1&=x+6+1-2\sqrt{x+6}\\ -6&=-2\sqrt{x+6}\\ 3&=\sqrt{x+6}\\ 9&=x+6\\ 3&=x\end{aligned}$$

(32) $8^{2x+3}=4\cdot 2^{x+1}$
$$\begin{aligned} 2^{3(2x+3)}&=2^2\cdot 2^{x+1}\\ 2^{6x+9}&=2^{x+3}\\ 6x+9&=x+3\\ 5x&=-6\\ x&=-6/5\end{aligned}$$

(33) $\frac{x+1}{3(x-2)}=\frac{5x}{3\cdot 2}+\frac{1}{x-2}$ $\quad\cdot 3\cdot 2(x-2)$
$$\begin{aligned} 2x+2&=5x^2-10x+6\\ 0&=5x^2-12x+4\\ &(5x-2)(x-2)\end{aligned}$$
$x=\frac{2}{5}$ ~~$x=2$~~ extraneous

(34) $2x^2=x$
$$\begin{aligned} 2x^2-x&=0\\ x(2x-1)&=0\\ x=0\quad x&=\tfrac{1}{2}\end{aligned}$$

(35) $\sqrt{x-5}=2\sqrt{x}$
$$\begin{aligned} x-5&=4x\\ -5&=3x\\ -\tfrac{5}{3}&=x\end{aligned}$$
$\varnothing$

(36) $\frac{x(x+2a)}{2a-x} \cdot \frac{(x-2a)(x-a)}{(a-x)(a+x)} \cdot \frac{x+a}{x+2a} = x$

$-1(x-2a)$ $\quad -1(x-a)$

(37) $\frac{\frac{1}{x}-\frac{1}{x+1}}{\frac{1}{x}-\frac{1}{x-1}} \cdot \frac{x(x-1)(x+1)}{x(x-1)(x+1)} = \frac{x^2-1-x^2+1}{x^2-1-x^2-1} = 0$

(38) $\frac{3-\sqrt{2}}{2\sqrt{3}+5} \cdot \frac{2\sqrt{3}-5}{2\sqrt{3}-5} = \frac{6\sqrt{3}-15-2\sqrt{6}+5\sqrt{2}}{12-25} = -\frac{(5\sqrt{2}+6\sqrt{3}-2\sqrt{6}-15)}{13}$

(39) $4i \cdot 13i = -52$ (40) 3 (41) 125 (42) x^2

(43) b) $x-3 \neq 0$ $x=3$ V.A. d) $2x^2+5x-3 \neq 0$ $(2x-1)(x+3) \neq 0$ $x=\frac{1}{2}$ $x=-3$ V.A.

(44a) ~~0=0~~ $x = \pm\sqrt{-y^2}$ $y = \pm\sqrt{-x^2}$

(44b) $2xy-3y=0$ $y(2x-3)=0$

(45a) -11 complex (b) 1 real (c) 2 rational

(46) $(x-6)(x+6)$ (47) $(x-(1+i\sqrt{7}))(x-(1-i\sqrt{7}))$ (48) $x(x+10)(x-2)$

(49) $3y(y-8)(y+2)$ (50) $(5+x)(3x-7)$ (51) $3(x^2+2xy+y^2)+(x+y)$ $3(x+y)^2+(x+y)$ $(x+y)(3(x+y)+1)$ $(x+y)(3x+3y+1)$

(52) $3-2m=3m+1$ $2=5m$ $\frac{2}{5}=m$

(53) $\frac{1}{3}x = 2-\frac{2}{3}x$ $x=6-2x$ $3x=6$ $x=2$

(54) $x^2(x-2)-4(x-2)=0$ $(x^2-4)(x-2)=0$ $(x-2)(x+2)(x-2)=0$ $x=2$ double root $x=-2$

(55) $2x^2+5x-3=0$ $(2x-1)(x+3)=0$ $x=\frac{1}{2}$ $x=-3$

(56) $x^2+10x+25=36$ $(x+5)^2=36$ $x=-5+6=1$ $x=-5-6=-11$

(57) $x^2-14x=15$ $x^2-14x+49=64$ $(x-7)^2=64$ $x=7+8=15$ $x=7-8=-1$