

AP Calculus AB Summer Review Packet

Welcome to AP Calculus AB. This packet is a set of problems that you should be able to do before entering this course. You are not required to do EVERY problem in this packet, but you are responsible for all content. We will not collect these problems, but you will have an online quiz on these questions during the first week of school. We'll give you a few days to ask us questions and then you'll have a graded test on this background material. Make sure you circle any questions that you are unsure about so that you can get clarification during the Q & A period before the test is given.

We would recommend reviewing instructional videos on any topics that you may find unclear. We'd recommend Khan Academy, but there are numerous, excellent web sites that can help you. You need to be an active learner in Calculus AB, this is good practice for you. At the end of the problems, We've included an answer key for you to do some self-check.

Probably the biggest difficulty for students is the limitations placed on the use of a calculator. Approximately 70% of the AP exam is without a calculator, so that is the way the course is taught. Approximately 70% of your evaluations will be without a calculator. (That includes a four function calculator, no calculator is allowed at all). When tested on this material, it will be without a calculator.

Finally, we cannot stress enough the importance of these background skills. Calculus is easy, it's the algebra that is hard. Most of the time, you'll understand the calculus concept being taught, but will struggle to get the correct answer because of your background skills. Please be diligent and figure out what you need help with so that we can clarify for you. A little extra work this summer will go a long way to help you succeed in the upcoming year.

We look forward to having you in class and working together to accomplish your goals. We think you'll find this course challenging but enjoyable at the same time. Should you have any questions throughout the summer, please feel free to contact us via email at james_m_kuhn@mcpsmd.org.

Hope you have a great summer!

Mr. Kuhn and Ms. Thatcher

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Simplify

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12}$$

$$2. \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{9 - x^{-2}}{3 - x^{-1}}$$

Rationalize the denominator

$$5. \frac{2}{\sqrt{3} + \sqrt{2}}$$

$$6. \frac{4}{1 - \sqrt{5}}$$

$$7. \frac{1 - \sqrt{5}}{1 + \sqrt{3}}$$

Write each of the following expressions in the form of ca^pb^q where c , p , and q are numbers

$$8. \frac{(2a^2)^3}{b}$$

$$10. \frac{a(2/b)}{3/a}$$

$$12. \frac{a^{-1}}{(b^{-1})\sqrt{a}}$$

$$9. \sqrt{9ab^3}$$

$$11. \frac{ab - a}{b^2 - b}$$

$$13. \left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$$

Solve for x . Do not use a calculator

$$14. 5^{(x+1)} = 25$$

$$16. \log_2 x = 3$$

$$15. \frac{1}{3} = 3^{2x+2}$$

$$17. \log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$$

Simplify

$$18. \log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$$

$$19. 2 \log_4 9 - \log_2 3$$

$$20. 3^{2 \log_3 5}$$

Simplify

$$21. \log_{10} 10^{1/2}$$

$$22. \log_{10} \frac{1}{10^x}$$

$$23. 2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$$

Solve the following equations for the indicated variable

$$24. \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \text{ for } a$$

$$25. V = 2(ab + bc + ca), \text{ for } a$$

$$26. A = 2\pi r^2 + 2\pi rh, \text{ for positive } h$$

$$27. A = P + \pi rP, \text{ for } P$$

$$28. 2x - 2yd = y + xd, \text{ for } d$$

$$29. \frac{2x}{4\pi} + \frac{1-x}{2} = 0, \text{ for } x$$

For each equation complete the square and reduce to one of the standard forms $y - y_1 = A(x - x_1)^2$ or $x - x_1 = (y - y_1)^2$

$$30. y = x^2 + 4x + 3$$

$$31. 3x^2 + 3x + 2y = 0$$

$$32. 9y^2 - 6y - 9 - x = 0$$

Factor completely

$$33. x^6 - 16x^4$$

$$34. 4x^3 - 8x^2 - 25x + 50$$

$$35. 8x^3 + 27$$

$$36. x^4 - 1$$

Find all real solutions

$$37. x^6 - 16x^4 = 0$$

$$38. 4x^3 - 8x^2 - 25x + 50 = 0$$

$$39. 8x^3 + 27 = 0$$

Solve for x

$$40. 3\sin^2 x = \cos^2 x; \quad 0 \leq x < 2\pi$$

$$41. \cos^2 x - \sin^2 x = \sin x; \quad -\pi < x \leq \pi$$

$$42. \tan x + \sec x = 2 \cos x; \quad -\infty < x < \infty$$

Without using a calculator, evaluate the following:

$$43. \cos 210^\circ$$

$$44. \sin \frac{5\pi}{4}$$

$$45. \tan^{-1}(-1)$$

$$46. \sin^{-1}(-1)$$

$$47. \cos \frac{9\pi}{4}$$

$$48. \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$49. \tan\left(\frac{7\pi}{6}\right)$$

$$50. \cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$$

Given the graph of $y = \sin x$, sketch the graphs of:

