Key

1. a) $\frac{x^3-9x}{x^2-7x+12} = \frac{x(x^2-9)}{(x-3)(x-4)} = \frac{x(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x-4)} = \frac{x^2+3x}{x-4}$

b) $\frac{x^2-2x-8}{x^3+x^2-2x} = \frac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)} = \frac{x-4}{x(x-1)}$

c) $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}} = \frac{\cancel{\frac{1}{x}-\frac{1}{5}}}{\cancel{(\frac{1}{x}-\frac{1}{5})}(\frac{1}{x}+\frac{1}{5})} = \frac{1}{\frac{1}{x}+\frac{1}{5}} = \frac{1}{\frac{5+x}{5x}} = \frac{5x}{5+x}$

d) $\frac{9-\frac{1}{x^2}}{3+\frac{1}{x}} = \frac{\frac{9x^2-1}{x^2}}{\cancel{\frac{3x+1}{x}}} \cdot \frac{x}{3x+1} \Rightarrow \frac{(3x-1)\cancel{(3x+1)}}{x\cancel{(3x+1)}} = \frac{3x-1}{x}$

2. a) $\frac{2}{\sqrt{3}+\sqrt{2}} \cdot \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \frac{2\sqrt{3}-2\sqrt{2}}{3-2} = 2\sqrt{3}-2\sqrt{2}$

b) $\frac{4}{1-\sqrt{5}}\left(\frac{1+\sqrt{5}}{1+\sqrt{5}}\right) = \frac{4+4\sqrt{5}}{1-5} = \frac{4+4\sqrt{5}}{-4} = -1-\sqrt{5}$

c) $\frac{1}{1+(\sqrt{3}-\sqrt{5})} \cdot \left(\frac{1-(\sqrt{3}-\sqrt{5})}{1-(\sqrt{3}-\sqrt{5})}\right) = \frac{1-\sqrt{3}+\sqrt{5}}{1-(3-2\sqrt{15}+5)}$

$\frac{1-\sqrt{3}+\sqrt{5}}{(-7+2\sqrt{15})} \cdot \frac{(-7-2\sqrt{15})}{(-7-2\sqrt{15})} = \frac{-7-2\sqrt{15}+7\sqrt{3}+\overset{6\sqrt{5}}{2\sqrt{45}}-7\sqrt{5}-\overset{10\sqrt{3}}{2\sqrt{15\cdot5}}}{49-60}$

$= \frac{-7-2\sqrt{15}-3\sqrt{3}-\sqrt{5}}{-11}$ or $\frac{7+2\sqrt{15}+3\sqrt{3}+\sqrt{5}}{11}$

3. a) $\left(\frac{2a^3}{b}\right)^3 = 8a^6b^{-1}$

b) $\sqrt{9ab^3} = 3a^{1/2}b^{3/2}$

c) $\frac{a(2/b)}{3/a} = \frac{2a}{b} \cdot \frac{a}{3} = \frac{2}{3}a^2b^{-1}$

d) $\frac{ab-a}{b^2-b} = \frac{a\cancel{(b-1)}}{b\cancel{(b-1)}} = ab^{-1}$

e) $\frac{a^{-1}}{b^{-1}\sqrt{a}} = a^{-3/2}b$

f) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right) = \frac{a^{4/3}}{b} \cdot \frac{b^{3/2}}{a^{1/2}} = a^{5/6}b^{1/2}$

4. a) $5^{x+1} = 25$

$5^{x+1} = 5^2$

$x+1 = 2$

$x = 1$

b) $\frac{1}{3} = 3^{2x+2}$

$3^{-1} = 3^{2x+2}$ $\quad -1 = 2x+2 \quad 2x = -3 \quad x = -3/2$

c) $\log_2 x = 3 \quad 2^3 = x \quad x = 8$

d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

$\log_3 x^2 = \log_3 16 - \log_3 5^4$

$\log_3 x^2 = \log_3\left(\frac{16}{5^4}\right)$

$\sqrt{x^2} = \sqrt{\frac{16}{5^4}} \qquad x = \pm\frac{4}{25}$

5. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

a) $\log_2 5 + \log_2\left(\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}\right)$

