CK-12 FlexBook

MCPS C2.0 Geometry Unit 1 Topic 3 Flexbook

MCPS

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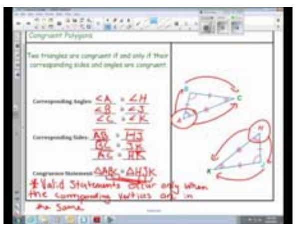
Contents

CONCEPT 1 SLT 21, 22, & 23 Use the definition of congruence to explain why two figures are congruent.

What does congruence have to do with rigid transformations?

Watch This



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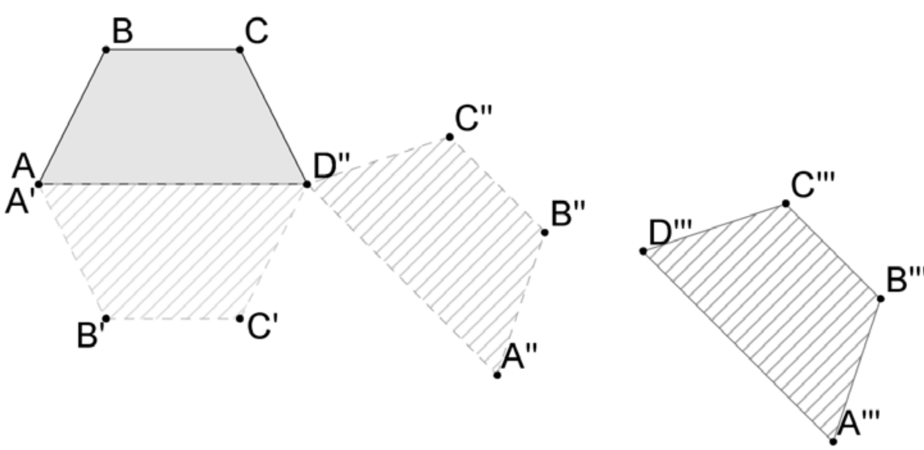
http://www.youtube.com/watch?v=qEFI5EADteE

Guidance

When a figure is transformed with one or more rigid transformations, an image is created that is **congruent** to the original figure. In other words, two figures are **congruent** if a sequence of rigid transformations will carry the first figure to the second figure. In the picture below, trapezoid *ABCD* has been reflected, then rotated, and then translated. All four trapezoids are congruent to one another.



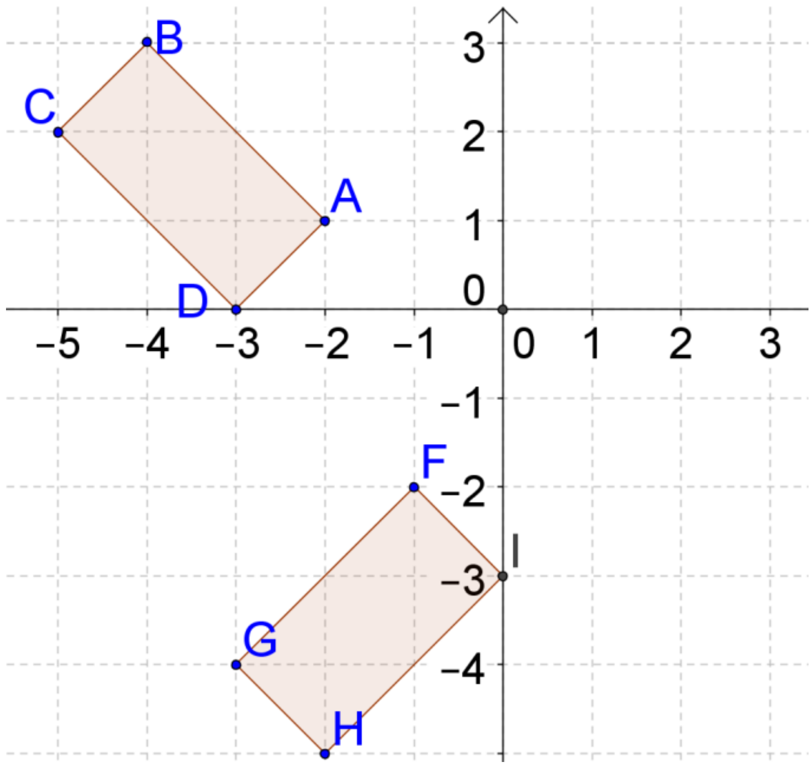
Recall that rigid transformations preserve distance and angles. This means that **congruent figures** will have corresponding angles and sides that are the same measure and length.

In order to determine if two shapes are congruent, you can:

1. Carefully describe the sequence of rigid transformations necessary to carry the first figure to the second. AND/OR
2. Verify that all corresponding pairs of sides and all corresponding pairs of angles are congruent.

Example A

Are the two rectangles congruent? Explain.

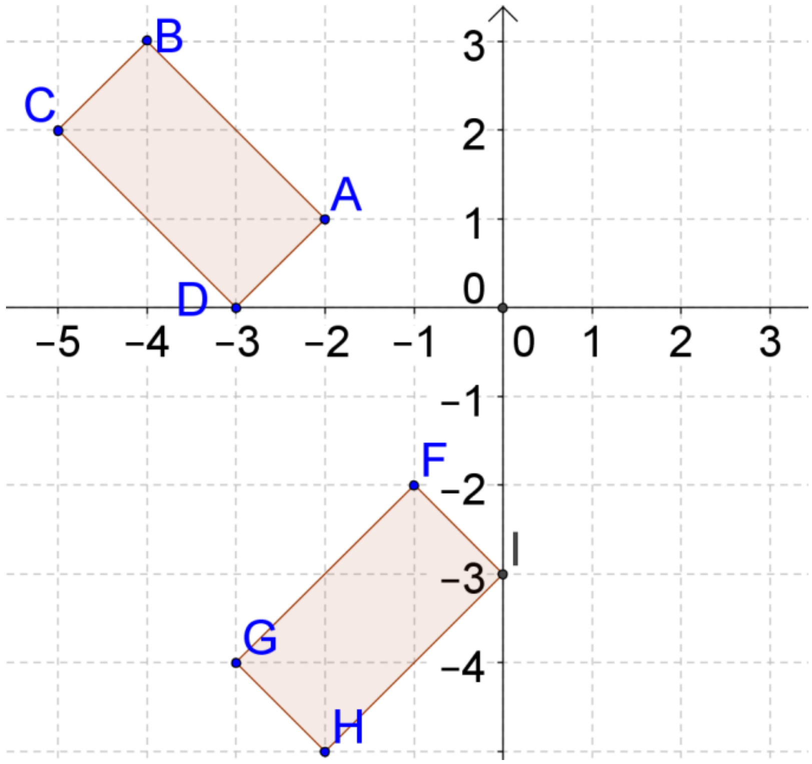


Solution: One way to determine whether or not the rectangles are congruent is to consider if transformations to rectangle $ABCD$ would produce rectangle $FGHI$. Just from looking at the rectangles it appears that if rectangle $ABCD$ were rotated 90° counterclockwise about the origin it would produce rectangle $FGHI$. To verify this, you can check the points and notice that $(x,y) \rightarrow (-y,x)$ for rectangle $ABCD$ to rectangle $FGHI$, so this is in fact a 90° counterclockwise rotation about the origin.

Because a rigid transformation on rectangle $ABCD$ produces rectangle $FGHI$, the two rectangles are congruent.

Example B

Give another explanation for why the two rectangles from Example A are congruent.



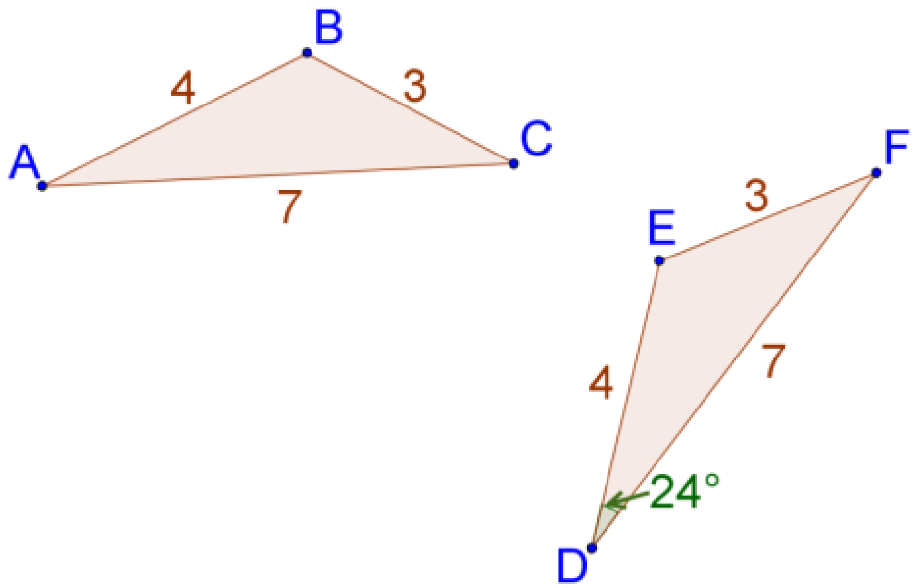
Solution: To verify that the rectangles are congruent, you could also verify that all corresponding angles and sides are congruent. Notice that the slopes of each line segment making up the rectangles is either +1 or -1. All adjacent sides have opposite reciprocal slopes and are therefore perpendicular. This means that all angles are 90° . All pairs of angles are congruent since all angles are 90° . To find the length of the line segments, you can use the Pythagorean Theorem (which is the same as the distance formula).

- $AD = BC = FI = GH = \sqrt{1^2+1^2} = \sqrt{2}$, so $\overline{AD} \cong \overline{FI}$ and $\overline{BC} \cong \overline{GH}$
- $CD = BA = FG = IH = \sqrt{2^2+2^2} = 2\sqrt{2}$, so $\overline{CD} \cong \overline{HI}$ and $\overline{AB} \cong \overline{FG}$

Because all corresponding pairs of sides are congruent and all corresponding pairs of angles are congruent, the rectangles are congruent.

Example C

The triangles below are congruent. What does that tell you about $\angle A$?



Solution: Because the triangles are congruent, corresponding sides and angles are congruent. By looking at the sides, you can see that $\angle A$ corresponds to $\angle D$, because both of these angles are in between the sides of lengths 4 and 7. Since $\angle D$ is 24° , $\angle A$ must also be 24° .

Concept Problem Revisited

Rigid transformations create congruent figures. You might think of congruent figures as shapes that "look exactly the same", but congruent figures can always be linked to rigid transformations as well. If two figures are congruent, you will always be able to perform a sequence of rigid transformations on one to create the other.