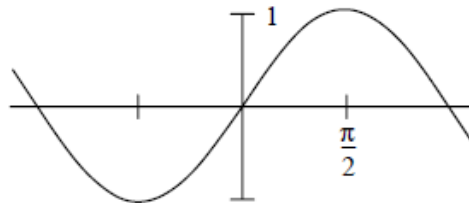
51. $\sin\left(x - \frac{\pi}{4}\right)$

52. $\sin\left(\frac{x}{2}\right)$

53. $2 \sin x$

54. $\cos x$

55. $\frac{1}{\sin x}$



Solve the equations

56. $4x^2 + 12x + 3 = 0$

58. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

57. $2x + 1 = \frac{5}{x+2}$

Find the remainders on division of

59. $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$

60. $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

61. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

62. Solve for x , the equation $12x^3 + 8x^2 - x - 1 = 0$ (all solutions are rational and between ± 1)

Solve the inequalities. Give the solution in interval notation

63. $x^2 + 2x - 3 \leq 0$

64. $\frac{2x-1}{3x-2} \leq 1$

65. $\frac{2}{2x+3} > \frac{2}{x-5}$

Solve for x . Give the solution in interval notation

66. $|-x + 4| \leq 1$

67. $|5x - 2| = 8$

68. $|2x + 1| > 3$

Determine the equation of the following lines

69. The line through $(-1, 3)$ and $(2, -4)$

70. The line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$

71. The line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$

72. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

73. Shade the region in the xy -plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$

Find the equations of the following circles:

74. The circle with center at $(1, 2)$ that passes through the point $(-2, -1)$

75. The circle that passes through the origin and has intercepts equal to 1 and 2 on the x and y axes respectively.

76. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$ find the center and the radius

77. Find the domain of $\frac{3x+1}{\sqrt{x^2+x-2}}$

Find the domain and range of:

78. $f(x) = 7$

79. $g(x) = \frac{5x-3}{2x+1}$

80. $f(x) = \frac{|x|}{x}$

Simplify $\frac{f(x+h)-f(x)}{h}$ when

81. $f(x) = 2x + 3$

82. $f(x) = \frac{1}{x+1}$

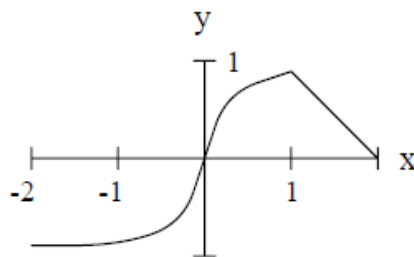
83. $f(x) = 3x^2 - x + 5$

The graph of the functions $y = f(x)$ is given as follows: Determine the graphs of the functions:

84. $f(x + 1)$

85. $f(-x)$

86. $|f(x)|$



Sketch the graphs of the functions

87. $g(x) = |3x + 2|$

88. $h(x) = |x(x - 1)|$

89. The graph of a quadratic function has x-intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4 . Determine an expression for the function.

90. Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$

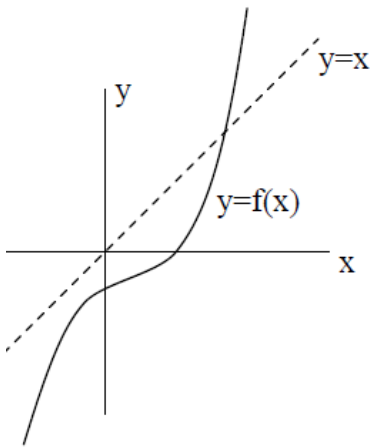
Find the inverse of the functions

91. $f(x) = 2x + 3$

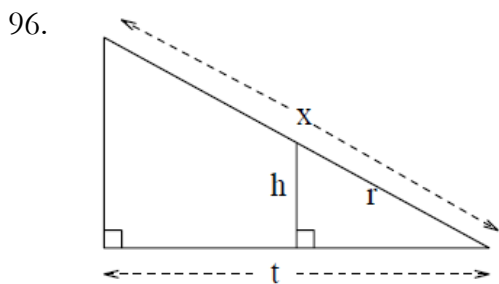
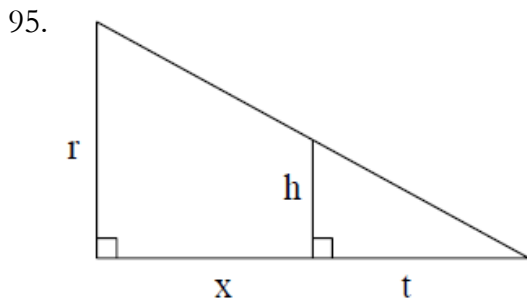
93. $f(x) = x^2 - 2x - 1, x > 0$