$\log_2(5(x+1))$

b) $2\log_4 9 - \log_2 3$

$\log_4 81 - \log_2 3$

get same base $\quad \log_2 9 - \log_2 3$

$\log_2(3)$

$\log_4 81 \Rightarrow 4^x = 81$

$(4^x)^{1/2} = 81^{1/2}$

$(4^{1/2})^x = 9$

$2^x = 9$

$\therefore \log_2 9 = x$

c) $3^{2\log_3 5} = 3^{\log_3 25} = 25$

$3^{\log_3 25} = x \qquad \log_3 x = \log_3 25$

$\Delta^{\circ} = \square \qquad \log_\Delta \square = \circ \qquad x = 25$

6. a) $\log_{10} 10^{1/2} = \frac{1}{2}\log_{10} 10 = \frac{1}{2}(1) = \frac{1}{2}$

b) $\log_{10}\left(\frac{1}{10^x}\right) = \log_{10} 10^{-x} = -x\log_{10} 10 = -x$

c) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

$= \log_{10}(\sqrt{x})^2 + \log_{10}(x^{1/3})^3$

$= \log_{10} x + \log_{10} x = 2\log_{10} x$

7a. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\frac{x}{a}=\frac{1}{1}-\frac{y}{b}-\frac{z}{c}$