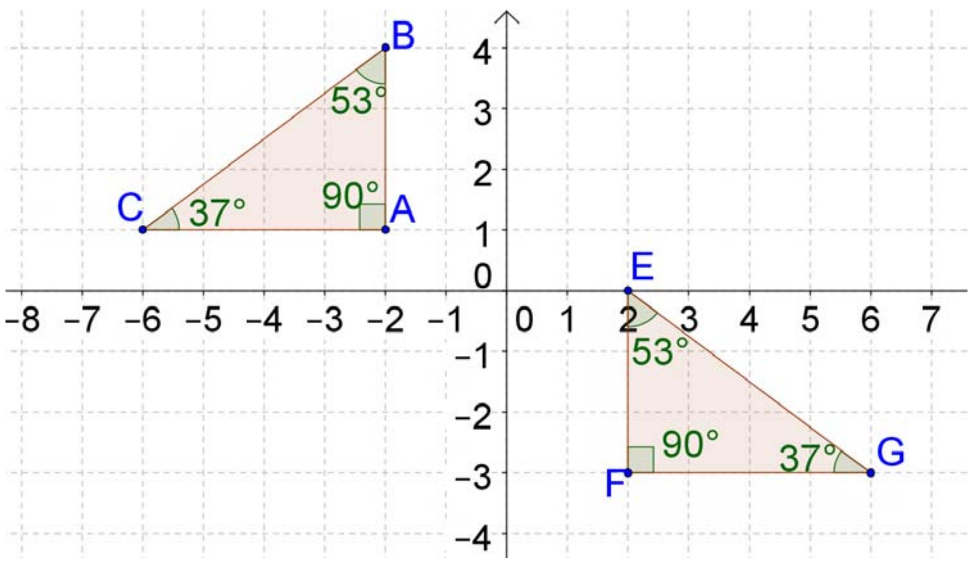
Vocabulary

Rigid transformations are transformations that preserve distance and angles. The rigid transformations are reflections, rotations, and translations.

Two figures are ***congruent*** if a sequence of rigid transformations will carry one figure to the other. ***Congruent figures*** will always have corresponding angles and sides that are congruent as well.

Guided Practice

1. Are the two triangles congruent? Explain.



2. Give another explanation for why the two triangles from #1 are congruent.

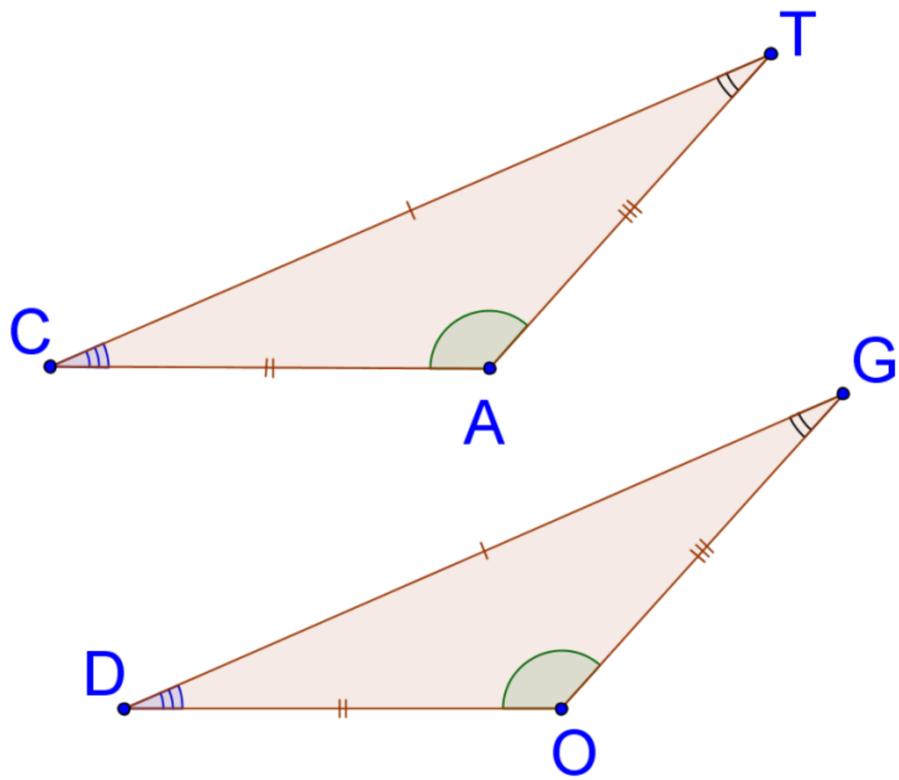
3. The symbol for congruence is $\cong$. $\Delta ABC \cong \Delta DEF$ means "triangle ABC is congruent to triangle DEF". The order of the letters matters. When you say $\Delta ABC \cong \Delta DEF$ it means that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. Suppose $\Delta CAT \cong \Delta DOG$. Draw a picture that matches this situation.

Answers :

1. ΔABC can be reflected across the y -axis and then translated over one unit to the right and down four units to create ΔEFG. Therefore, the triangles are congruent.

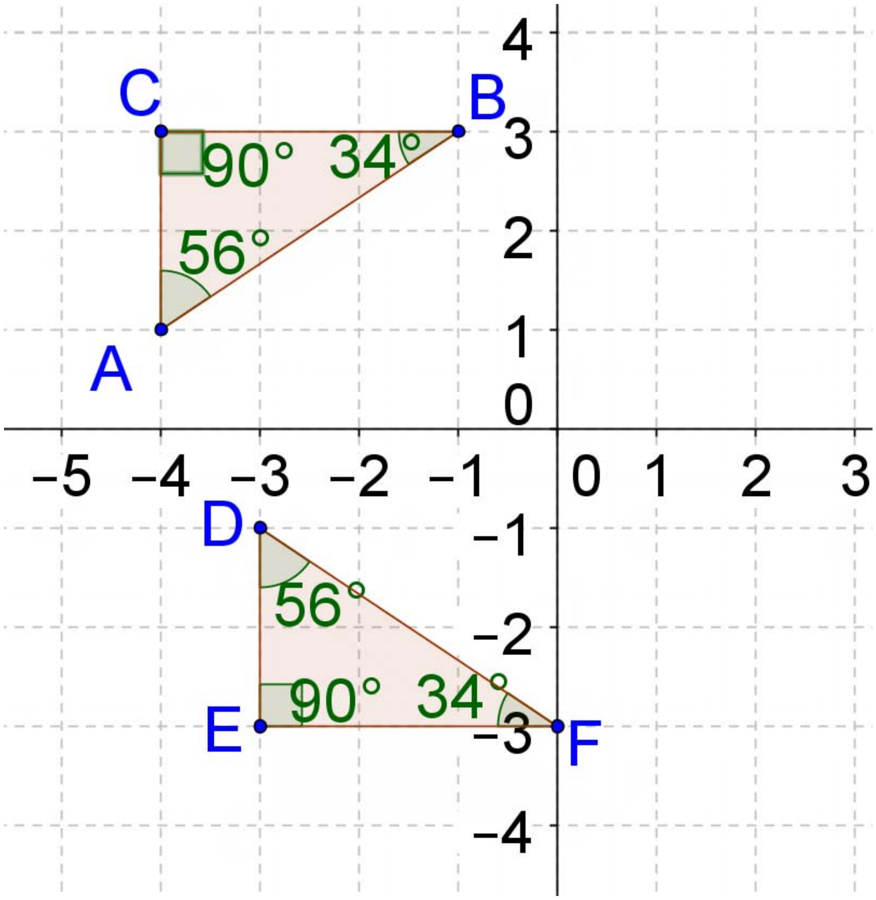
2. You can see that $\angle A \cong \angle F$, $\angle C \cong \angle G$, $\angle B \cong \angle E$. You can also see that from A to B is 3 units and from E to F is 3 units so $\overline{AB} \cong \overline{EF}$. Similarly, from A to C is 4 units and from F to G is 4 units so $\overline{AC} \cong \overline{FG}$. Using the 3, 4, 5 Pythagorean triple you know that both $\overline{BC}$ and $\overline{EG}$ must be 5 units, so $\overline{BC} \cong \overline{EG}$. Because all pairs of corresponding angles and sides are congruent, the triangles are congruent.

3. Remember that to denote that two sides are congruent, you can either mark them as being the same length (e.g., each 7 units), or use corresponding tick marks. It works the same way with angles. Corresponding angle markings mean congruent angles.


T
C
A
G
D
O

Practice

Use the triangles below for #1 - #3.

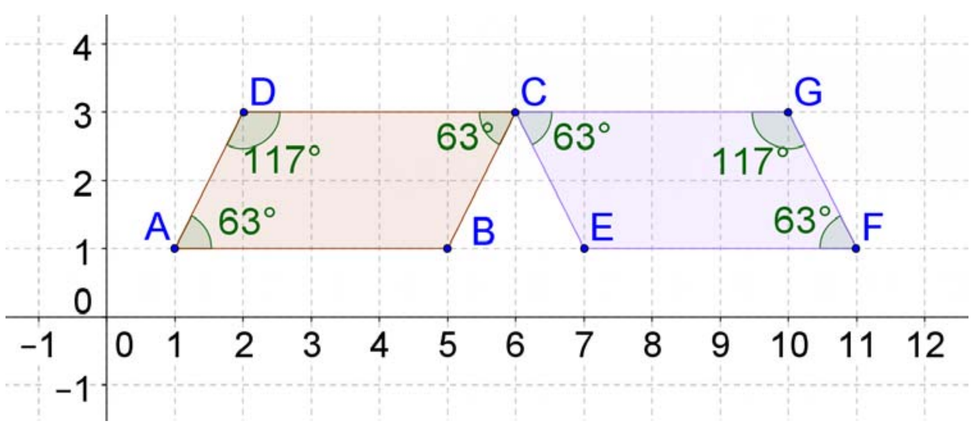


1. Explain why the triangles are congruent in terms of rigid transformations.

2. Explain why the triangles are congruent in terms of corresponding angles and sides.

3. Use notation like $\Delta CAT \cong \Delta DOG$ to state how the triangles are congruent. *Note that there are multiple correct ways to write this!*

Use the parallelograms below for #4 - #6.



4. Explain why the parallelograms are congruent in terms of rigid transformations.

5. Explain why the parallelograms are congruent in terms of corresponding angles and sides.

6. Use notation like $ABCD \cong A'B'C'D'$ to state how the parallelograms are congruent. *Note that there are multiple correct ways to write this!*

$\Delta MRG \cong \Delta KPS$

7. Draw a picture that matches this situation.

8. $\angle R \cong \angle$_____

9. $\overline{RG} \cong$ _____

10. $\overline{SK} \cong$ _____

11. $m\angle M = 60^\circ$ and $m\angle S = 20^\circ$. What does this tell you about $m\angle R$?

12. ΔDLP is reflected across the $x-axis$, then rotated 90° clockwise to create ΔMRK . How are the two triangles related?

13. Why will rigid transformations always produce congruent figures? Could non-rigid transformations also produce congruent figures?

14. If you know that all pairs of corresponding angles for two triangles are congruent, must the triangles be congruent? Explain and provide a counterexample if relevant.

15. If you know that two pairs of corresponding angles and all pairs of corresponding sides for two triangles are congruent, must the triangles be congruent? Explain and provide a counterexample if relevant.

References

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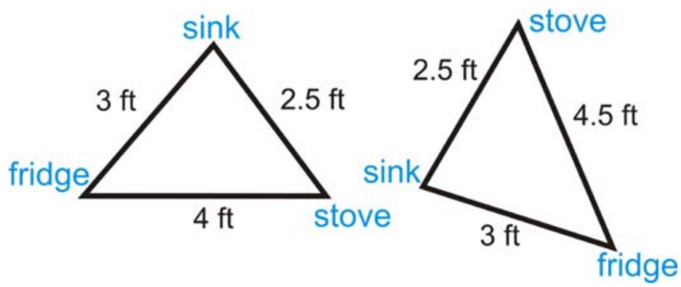
CONCEPT 2 SLT 24 Explore and apply Side Side Side (SSS) criteria to prove triangle congruence.