92. $f(x) = \frac{x+2}{5x-1}$

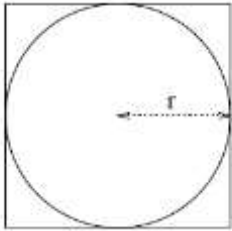
94. A function $f(x)$ has the graph below. Sketch the graph of the inverse function $f^{-1}(x)$.



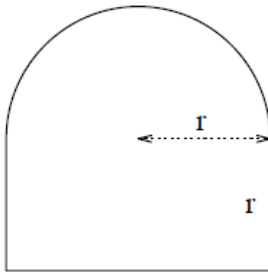
For problems 96 and 97, express x in terms of the other variables in the picture:



97. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below



98. Find the formula for the perimeter of the window of the shape in the picture below



99. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is **10m** high and has a radius of **3 m** as the top. If the water is **5 m** deep (in the middle) what is the surface area of the top of the water?
100. Two cars start moving from the same point. One travels south at **100 km/hr**, the other west at **50 km/hr**. How far apart are they two hours later?
101. A kite is **100 m** above the ground. If there are **200 m** of string out, what is the angle between the sting and the horizontal. (Assume that the string is perfectly straight.)

If $f(x) = 2x - 3$ and $g(x) = \sqrt{3x - 1}$, Find:

102. $f(g(x))$

103. $g(f(x))$

104. If $f(x) = \frac{3}{x}$ and $g(x) = \frac{x}{2x-1}$, Find $f(g(x))$ and state its domain.

Decompose each composition function into individual function. (If $y = f(u)$, identify u and rewrite y in terms of u)

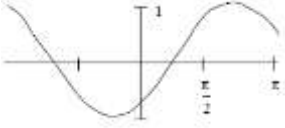
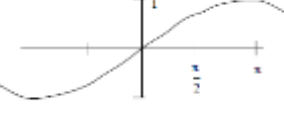
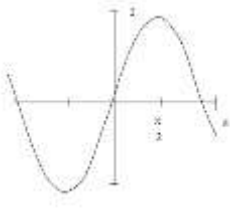

105. $y = \sin 3x$

107. $y = (x^2 - 2x + 5)^5$

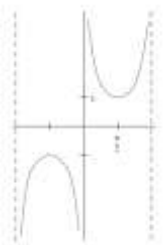
106. $y = \sqrt[5]{2x + 1}$

108. $y = \cos^2 x$

Answers

1. $\frac{x^2+3x}{x-4}$
2. $\frac{x^2-x}{5x}$
3. $\frac{x+5}{3x+1}$
4. $\frac{x}{x}$
5. $2(\sqrt{3} - \sqrt{2})$
6. $-1 - \sqrt{5}$
7. $\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}$
8. $8a^6b^{-1}$
9. $3a^{1/2}b^{3/2}$
10. $\frac{2}{3}a^2b^{-1}$
11. ab^{-1}
12. $a^{-3/2}b$
13. $a^{5/6}b^{1/2}$
14. 1
15. $-\frac{3}{2}$
16. 8
17. $\pm \frac{4}{25}$
18. $\log_2(5(x+1))$
19. $\log_2 3$
20. 25
21. $\frac{1}{2}$
22. $-x$
23. $2 \log_{10} x$
24. $\frac{bcx}{bc-cy-bz}$
25. $\frac{V-2bc}{2(b+c)}$
26. $\frac{A-2\pi r^2}{2\pi r}$
27. $\frac{1+\pi r}{A}$
28. $\frac{2x-y}{x+2y}$
29. $\frac{\pi}{\pi-1}$
30. $y+1 = (x+2)^2$
31. $y - \frac{3}{8} = -\frac{3}{2}\left(x + \frac{1}{2}\right)^2$
32. $x+10 = 9\left(y - \frac{1}{3}\right)^2$
33. $x^4(x-4)(x+4)$
34. $(x-2)(2x-5)(2x+5)$
35. $(2x+3)(4x^2-6x+9)$
36. $(x-1)(x+1)(x^2+1)$
37. $0, \pm 4$
38. $2, \pm \frac{5}{2}$
39. $-\frac{3}{2}$
40. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
41. $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
42. $\frac{\pi}{6} + 2k\pi$ and $\frac{5\pi}{6} + 2k\pi$ where $k \in I$
43. $-\frac{\sqrt{3}}{2}$
44. $-\frac{\sqrt{2}}{2}$
45. $-\frac{\pi}{2}$
46. $-\frac{4}{2}$
47. $\frac{\sqrt{2}}{2}$
48. $\frac{\pi}{3}$
49. $\frac{\sqrt{3}}{3}$
50. $\frac{3\pi}{4}$
51. 
52. 
53. 
54. 