$\frac{x}{a}=\frac{bc-cy-bz}{bc}$

$\frac{a}{x}=\frac{bc}{bc-cy-bz}$

$a=\frac{xbc}{bc-cy-bz}$

b. $V=2(ab+bc+ca)$

$\frac{V}{2}=a(b+c)+bc$

$\frac{V}{2}-bc=a(b+c)$

$a=\frac{V-2bc}{2(b+c)}$

c. $A=2\pi r^2+2\pi rh$

$(2\pi)r^2+(2\pi h)r-A=0$

$a=2\pi \quad b=2\pi h \quad c=-A$

$r=\frac{-2\pi h\pm\sqrt{(2\pi h)^2-4(2\pi)(-A)}}{2(2\pi)}$

$=\frac{-2\pi h\pm\sqrt{4\pi^2h^2+8\pi A}}{4\pi}$

$=\frac{-2\pi h\pm\sqrt{4(\pi^2h^2+2\pi A)}}{4\pi}$

$r=\frac{-\pi h\pm\sqrt{\pi^2h^2+2\pi A}}{2\pi}$

d. $A=P+nrP$

$A=P(1+nr)$

$P=\frac{A}{1+nr}$

e. $2x-2yd=y+xd$

$2x-y=2yd+xd$

$2x-y=d(2y+x)$

$d=\frac{2x-y}{2y+x}$

f. $\frac{2x}{4\pi}+\frac{1-x}{2}=0$

$\frac{2x+2\pi(1-x)}{4\pi}=0$

$2x+2\pi-2\pi x=0$

$x=\frac{-2\pi}{(2-2\pi)}=\frac{-\pi}{1-\pi}$ or $\frac{\pi}{\pi-1}$

8. $y=x^2+4x+3$

a. $y-3+(2)^2=x^2+4x+(2)^2$

$y+1=(x+2)^2$ or $y-(-1)=(x-(-2))^2$

b. $3x^2+3x+2y=0$

$3(x^2+x+(1/2)^2)=-2y+3(1/2)^2$

$3(x+1/2)^2=-2y+3/4$

$\overline{-2} \quad \overline{-2} \quad \overline{-2}$

$y-3/8=-\frac{3}{2}(x-(-1/2))^2$

c. $9y^2-6y-9-x=0$

$9(y^2-\frac{2}{3}y+(1/3)^2)=x+9+9(1/3)^2$

$9(y-1/3)^2=x+10$

$x-(-10)=9(y-1/3)^2$

9. a) x^6-16x^4

$x^4(x^2-16)=x^4(x+4)(x-4)$

b) $4x^3-8x^2-25x+50$

$4x^2(x-2)-25(x-2)$

$(4x^2-25)(x-2)$

$(2x-5)(2x+5)(x-2)$

c) $8x^3+27$

$(2x)^3+3^3$

$a^3+b^3=$ CCSOPS

$(a+b)(a^2-ab+b^2)$

$(2x+3)(4x^2-6x+9)$

d) x^4-1

$(x^2+1)(x^2-1)=(x^2+1)(x+1)(x-1)$

10. a) see above $x=0,-4,4$

b) $x=5/2,-5/2,2$

c) $x=-3/2$

11. a) $3\sin^2x=\cos^2x$

$\frac{\sin^2x}{\cos^2x}=\frac{1}{3}$

$\tan^2x=1/3$

$\tan x=\pm\sqrt{1/3}=\pm\frac{\sqrt{3}}{3}$

2 60 1

30 $\sqrt{3}$

$\therefore$ 30° or $\pi/6$ I, II, III, IV

$\pi/6,\ 5\pi/6,\ 7\pi/6,\ \frac{11\pi}{6}$

b) $\cos^2x-\sin^2x=\sin x$ $\quad -\pi<x\le\pi$

$1-\sin^2x-\sin^2x=\sin x$

$1-2\sin^2x=\sin x$

$2x^2+x-1$

$(2x-1)(x+1)$

$2\sin^2x+\sin x-1=0$

$(2\sin x-1)(\sin x+1)=0$

$\sin x=1/2$: $\pi/6,\ \frac{5\pi}{6}$

$\sin x=-1$: $-\pi/2$

11c $\tan x + \sec x = 2\cos x$

$\cos x \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x \right)$

$\cos x \neq 0$, $x \neq \pi/2, 3\pi/2$

$\sin x + 1 = 2\cos^2 x$

$\sin x + 1 = 2(1-\sin^2 x)$

$\sin x + 1 = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

lostpts ans. $\pi/6, 5\pi/6, \cancel{-\pi/2}$

$\therefore\ \pi/6 + 2\pi n$

$\frac{5\pi}{6} + 2\pi n$

n

12. a) $\cos 210^\circ = \cos 30^\circ$ III $= -\frac{\sqrt{3}}{2}$

b) $\sin 5\pi/4 = \sin \pi/4$ III $= -\sqrt{2}/2$

c) $\tan^{-1}(-1) \Rightarrow \tan\theta = -1 = -\pi/4$

d) $\sin^{-1}(-1) \Rightarrow \sin\theta = -1 = \cancel{3\pi/2}$ or $-\pi/2$ I or IV quad

e) $\cos 9\pi/4 \Rightarrow \cos \pi/4 = \sqrt{2}/2$

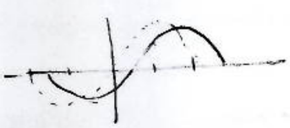
f) $\sin^{-1}\sqrt{3}/2 \Rightarrow \sin\theta = \sqrt{3}/2 \Rightarrow 60^\circ, 120^\circ$ I, IV quad

$\pi/3$, $\cancel{2\pi/3}$

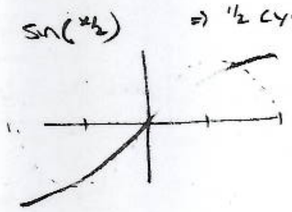
g) $\tan 7\pi/6 \Rightarrow \tan \pi/6$ III $\Rightarrow \sqrt{3}/3$

h) $\cos^{-1}(-1) \Rightarrow \cos\theta = -1$ I or II $= \pi$

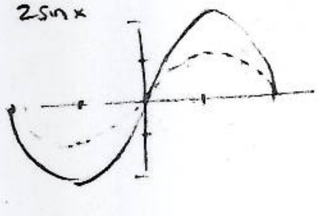
13. a) $\sin(x - \pi/4)$ $\pi/4 \rightarrow$



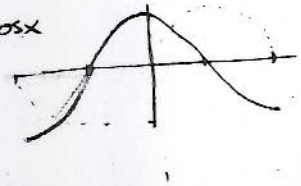
b) $\sin(x/2)$ ⇒ ½ cycle during same pd.