Here you'll learn how to prove that two triangles are congruent given only information about the side lengths of the triangles.

What if your parents were remodeling their kitchen so that measurements between the sink, refrigerator, and oven were as close to an equilateral triangle as possible? The measurements are in the picture at the left, below. Your neighbor's kitchen has the measurements on the right, below. Are the two triangles congruent? After completing this Concept, you'll be able to determine whether or not two triangles are congruent given only their side lengths.



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CK-12 Foundation: Chapter4SSSTriangleCongruenceA

Guidance

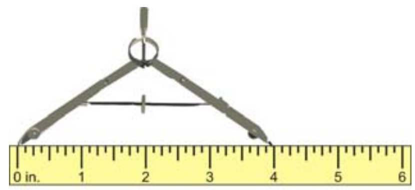
Consider the question: If I have three lengths, 3 in, 4 in, and 5 in, can I construct more than one triangle with these measurements? In other words, can I construct two different triangles with these same three lengths?

Investigation: Constructing a Triangle Given Three Sides

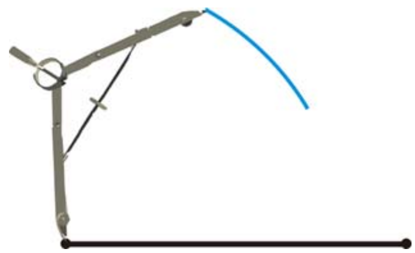
Tools Needed: compass, pencil, ruler, and paper

1. Draw the longest side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.*

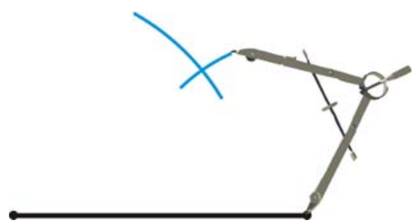
2. Take the compass and, using the ruler, widen the compass to measure 4 in, the next side.



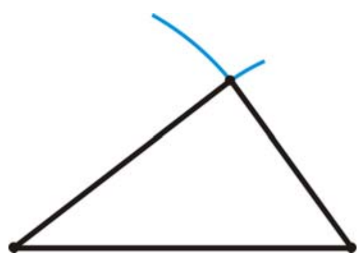
3. Using the measurement from Step 2, place the pointer of the compass on the left endpoint of the side drawn in Step 1. Draw an arc mark above the line segment.



4. Repeat Step 2 with the last measurement, 3 in. Then, place the pointer of the compass on the right endpoint of the side drawn in Step 1. Draw an arc mark above the line segment. Make sure it intersects the arc mark drawn in Step 3.



5. Draw lines from each endpoint to the arc intersections. These lines will be the other two sides of the triangle.



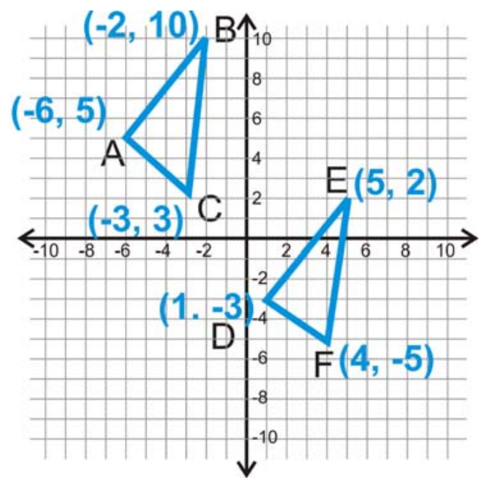
Can you draw another triangle, with these measurements that looks different? The answer is NO. Only one triangle can be created from any given three lengths.

An animation of this investigation can be found at: http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html

Side-Side-Side (SSS) Triangle Congruence Postulate: If three sides in one triangle are congruent to three sides in another triangle, then the triangles are congruent.

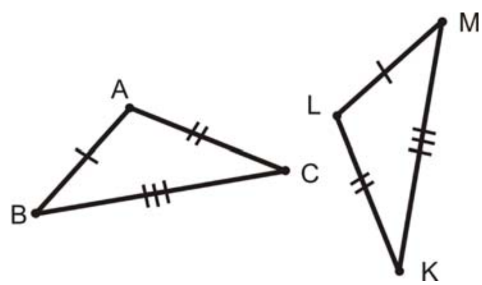
Now, we only need to show that all three sides in a triangle are congruent to the three sides in another triangle. This is a postulate so we accept it as true without proof. Think of the SSS Postulate as a shortcut. You no longer have to show 3 sets of angles are congruent and 3 sets of sides are congruent in order to say that the two triangles are congruent.

In the coordinate plane, the easiest way to show two triangles are congruent is to find the lengths of the 3 sides in each triangle. Finding the measure of an angle in the coordinate plane can be a little tricky, so we will avoid it in this text. Therefore, you will only need to apply SSS in the coordinate plane. To find the lengths of the sides, you will need to use the distance formula, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.



Example A

Write a triangle congruence statement based on the diagram below:

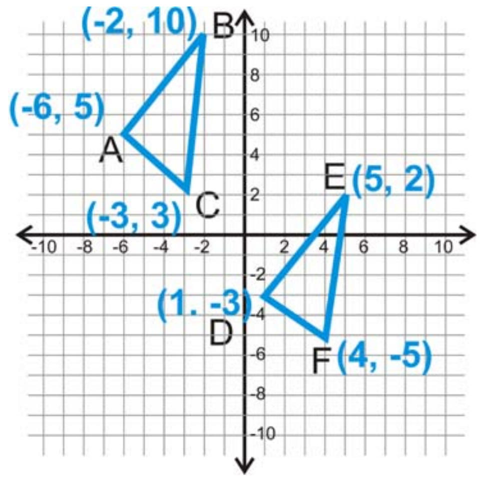


From the tic marks, we know $\overline{AB} \cong \overline{LM}, \overline{AC} \cong \overline{LK}, \overline{BC} \cong \overline{MK}$. Using the SSS Postulate we know the two triangles are congruent. Lining up the corresponding sides, we have $\triangle ABC \cong \triangle LMK$.

Don't forget ORDER MATTERS when writing triangle congruence statements. Here, we lined up the sides with one tic mark, then the sides with two tic marks, and finally the sides with three tic marks.

Example B

Find the distances of all the line segments from both triangles to see if the two triangles are congruent.



Begin with $\triangle ABC$ and its sides.

$$\begin{aligned} AB &= \sqrt{(-6-(-2))^2+(5-10)^2} \\ &= \sqrt{(-4)^2+(-5)^2} \\ &= \sqrt{16+25} \\ &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-2-(-3))^2+(10-3)^2} \\ &= \sqrt{(1)^2+(7)^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-6-(-3))^2+(5-3)^2} \\ &= \sqrt{(-3)^2+(2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

Now, find the distances of all the sides in $\triangle DEF$.

$$\begin{aligned}DE &= \sqrt{(1-5)^2+(-3-2)^2}\\ &= \sqrt{(-4)^2+(-5)^2}\\ &= \sqrt{16+25}\\ &= \sqrt{41}\end{aligned}$$

$$\begin{aligned}EF &= \sqrt{(5-4)^2+(2-(-5))^2}\\ &= \sqrt{(1)^2+(7)^2}\\ &= \sqrt{1+49}\\ &= \sqrt{50} = 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}DF &= \sqrt{(1-4)^2+(-3-(-5))^2}\\ &= \sqrt{(-3)^2+(2)^2}\\ &= \sqrt{9+4}\\ &= \sqrt{13}\end{aligned}$$

We see that $AB = DE, BC = EF$, and $AC = DF$. Recall that if two lengths are equal, then they are also congruent. Therefore, $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. Because the corresponding sides are congruent, we can say that $\triangle ABC \cong \triangle DEF$ by SSS.

Concept Problem Revisited

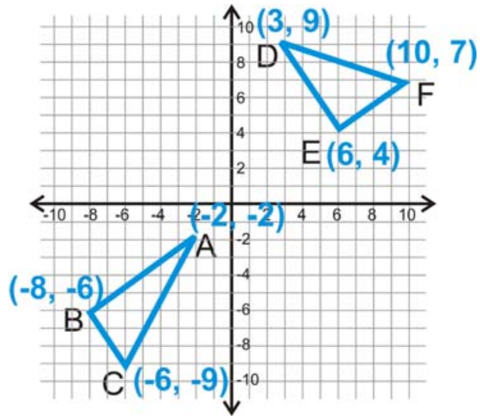
From what we have learned in this section, the two triangles are not congruent because the distance from the fridge to the stove in your house is 4 feet and in your neighbor's it is 4.5 ft. The SSS Postulate tells us that all three sides have to be congruent.

Vocabulary

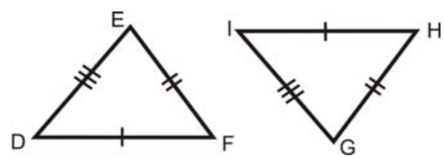
Two figures are ***congruent*** if they have exactly the same size and shape. By definition, two triangles are ***congruent*** if the three corresponding angles and sides are congruent. The symbol $\cong$ means congruent. There are shortcuts for proving that triangles are congruent. The ***SSS Triangle Congruence Postulate*** states that if three sides in one triangle are congruent to three sides in another triangle, then the triangles are congruent.

Guided Practice

1. Determine if the two triangles are congruent.



2. Is the pair of triangles congruent? If so, write the congruence statement and why.



Answers:

1. Start with $\triangle ABC$.

$$\begin{aligned} AB &= \sqrt{(-2-(-8))^2+(-2-(-6))^2} \\ &= \sqrt{(6)^2+(4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-8-(-6))^2+(-6-(-9))^2} \\ &= \sqrt{(-2)^2+(3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-2-(-6))^2+(-2-(-9))^2} \\ &= \sqrt{(4)^2+(7)^2} \\ &= \sqrt{16+49} \\ &= \sqrt{65} \end{aligned}$$

Now find the sides of $\triangle DEF$.

$$\begin{aligned}DE &= \sqrt{(3-6)^2+(9-4)^2}\\&= \sqrt{(-3)^2+(5)^2}\\&= \sqrt{9+25}\\&= \sqrt{34}\end{aligned}$$

$$\begin{aligned}EF &= \sqrt{(6-10)^2+(4-7)^2}\\&= \sqrt{(-4)^2+(-3)^2}\\&= \sqrt{16+9}\\&= \sqrt{25}=5\end{aligned}$$

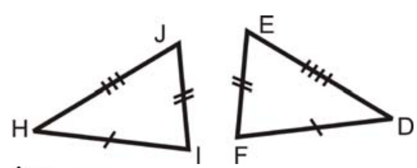
$$\begin{aligned}DF &= \sqrt{(3-10)^2+(9-7)^2}\\&= \sqrt{(-7)^2+(2)^2}\\&= \sqrt{49+4}\\&= \sqrt{53}\end{aligned}$$

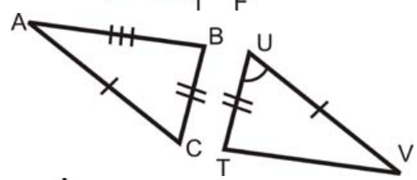
No sides have equal measures, so the triangles are not congruent.