55.



56. $\frac{-3 \pm \sqrt{6}}{2}$

57. $\frac{1}{2}$ or -3

58. $-\frac{1}{2}$

59. -89

60. $x^2 + 3$

61. $-\frac{1}{3}$ or $\frac{1}{4}$

62. $-\frac{1}{2}, \frac{1}{3}, -\frac{1}{2}$

63. $[-3, 1]$

64. $(-\infty, \frac{2}{3}) \cup [1, \infty)$

65. $(-\infty, -8) \cup (-\frac{3}{2}, 5)$

66. $[3, 5]$

67. 2 and $-\frac{6}{5}$

68. $(-\infty, -2) \cup (1, \infty)$

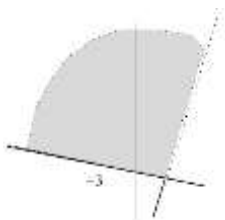
69. $7x + 3y = 2$

70. $3x + 2y = 1$

71. $y = 3$

72. $(2, -1)$

73.



74. $(x - 1)^2 + (y - 2)^2 = 18$

75. $(x - \frac{1}{2})^2 + (y - 1)^2 = \frac{5}{4}$

76. Center = $(-3, 2)$, radius = $\sqrt{10}$

77. $(-\infty, -2) \cup (1, \infty)$

78. Domain $(-\infty, \infty)$ Range $\{7\}$

79. Domain $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

Range $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

80. Domain $(-\infty, 0) \cup (0, \infty)$

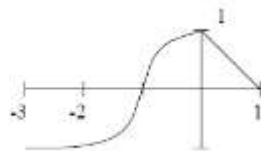
Range $\{-1, 1\}$

81. 2

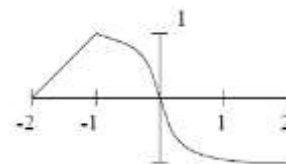
82. $\frac{-1}{(x+1)(x+h+1)}$

83. $6x + 3h - 1$

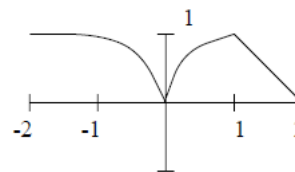
84.



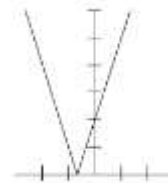
85.



86.



87.



88.



89. $y = -x^2 + 2x + 3$

90.

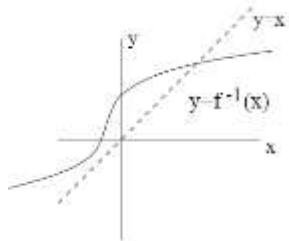


91. $f^{-1} = \frac{x-3}{2}$

92. $f^{-1} = \frac{x+2}{5x-1}$

93. $f^{-1} = 1 + \sqrt{x+2}$ for $x > -1$

94.



95. $x = t \left(\frac{r-h}{h} \right)$

96. $x = \frac{rt}{\sqrt{r^2-h^2}}$

97. $1 - \frac{\pi}{4}$

98. $4r + \pi r$

99. $\frac{9\pi}{4}$

100. $100\sqrt{5} \text{ KM}$

101. $\frac{\pi}{6}$

102. $2\sqrt{3x-1} - 3$

103. $\sqrt{6x-10}$

104. $\frac{6x-3}{x}$

Domain $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

105. Let $u = 3x$, then $y = \sin u$

106. Let $u = 2x + 1$, then $y = \sqrt[5]{u}$

107. Let $u = x^2 - 2x + 5$,

then $y = u^5$

108. Let $u = \cos x$, then $y = u^2$