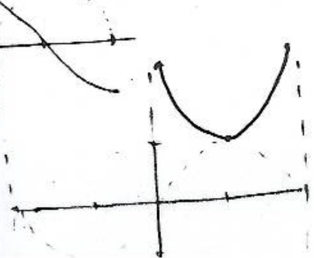
c) $2\sin x$



d) $\cos x$



e) $\frac{1}{\sin x} = \csc x$



14. $4x^2 + 12x + 3 = 0$

a) $x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(3)}}{2(4)}$

$= \frac{-12 \pm \sqrt{144-48}}{8}$

$= \frac{-12 \pm \sqrt{96}}{8}$

$= \frac{-12 \pm 4\sqrt{6}}{8}$

$= \frac{-3 \pm \sqrt{6}}{2}$

$144 - 48$; $96 \div 4 = 24$; $96 = 8 \cdot 12 = 16 \cdot 6$

b) $2x + 1 = \frac{5}{x+2}$ $x \neq -2$

$(2x+1)(x+2) = 5$

$2x^2 + 5x + 2 = 5$

$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$ $x = \frac{1}{2}, -3$

c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

$x \neq 0, -1$

$(x+1)^2 - x^2 = 0$

$\cancel{x^2} + 2x + 1 - x^2 = 0$

$2x + 1 = 0$ $x = -\frac{1}{2}$

15. a)

$$\begin{array}{r} x^4 - 6x^3 + 13x^2 - 26x + 45 \\ x+2 \overline{) x^5 - 4x^4 + x^3 + 0x^2 - 7x + 1} \\ -(x^5 + 2x^4) \\ -6x^4 + x^3 \\ -(-6x^4 - 12x^3) \\ 13x^3 + 0x^2 \\ -(13x^3 + 26x^2) \\ -26x^2 - 7x \\ -(-26x^2 - 52x) \\ 45x + 1 \\ -(45x + 90) \\ -89 \end{array}$$

$r = -89$

-2	1	-4	1	0	-7	1
		-2	12	-26	52	-90
	1	-6	13	-26	45	-89

b)

$$\begin{array}{r} x^2 - x + 1 \\ x^3 + 0x^2 + 0x + 1 \overline{) x^5 - x^4 + x^3 + 2x^2 - x + 4} \\ -(x^5 + 0x^4 + 0x^3 + x^2) \\ -x^4 + x^3 + x^2 - x \\ -(-x^4 - 0x^3 - 0x^2 - x) \\ x^3 + x^2 + 4 \\ -(x^3 \qquad + 1) \\ x^2 + 3 \end{array}$$

16. a)

$$\begin{array}{r|rrrr} 2 & 12 & -23 & -3 & 2 \\ & & 24 & 2 & -2 \\ \hline & 12 & 1 & -1 & 0 \end{array}$$

$12x^2 + x - 1 = 0$

$(4x-1)(3x+1) = 0$

$x = 1/4, -1/3$

b) $12x^3 + 8x^2 - x - 1 = 0$

$p = \pm 1$

$q = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$

use calc to get hint

$$\begin{array}{r|rrrr} 1/3 & 12 & 8 & -1 & -1 \\ & & 4 & 4 & 1 \\ \hline & 12 & 12 & 3 & 0 \end{array}$$