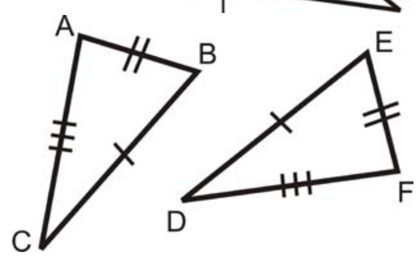
2. The triangles are congruent because they have three pairs of sides congruent. $\triangle DEF \cong \triangle IGH$.

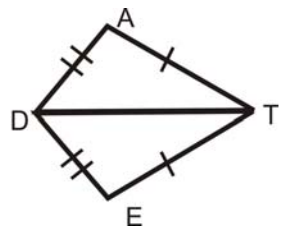
Practice

Are the pairs of triangles congruent? If so, write the congruence statement and why.

1. 

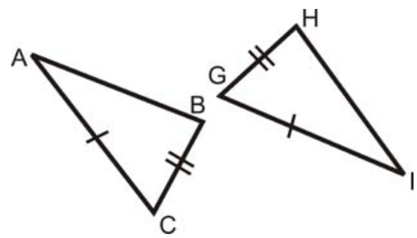
2. 

3. 

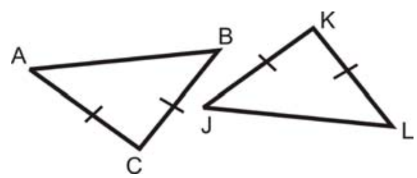
4. 

State the additional piece of information needed to show that each pair of triangles is congruent.

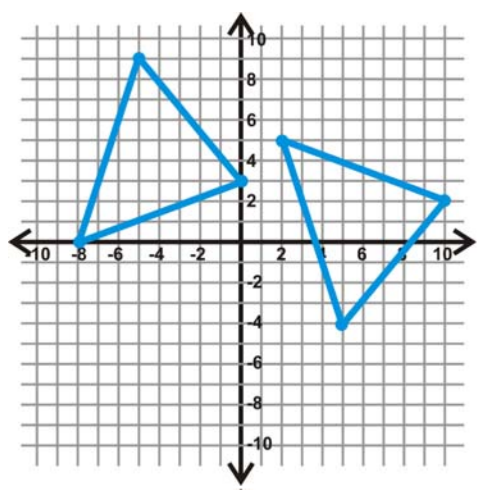
5. Use SSS

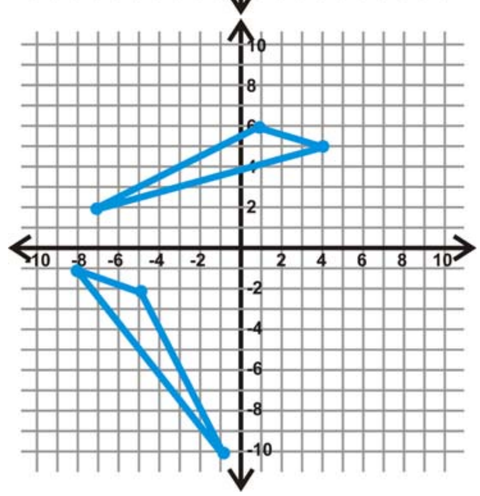


6. Use SSS



Find the lengths of the sides of each triangle to see if the two triangles are congruent.

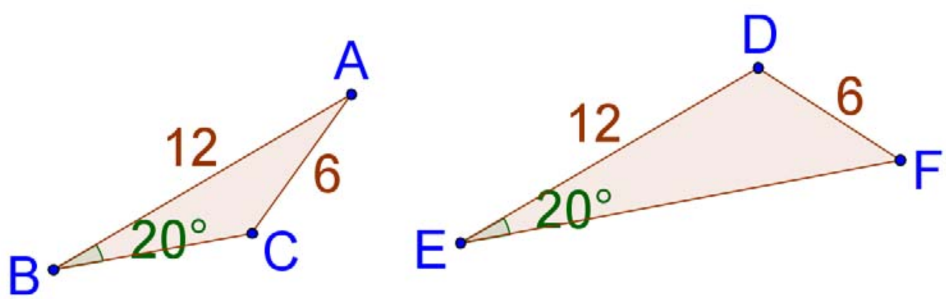
8. 

9. 

10. $\triangle ABC: A(-1,5), B(-4,2), C(2,-2)$ and $\triangle DEF: D(7,-5), E(4,2), F(8,-9)$
11. $\triangle ABC: A(-8,-3), B(-2,-4), C(-5,-9)$ and $\triangle DEF: D(-7,2), E(-1,3), F(-4,8)$
12. $\triangle ABC: A(0,5), B(3,2), C(1,4)$ and $\triangle DEF: D(1,2), E(4,4), F(7,1)$
13. $\triangle ABC: A(1,7), B(2,2), C(4,6)$ and $\triangle DEF: D(4,10), E(5,5), F(7,9)$
14. Draw an example to show why SS is not enough to prove that two triangles are congruent.

CONCEPT 3 SLT 25 Explore and apply Side Angle Side (SAS) criteria to prove triangle congruence.

When two triangles have two pairs of sides and their included angles congruent, the triangles are congruent. What if the angles aren't included angles?



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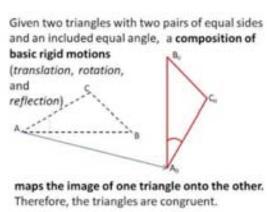

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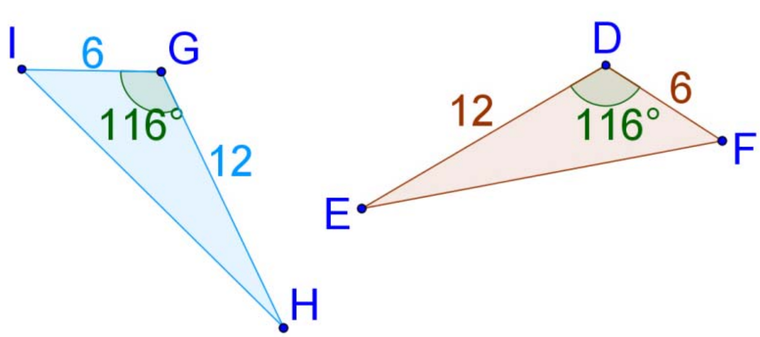
http://www.youtube.com/watch?v=30dOn3QARVU

Guidance

If two triangles are **congruent** it means that all corresponding angle pairs and all corresponding sides are congruent. However, in order to be sure that two triangles are congruent, you do not necessarily need to know that all angle pairs and side pairs are congruent. Consider the triangles below.



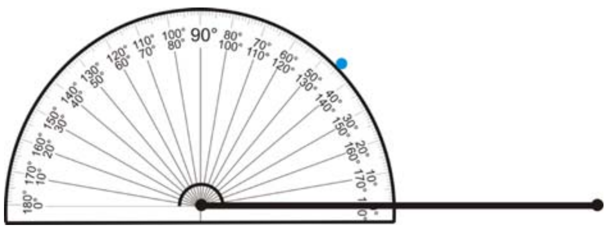
In these triangles, you can see that $\angle G \cong \angle D$, $\overline{IG} \cong \overline{FD}$, and $\overline{GH} \cong \overline{DE}$. The information you know about the congruent corresponding parts of these triangles is a **s** ide, an **a** ngle, and then another **s** ide. This is commonly referred to as "side-angle-side" or "SAS".

Consider the question: If I have two sides of length 2 in and 5 in and the angle between them is 45° , can I construct only one triangle?

Investigation: Constructing a Triangle Given Two Sides and Included Angle

Tools Needed: protractor, pencil, ruler, and paper

1. Draw the longest side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.* 
2. At the left endpoint of your line segment, use the protractor to measure a 45° angle. Mark this measurement.



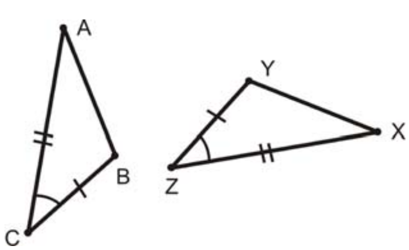
3. Connect your mark from Step 2 with the left endpoint. Make your line 2 in long, the length of the second side.

4. Connect the two endpoints by drawing the third side.

Can you draw another triangle, with these measurements that looks different? The answer is NO. Only one triangle can be created from any given two lengths and the INCLUDED angle.

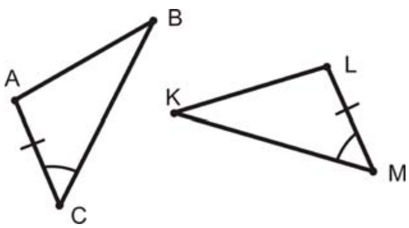
Side-Angle-Side (SAS) Triangle Congruence Postulate: If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the two triangles are congruent.

The markings in the picture are enough to say that $\triangle ABC \cong \triangle XYZ$.



Example A

What additional piece of information would you need to prove that these two triangles are congruent using the SAS Postulate?



a) $\angle ABC \cong \angle LKM$

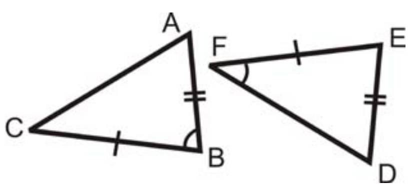
b) $\overline{AB} \cong \overline{LK}$

c) $\overline{BC} \cong \overline{KM}$

d) $\angle BAC \cong \angle KLM$

For the SAS Postulate, you need two sides and the included angle in both triangles. So, you need the side on the other side of the angle. In $\triangle ABC$, that is $\overline{BC}$ and in $\triangle LKM$ that is $\overline{KM}$. The correct answer is c.

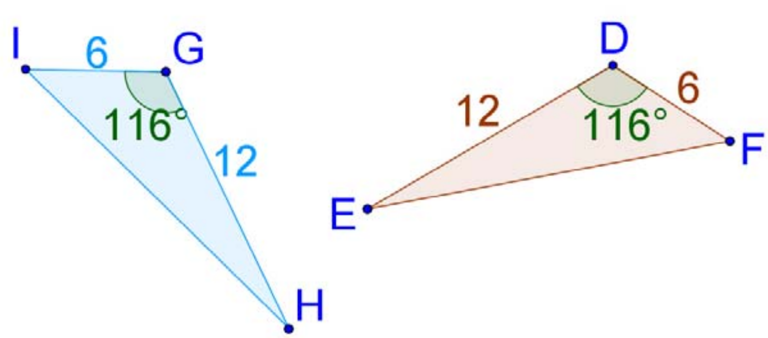
Is the pair of triangles congruent? If so, write the congruence statement and why.