$12x^2 + 12x + 3 = 0$

$3(4x^2 + 4x + 1) = 0$

$3(2x+1)(2x+1) = 0$

$x = 1/3, -1/2$ dr

17. $x^2 + 2x - 3 \le 0$

a) $(x+3)(x-1) \le 0$ ∴ $-3 \le x \le 1$

b) $\frac{2x-1}{3x-2} \le 1$ $\quad 2x - 1 \le 3x - 2$

$1 \le x$

$x \ne 2/3$

graph rational

& $x < 2/3$

or $\frac{2x-1}{3x-2} - 1 \le 0$

$\frac{2x-1-3x+2}{3x-2} \le 0$

$\frac{-x+1}{3x-2} \le 0$

check intervals

$- - - \;|\; + + + \;|\; - - -$

$\quad\quad 2/3 \quad\quad 1$

c) $x^2 + x + 1 > 0$

graph

zeros

$\frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2a}$

imaginary

∴ never hits x-axis

∴ all $\mathbb{R}$

18. $|-x+4| \le 1$

a) $-x + 4 \le 1$ & $-(-x+4) \le 1$

$-x \le -3$ & $x - 4 \le 1$

$x \ge 3$ & $x \le 5$

$3 \le x \le 5$

b) $|5x-2| = 8$

$5x - 2 = 8$ $\quad -(5x-2) = 8$

$5x = 10$ $\quad 5x - 2 = -8$

$x = 2$ $\quad 5x = -6$

$x = -6/5$

c) $|2x+1| = x + 3$

$2x + 1 = x + 3$ $\quad -(2x+1) = x + 3$

$x = 2$ $\quad -2x - 1 = x + 3$

$-4 = 3x$

$-4/3 = x$

19. a) line $(-1,3)$ $(2,-4)$

$m = \frac{-4-3}{2-(-1)} = \frac{-7}{3} \Rightarrow y - 3 = -7/3(x+1)$

$3y - 9 = -7x - 7$

$7x + 3y = 2$

b) $(-1,2)$ $\perp$ $2x - 3y + 5 = 0$

$m = 2/3$ $\quad m_\perp = -3/2$

$y - 2 = -3/2(x+1)$

$2y - 4 = -3x - 3$

$3x + 2y = 1$

c) $(2,3)$ & midpt $(-1,4)$ $(3,2)$

$\left(\frac{-1+3}{2}, \frac{4+2}{2}\right)$

$(1,3)$

$m = \frac{0}{1} = 0$

∴ $y = 3$

20. $3x - y - 7 = 0$ $\quad x + 5y + 3 = 0$

a) $3x - y = 7$

$-3(x + 5y = -3)$

$3x - y = 7$

$-3x - 15y = 9$

$-16y = 16$

$y = -1$

$3x - (-1) = 7$

$3x + 1 = 7$

$3x = 6$

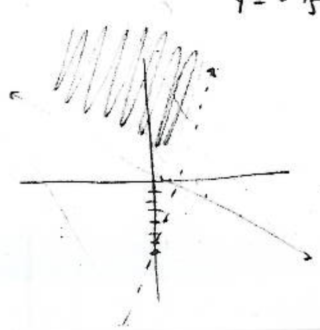
$x = 2$

$(2,-1)$

b) $3x - y - 7 < 0$ $\quad x + 5y + 3 \ge 0$

$y > 3x - 7$ $\quad 5y \ge -x - 3$

$y \ge -1/5x - 3/5$



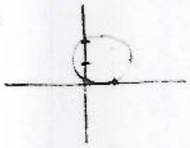
21 a) $(1,2)$ $(-2,-1)$

$r = \sqrt{(-2-1)^2 + (-1-2)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$

$(x-1)^2 + (y-2)^2 = 18$

b)



3pts ⇒ plug into $(x-h)^2 + (y-k)^2 = r^2$

$(0,0)$

(1) $h^2 + k^2 = r^2$

$(1,0)$

(2) $(1-h)^2 + k^2 = r^2$

$(0,2)$

(3) $h^2 + (2-k)^2 = r^2$

sub (1) into (2) & (3)

rewrite ::

$(1-h)^2 + k^2 = h^2 + k^2$

$1 - 2h + \cancel{h^2} + \cancel{k^2} = \cancel{h^2} + \cancel{k^2}$

$1 - 2h = 0 \Rightarrow h = 1/2$

$h^2 + (2-k)^2 = h^2 + k^2$

$h^2 + 4 - 4k + k^2 = h^2 + k^2$

$4 - 4k = 0 \Rightarrow k = 1$

$C(1/2, 1)$

$(1,1)$ $(0,0)$ $r = \sqrt{(1/2 - 0)^2 + (1-0)^2} = \sqrt{5/4}$

$\therefore (x - 1/2)^2 + (y-1)^2 = 5/4$

22. $x^2 + y^2 + 6x - 4y + 3 = 0$

a) $x^2 + 6x + (3)^2 + y^2 - 4y + (2)^2 = -3 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 10$

$C(-3,2)$ $r = \sqrt{10}$

b) tan line @ $(-2,5)$

find slope $(-3,2)$ $(-2,5)$ $m = \frac{5-2}{-2-(-3)} = \frac{3}{1}$

$\therefore m_{\perp} = -1/3$

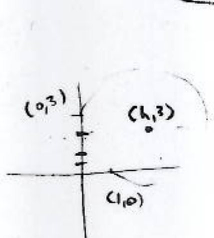
$y - 5 = -1/3(x+2)$

$3y - 15 = -x - 2$

$x + 3y = 13$

tan line has ⊥ slope to line that goes center to edge.

23.



$(x-h)^2 + (y-3)^2 = r^2$

pt $(0,3) \Rightarrow h^2 + (0)^2 = r^2 \Rightarrow h^2 = r^2$

$(1,0) \rightarrow (1-h)^2 + 9 = r^2$

$1 - 2h + h^2 + 9 = r^2$

$1 - 2h + h^2 + 9 = h^2$

$10 - 2h = 0$

$h = 5$

$\therefore (x-5)^2 + (y-3)^2 = r^2$

$C(5,3)$ ∴ radius $= 5$

$(x-5)^2 + (y-3)^2 = 25$

other inter.

$(x-5)^2 + (0-3)^2 = 25$

$(x-5)^2 + 9 = 25$

$\sqrt{(x-5)^2} = \sqrt{16}$

$x - 5 = \pm 4$

24. $\sqrt{(-1-x)^2 + (1-y)^2} = 3\sqrt{(2-x)^2 + (-1-y)^2}$

$(-1-x)^2 + (1-y)^2 = 9\left[(2-x)^2 + (-1-y)^2\right]$

$1 + 2x + x^2 + 1 - 2y + y^2 = 9(4 - 4x + x^2 + 1 + 2y + y^2)$

$2 + 2x - 2y + x^2 + y^2 = 45 - 36x + 18y + 9x^2 + 9y^2$

$8x^2 + 8y^2 - 38x + 20y + 43 = 0$

25. a) $f(x) = \frac{3x+1}{\sqrt{x^2 + x - 2}}$

den ≥ 0 $x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

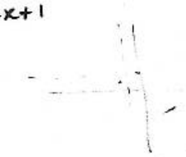
+++ | - - - | +++
-2 1

∴ ℝ except $-2 \le x \le 1$ or $x > 1$ or $x < -2$

b) $f(x) = 7$

D: ℝ
R: $\{7\}$

$g(x) = \frac{5x-3}{2x+1}$



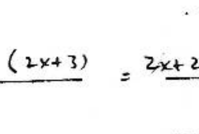
D: ℝ except $-1/2$
R: ℝ except $5/2$

26. $f(x) = \frac{|x|}{x}$ $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

$x \neq 0$ because den $= 0$

by def $x > 0 = \frac{x}{x} = 1$ ✓

$x < 0 = \frac{-(x)}{x} = -1$ ✓



D: ℝ except 0
R $\{-1, 1\}$

27 a) $\frac{f(x+h) - f(x)}{h}$ $f(x) = 2x + 3$

$\frac{2(x+h) + 3 - (2x+3)}{h} = \frac{2x + 2h + 3 - 2x - 3}{h}$

$= 2$

b) $f(x) = \frac{1}{x+1}$ $\frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h} = \frac{\frac{x+1-(x+h+1)}{(x+1)(x+h+1)}}{\frac{h}{1}} \cdot \frac{1}{h}$

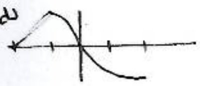
$\frac{-h}{h(x+1)(x+h+1)} = \frac{-1}{(x+h)(x+h+1)}$

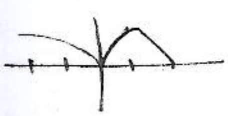
c) $f(x) = x^2$

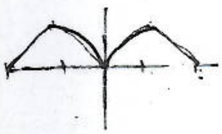
$\frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = 2x + h$