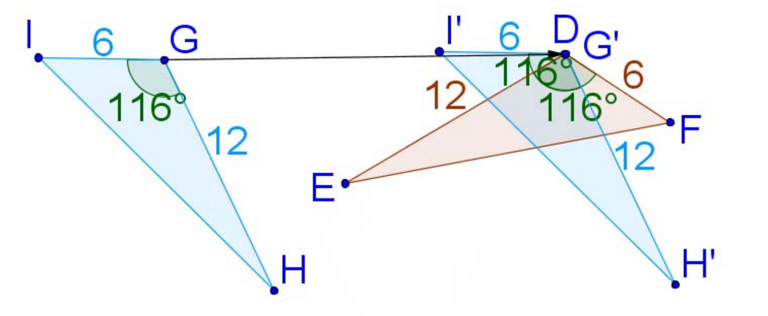
While the triangles have two pairs of sides and one pair of angles that are congruent, the angle is not in the same place in both triangles. The first triangle fits with SAS, but the second triangle is SSA. There is not enough information for us to know whether or not these triangles are congruent.

Example C

Perform a rigid transformation to bring point G to point D .

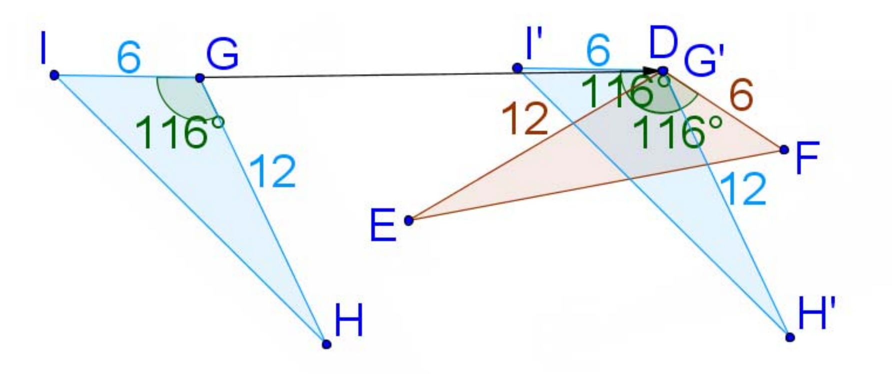


Solution: Draw a vector from point G to point D . Translate $\triangle GHI$ along the vector to create $\triangle G'H'I'$.

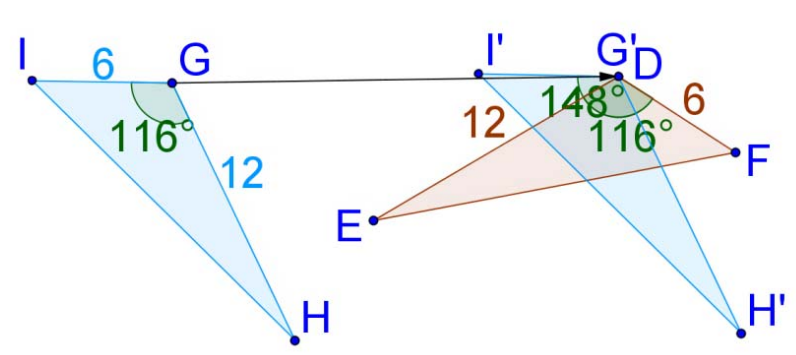


Example D

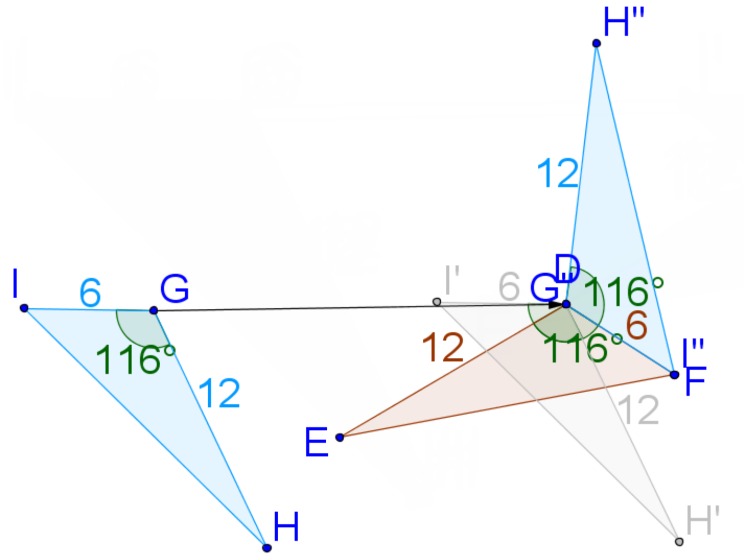
Rotate $\triangle G'H'I'$ to map to $\overline{G'I'}$ to $\overline{DF}$.



Solution: Measure $\angle I'DF$. In this case, $m\angle I'DF = 148°$.

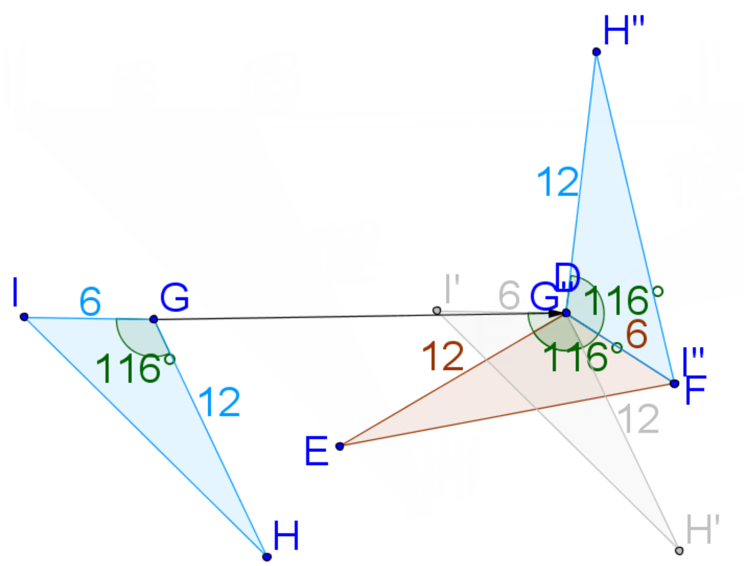


Rotate $\triangle G'H'I'$ counterclockwise that number of degrees about point G' to create $\triangle G''H''I''$. Note that because $\overline{GI} \cong \overline{DF}$ and rigid transformations preserve distance, $\overline{G''I''}$ matches up perfectly with $\overline{DF}$.

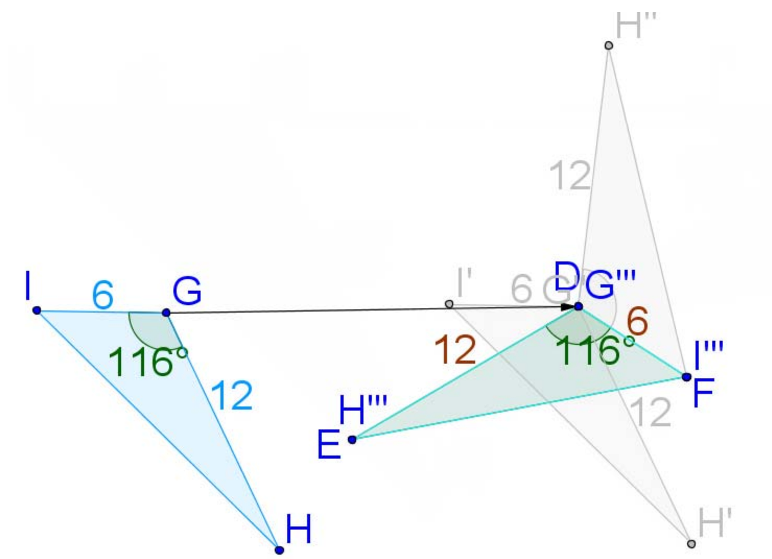


Example E

Reflect $\triangle G''H''I''$ to map it to $\triangle DEF$. Can you be confident that the triangles are congruent?



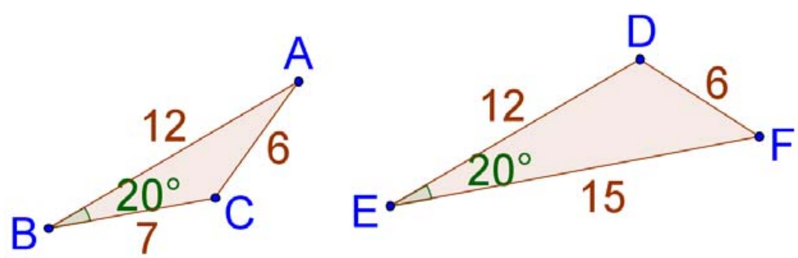
Solution: Reflect $\triangle G''H''I''$ across $\overline{G''I''}$ (which is the same as $\overline{DF}$).



Because $\angle EDF \cong \angle H''G''I''$ and $\overline{G''H''} \cong \overline{DE}$, the triangles must match up exactly (in particular, H''' must map to E), and the triangles are congruent.

This means that even though you didn't know all the side and angle measures, because you knew two pairs of sides and the included angles were congruent, the triangles had to be congruent overall. At this point you can use the SAS criterion for showing triangles are congruent without having to go through all of these transformations each time (but make sure you can explain why SAS works in terms of the rigid transformations!).

Concept Problem Revisited



Even though these triangles have two pairs of sides and one pair of angles that are congruent, the triangles are clearly not congruent. SSA is NOT a criterion for triangle congruence. In order to use two pairs of sides and one pair of angles to show that triangles are congruent, the pair of angles must be included between the pairs of congruent sides.

Vocabulary

SAS, or Side-Angle-Side, is a criterion for triangle congruence. The SAS criterion for triangle congruence states that if two triangles have two pairs of congruent sides and the included angle in one triangle is congruent to the included angle in the other triangle, then the triangles are congruent.

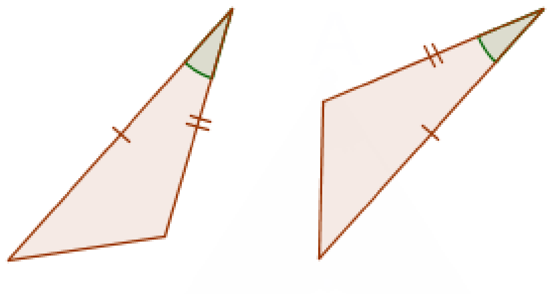
Rigid transformations are transformations that preserve distance and angles. The rigid transformations are reflections, rotations, and translations.

Two figures are ***congruent*** if a sequence of rigid transformations will carry one figure to the other. ***Congruent figures*** will always have corresponding angles and sides that are congruent as well.

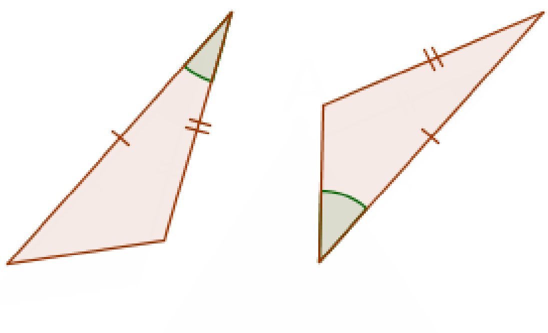
Guided Practice

Are the following triangles congruent? Explain.

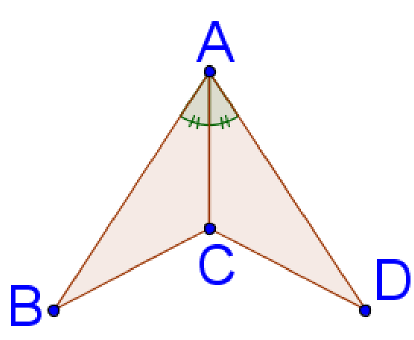
1.