28. $f(x) =$ 

a) $f(x+1)$ ← 1 

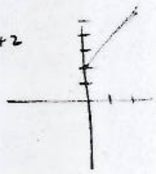
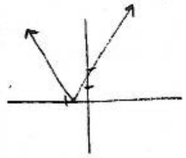
b) $f(-x)$ symmetry goes backwards 

c) $|f(x)|$ all above 

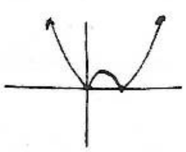
d) $f(|x|)$ 

29. $g(x) = |3x+2|$

2 cases
$y = 3x+2 \quad x > -2/3$
$y = -(3x+2) \quad x < -2/3$

a) $3x+2$  ⇒ 

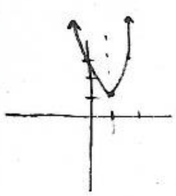
b) $h(x) = |x(x-1)|$

$x(x-1)$  ⇒ 

30. a) (1,4) 

$y = a(x-h)^2 + k$
$y = a(x-1)^2 + 4$
pt (3,0)
$0 = a(2)^2 + 4$
$-4 = a4$
$-1 = a \quad \Rightarrow \quad y = -(x-1)^2 + 4$
or
$y = -x^2 + 2x + 3$

$y = a(x-3)(x+1)$
pt (1,4)

b) $y = 2x^2 - 4x + 3$
$x = -b/2a = \frac{4}{4} = 1$
$V(1,1)$ 

31. a) $x = t+1 \quad t = x-1$
$y = t^2 - t$
$\Rightarrow y = (x-1)^2 - (x-1)$
$= x^2 - 2x + 1 - x + 1$
$y = x^2 - 3x + 2$

b) $x = \sqrt[3]{t} - 1 \quad t = (x+1)^3$
$y = t^2 - t$
$\Rightarrow y = ((x+1)^3)^2 - (x+1)^3$
$= (x+1)^6 - (x+1)^3$
$= (x+1)^3[(x+1)^3 - 1]$
$= (x+1)^3[x^3 + 3x^2 + 3x + 1 - 1]$
$(x+1)^3[x^3 + 3x^2 + 3x]$
$x(x+1)^3[x^2 + 3x + 3]$

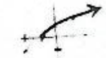
1
1 2 1
1 3 3 1

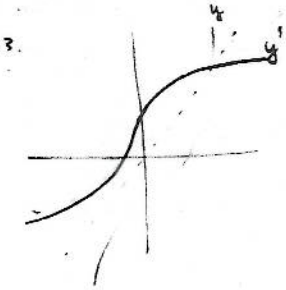
c) $x = \sin t$
$y = \cos t$
$\sin^2 t + \cos^2 t = 1$
$x^2 + y^2 = 1$

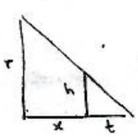
32. a) $f(x) = 2x+3$
$y = 2x+3$
inv ⇒ $x = 2y+3 \quad y^{-1} = \frac{x-3}{2}$

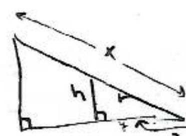
Switch x & y and s…

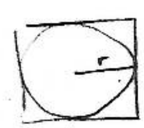
b) $f(x) = \frac{x+2}{5x-1} \Rightarrow x = \frac{y+2}{5y-1}$
$5xy - x = y + 2$
$y(5x-1) = x + 2$
$y^{-1} = \frac{x+2}{5x-1}$

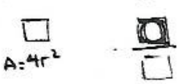
c) $f(x) = x^2 + 2x - 1 \quad x > 0 \quad x = -\frac{2}{2} = -1$
(−1, −2)  (0,−1)
$x = y^2 + 2y - 1$
$x + 1 + (1)^2 = y^2 + 2y + (1)^2$
$x + 2 = (y+1)^2$
$(y+1)^2 = x+2 \quad \Rightarrow \quad y + 1 = \pm\sqrt{x+2}$ 
$y^{-1} = \sqrt{x+2} - 1 \quad x > -1$

33.  inverse (mirror $y = x$)

34. a)  similar Δ's
$\frac{t}{h} = \frac{t+x}{r}$
$x = \frac{tr}{h} - t$
or
$x = t\left(\frac{r-h}{h}\right)$

b)  Similar Δ's
$y^2 + h^2 = r^2$
$y = \sqrt{r^2 - h^2}$
$\frac{x}{r} = \frac{t}{y}$
$\frac{x}{r} = \frac{t}{\sqrt{r^2 - h^2}}$
$x = \frac{rt}{\sqrt{r^2 - h^2}}$

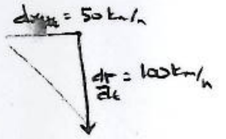
35. 

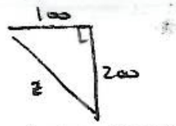
a) 
$A: 4r^2$ $= \frac{4r^2 - \pi r^2}{4r^2} = \frac{4-\pi}{4}$

b) 
$p = 4r + \frac{2\pi r}{2}$
$= 4r + \pi r$

c) 
$\frac{10}{5} = \frac{3}{l}$
$l = 1.5$
$\therefore SA = \pi(1.5)^2 = \frac{9\pi}{4}$

35 d





$100^2 + 200^2 = z^2$

$50000 = z^2$

$z = 223.607$ mi

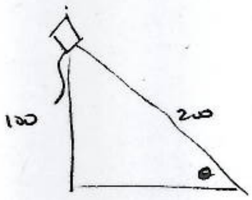
$z^2 = 50000$

$z = \sqrt{50000}$

$= \sqrt{5 \cdot 100 \cdot 100}$

$z = 100\sqrt{5}$

e.



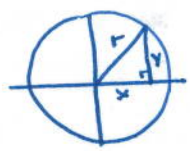
$\sin\theta = \frac{100}{200}$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$ or $\pi/6$

36. $\sin^2 x + \cos^2 x = 1$

a) $x = r\cos\theta \quad y = r\sin\theta$



$x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

assume unit circle $\therefore r = 1$

$x = \sin\theta \quad y = \cos\theta$

$x^2 + y^2 = r^2$ ← pythagorean & form for circle

by def $\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$\sin^2\theta + \cos^2\theta = 1$ ✓

b) $\sin 2x \equiv 2\sin x\cos x$

$\sin(x+x) = \sin x\cos x + \cos x\sin x$

$\sin 2x = 2\sin x\cos x$ ✓

c) $\cos 2x \equiv \cos^2 x - \sin^2 x$

$\cos(x+x) = \cos x\cos x - \sin x\sin x$

$\cos(2x) = \cos^2 x - \sin^2 x$ ✓

d) $\cos 2x = 2\cos^2 x - 1$

$\cos 2x = \cos^2 x - \sin^2 x$ ← from above

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$\cos 2x = 2\cos^2 x - 1$

e) $\cos 2x = 1 - 2\sin^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

f) $\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+\cos x}{2}}$

$\cos 2x = 2\cos^2 x - 1$

let $x \Rightarrow \frac{x}{2}$

$\cos 2\left(\frac{x}{2}\right) = 2\cos^2\left(\frac{x}{2}\right) - 1$

$\cos x = 2\cos^2(x/2) - 1$

$\sqrt{\cos^2(x/2)} = \sqrt{\frac{\cos x + 1}{2}}$

$|\cos(x/2)| = \sqrt{\frac{\cos x + 1}{2}}$ ✓

g) $\left|\sin\frac{x}{2}\right| \equiv \sqrt{\frac{1-\cos x}{2}}$

use $\cos 2x = 1 - 2\sin^2 x$

$\cos 2(x/2) = 1 - 2\sin^2(x/2)$

$\cos x = 1 - 2\sin^2(x/2)$

$\cos x - 1 = -2\sin^2(x/2)$

$\sqrt{\sin^2(x/2)} = \sqrt{\frac{1-\cos x}{2}}$

$|\sin x/2| = \sqrt{\frac{1-\cos x}{2}}$