2.



3. What additional information would you need in order to be able to state that the triangles below are congruent by SAS?



Answers:

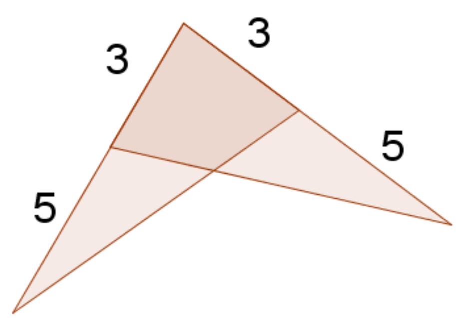
1. The triangles are congruent by SAS.
2. The triangles are not necessarily congruent. The given angle is not the included angle in both triangles.
3. You would need to know that $\overline{AB} \cong \overline{AD}$.

Practice

1. What does SAS stand for? What does it have to do with congruent triangles?
2. What does SSA stand for? What does it have to do with congruent triangles?
3. Draw an example of two triangles that must be congruent due to SAS.
4. Draw an example of two triangles that **are not congruent** because all you know is SSA.

For each pair of triangles below, state if they are congruent by SAS or if there is not enough information to determine whether or not they are congruent.

5.



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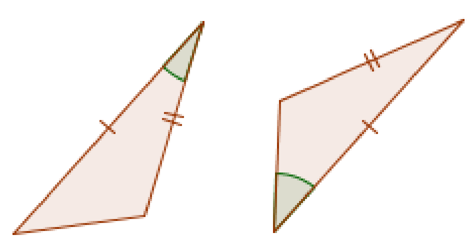

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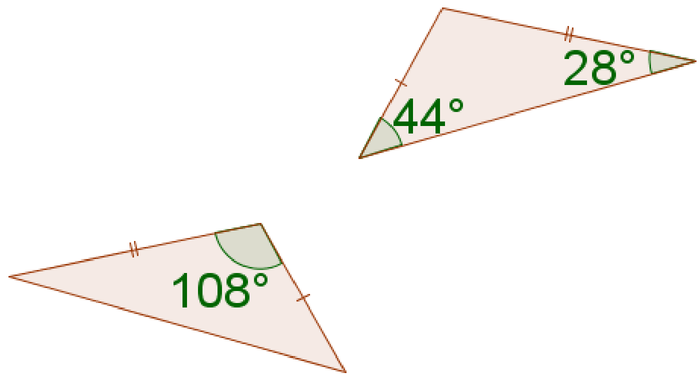

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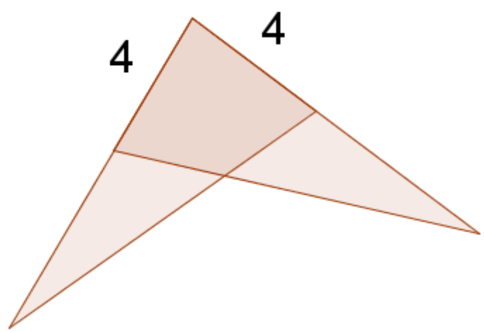

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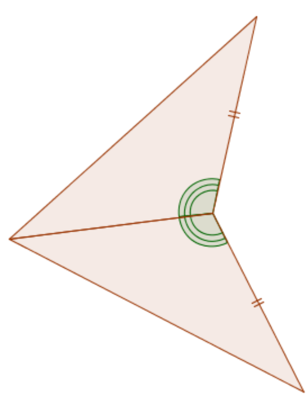
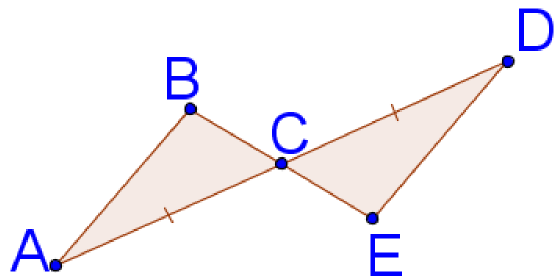
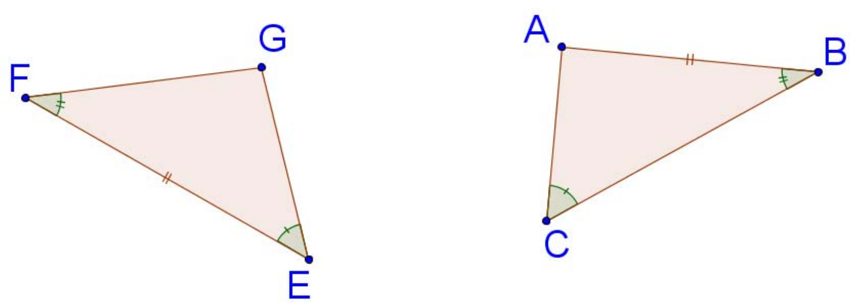


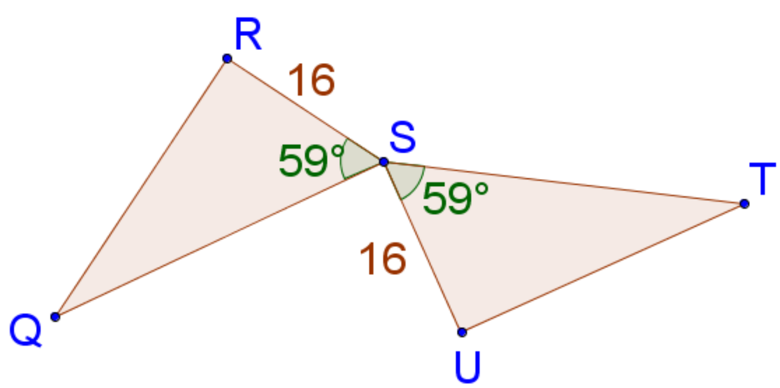
10. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by SAS? Assume that points B , C , and E are collinear.



11. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by SAS?



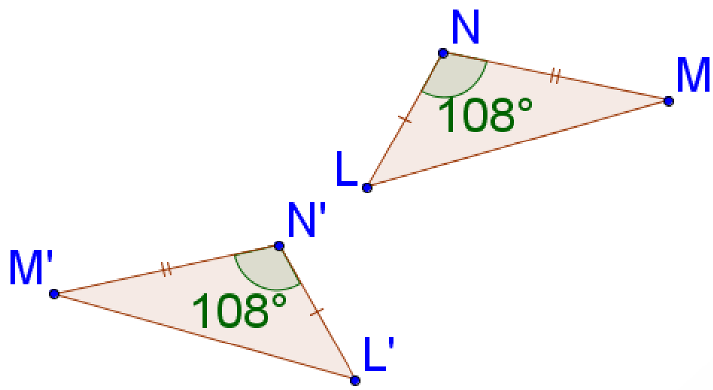
12. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by SAS?



13. Do you think you always need at least three pairs of congruent sides/angles to show that two triangles are congruent? Explain.

14. If the two pairs of legs are congruent on two right triangles, are the triangles congruent? Explain. Draw a picture to support your reasoning.

15. Show how the SAS criterion for triangle congruence works: use rigid transformations to help explain why the triangles below are congruent.



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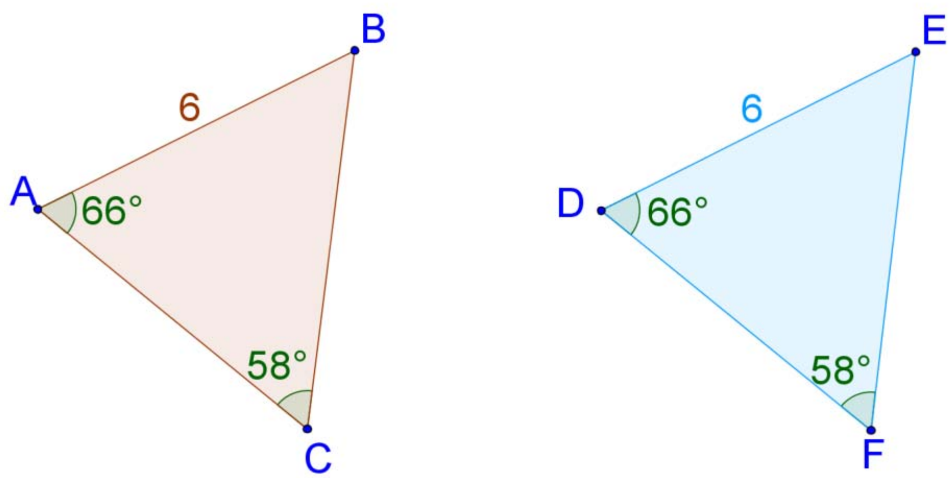
Concept 4 SLT 26 Explore and apply Angle Side Angle (ASA) criteria to prove triangle congruence.

The information for the triangles below looks to be "AAS". How could you use "ASA" to verify that the triangles are congruent?



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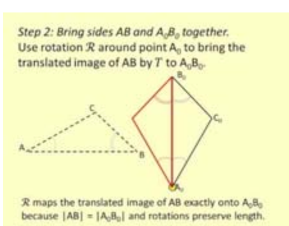

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http://www.youtube.com/watch?v=-yIZdenw5U4

Guidance

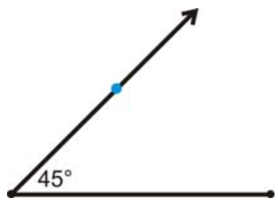
Consider the question: If I have two angles that are 45° and 60° and the side between them is 5 in, can I construct only one triangle? We will investigate it here.

Investigation: Constructing a Triangle Given Two Angles and Included Side

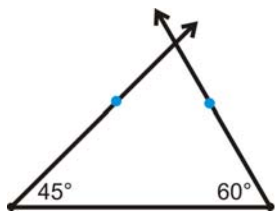
Tools Needed: protractor, pencil, ruler, and paper

1. Draw the side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.*

2. At the left endpoint of your line segment, use the protractor to measure the 45° angle. Mark this measurement and draw a ray from the left endpoint through the 45° mark.



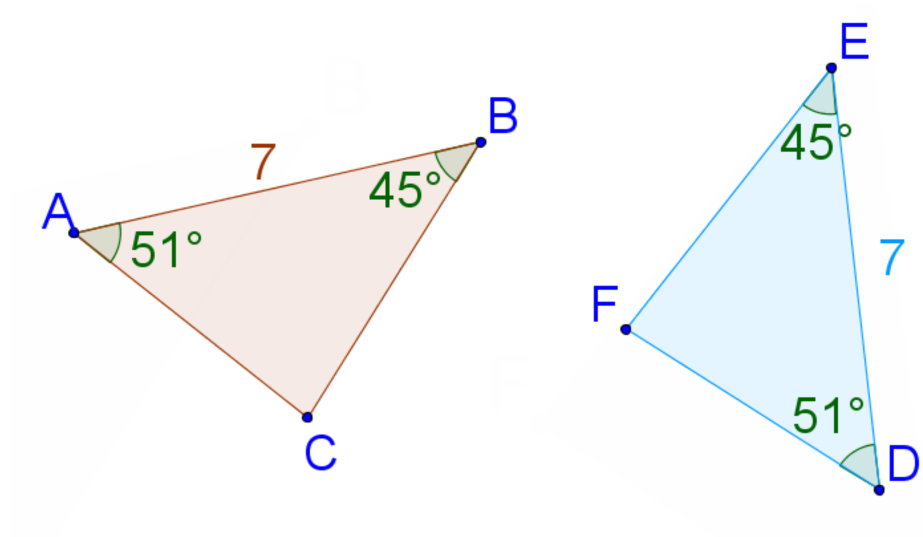
3. At the right endpoint of your line segment, use the protractor to measure the 60° angle. Mark this measurement and draw a ray from the left endpoint through the 60° mark. Extend this ray so that it crosses through the ray from Step 2.



4. Erase the extra parts of the rays from Steps 2 and 3 to leave only the triangle.

Can you draw another triangle, with these measurements that looks different? The answer is NO. Only one triangle can be created from any given two angle measures and the INCLUDED side.

If two triangles are **congruent** it means that all corresponding angle pairs and all corresponding sides are congruent. However, in order to be sure that two triangles are congruent, you do not necessarily need to know that all angle pairs and side pairs are congruent. Consider the triangles below.



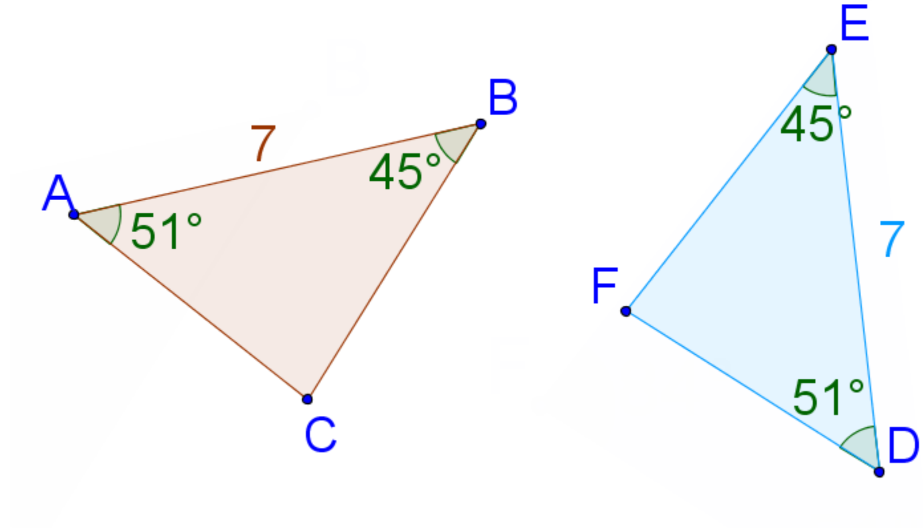
In these triangles, you can see that $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\overline{AB} \cong \overline{DE}$. The information you know about the congruent corresponding parts of these triangles is an **a** ngle, a **s** ide, and then another **a** ngle. This is commonly referred to as "angle-side-angle" or "ASA".

The ASA criterion for triangle congruence states that if two triangles have two pairs of congruent angles and the common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.

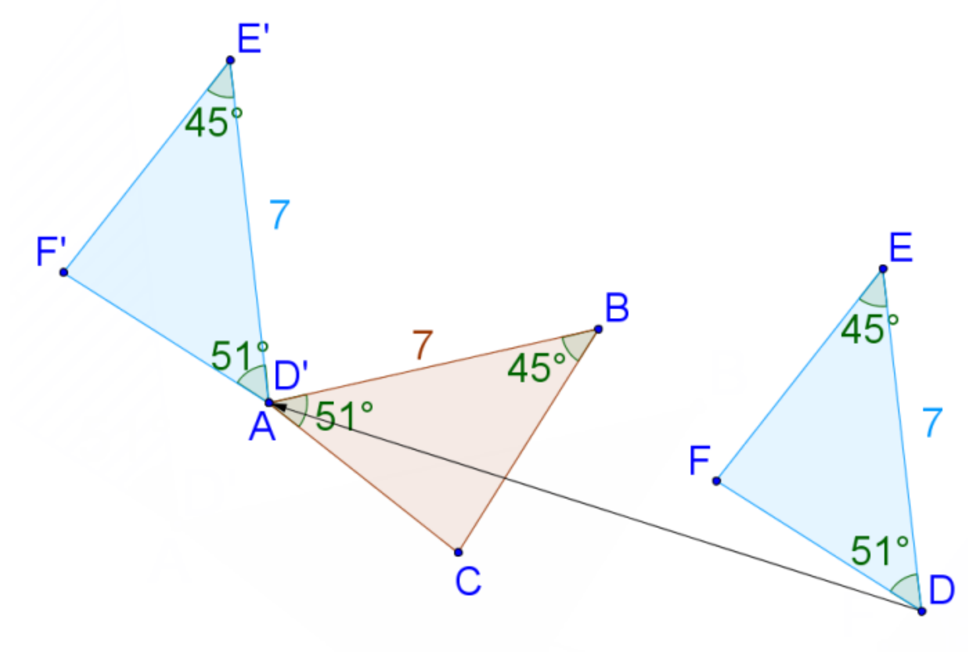
In the examples, you will use rigid transformations to show why the above ASA triangles must be congruent overall, even though you don't know the lengths of all the sides and the measures of all the angles.

Example A

Perform a rigid transformation to bring point D to point A .

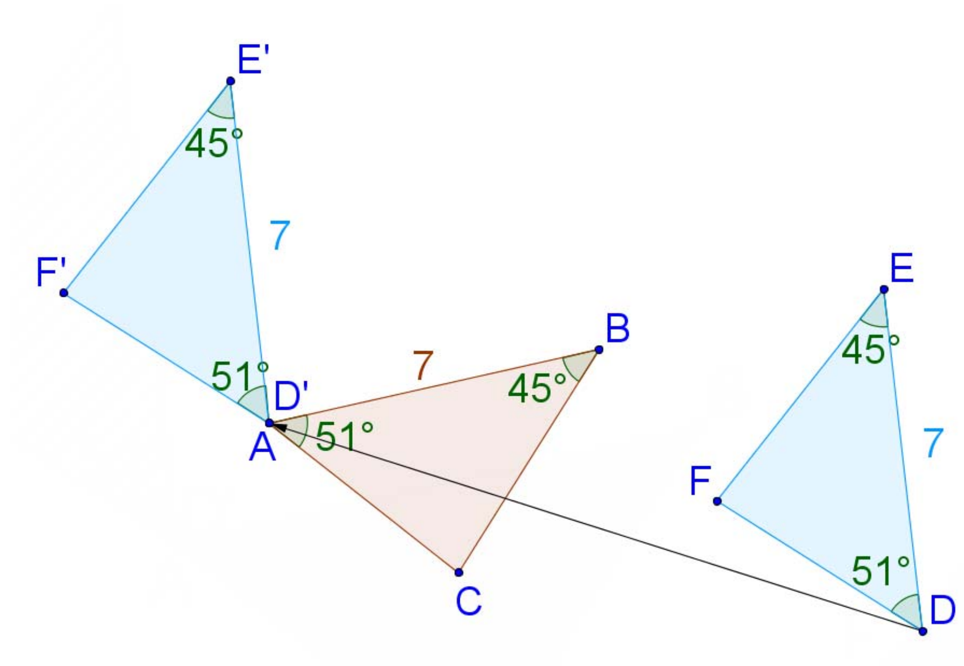


Solution: Draw a vector from point D to point A . Translate ΔDEF along the vector to create $\Delta D'E'F'$.

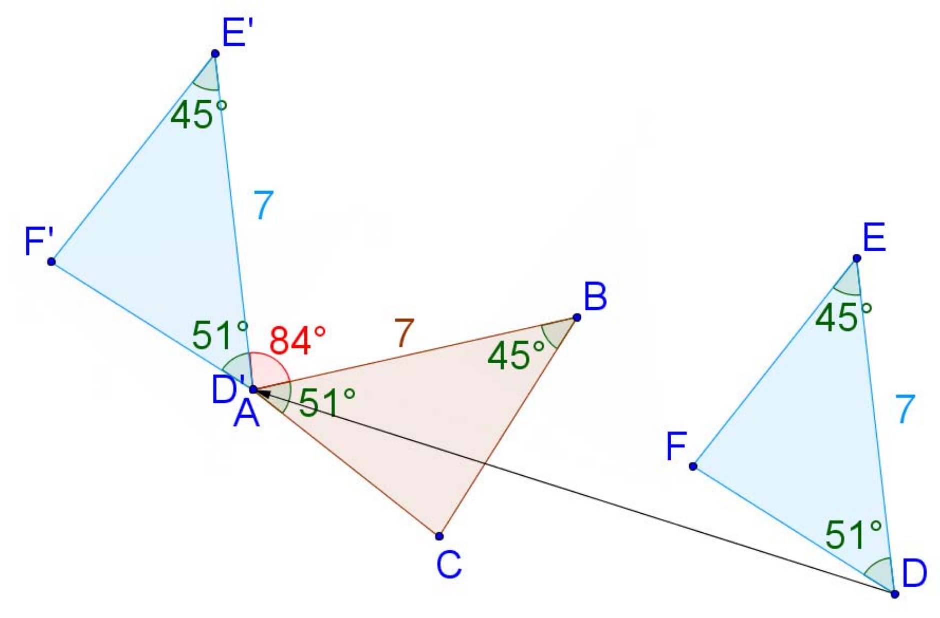


Example B

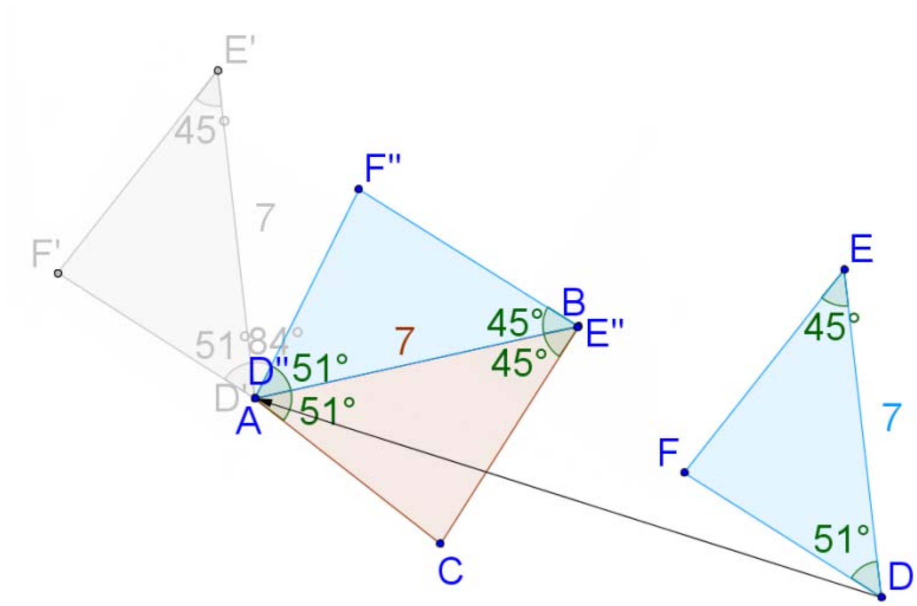
Rotate $\Delta D'E'F'$ to map $\overline{D'E'}$ to $\overline{AB}$.



Solution: Measure $\angle BD'E'$. In this case, $m\angle BD'E' = 84°$.

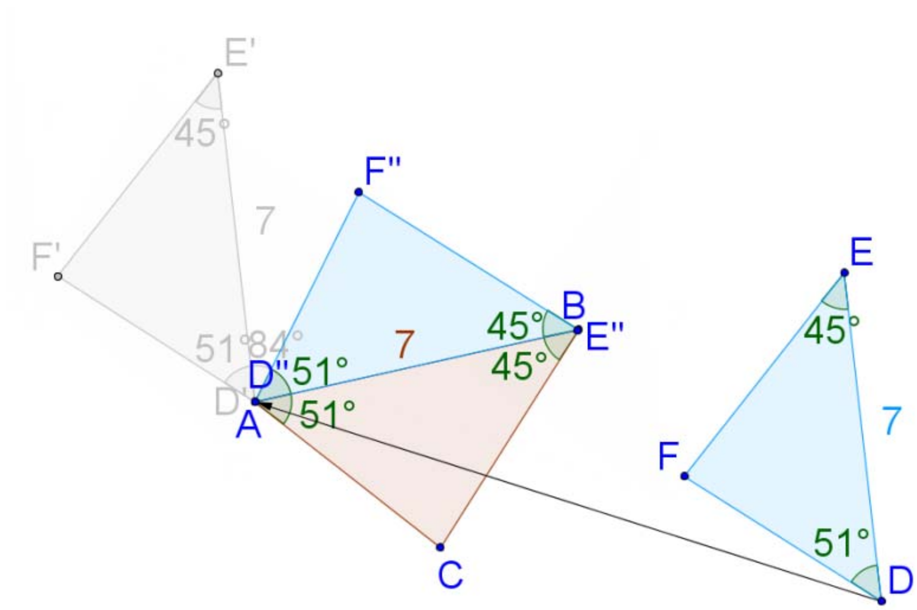


Rotate $\Delta D'E'F'$ clockwise that number of degrees about point D' to create $\Delta D''E''F''$. Note that because $\overline{DE} \cong \overline{AB}$ and rigid transformations preserve distance, $\overline{D''E''}$ matches up perfectly with $\overline{AB}$.

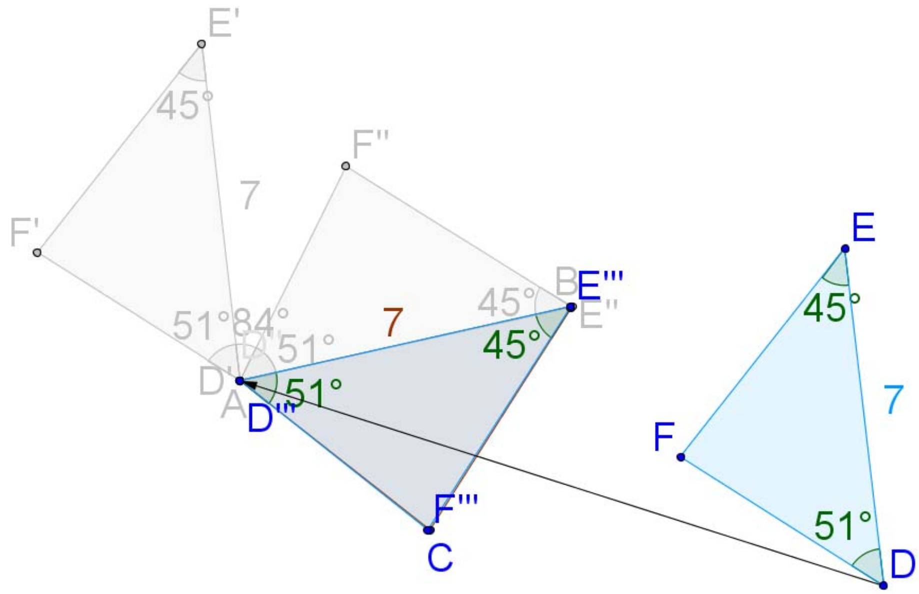


Example C

Reflect $\Delta D''E''F''$ to map it to ΔABC . Can you be confident that the triangles are congruent?



Solution: Reflect $\Delta D''E''F''$ across $\overline{D''E''}$ (which is the same as $\overline{AB}$).

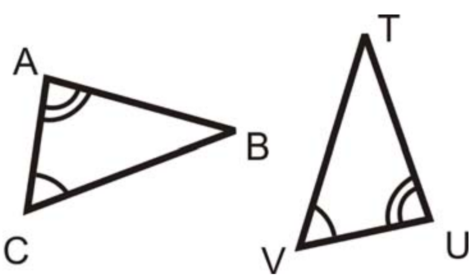


Because $\angle F''D''E'' \cong \angle CAB$ and $\angle F''E''D'' \cong \angle CBA$, the triangles must match up exactly (in particular, F''' must map to C), and the triangles are congruent.

This means that even though you didn't know all the side lengths and angle measures, because you knew two pairs of angles and the included sides were congruent, the triangles had to be congruent overall. At this point you can use the ASA criterion for showing triangles are congruent without having to go through all of these transformations each time (but make sure you can explain why ASA works in terms of the rigid transformations!).

Example D

What information would you need to prove that these two triangles are congruent using the ASA Postulate?



a) $\overline{AB} \cong \overline{UT}$

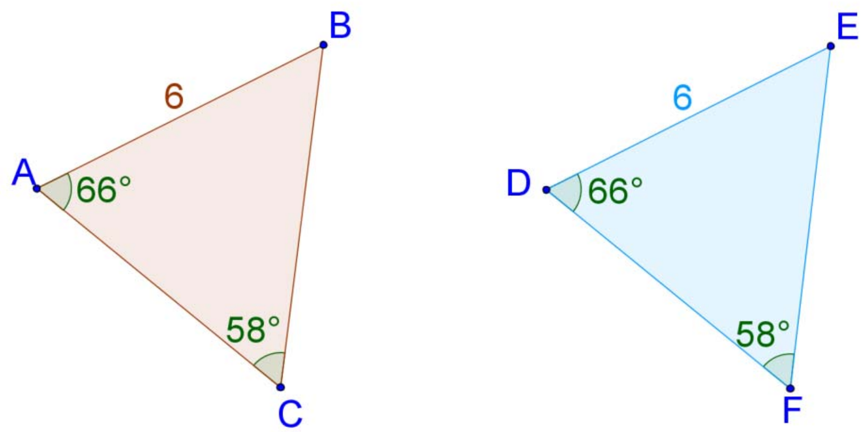
b) $\overline{AC} \cong \overline{UV}$

c) $\overline{BC} \cong \overline{TV}$

d) $\angle B \cong \angle T$

For ASA, we need the side between the two given angles, which is $\overline{AC}$ and $\overline{UV}$. The answer is b.

Concept Problem Revisited



Because the three angles of a triangle always have a sum of 180° , $m\angle B = 56^\circ$ and $m\angle E = 56^\circ$. Therefore, the triangles are congruent by ASA due to the fact that $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$.

Vocabulary

ASA, or Angle-Side-Angle is a criterion for triangle congruence **.** If two triangles have two pairs of congruent angles and the common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.

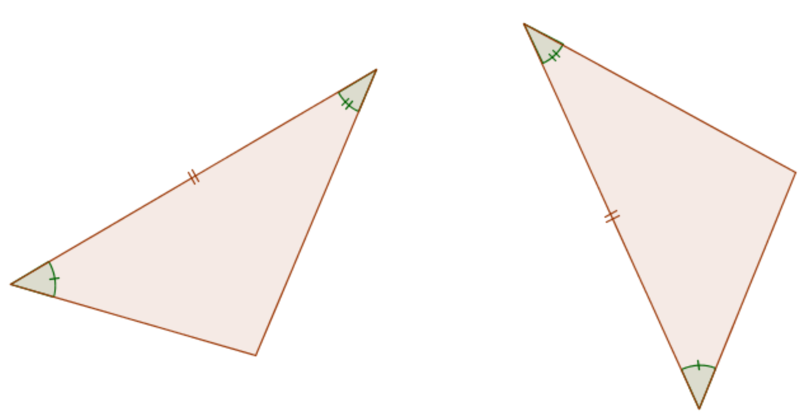
Rigid transformations are transformations that preserve distance and angles. The rigid transformations are reflections, rotations, and translations.

Two figures are ***congruent*** if a sequence of rigid transformations will carry one figure to the other. ***Congruent figures*** will always have corresponding angles and sides that are congruent as well.

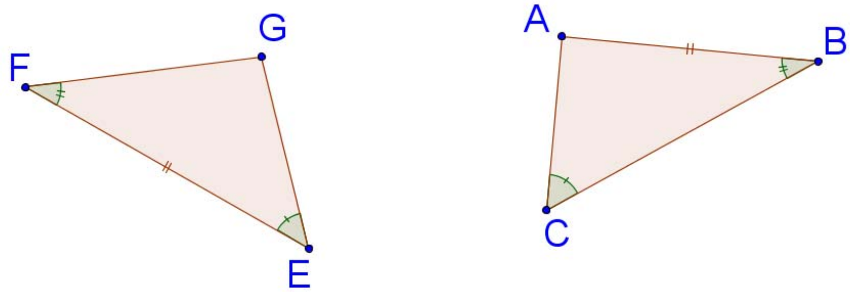
Guided Practice

Are the following triangles congruent? Explain.

1.



2.



Answers:

1. The triangles are congruent by ASA.

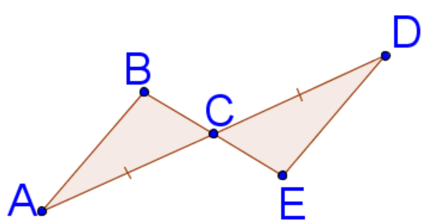
2. The triangles are not necessarily congruent. The information for ΔABC is AAS while the information for ΔEFG is ASA. There is not enough information about corresponding sides that are congruent.

Practice

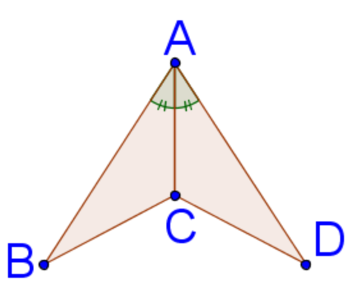
1. What does ASA stand for? How is it used?

2. Draw an example of two triangles that must be congruent due to ASA.

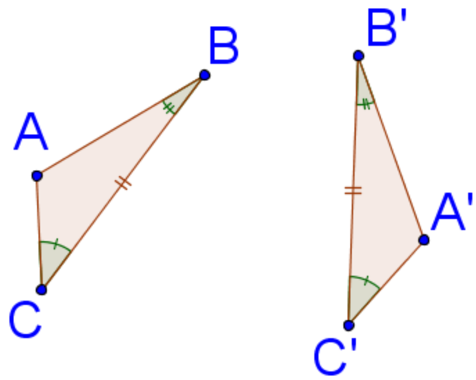
3. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by ASA? Assume that points B , C , and E are collinear.



4. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by ASA?



5. Show how the ASA criterion for triangle congruence works: use rigid transformations to help explain why the triangles below are congruent.



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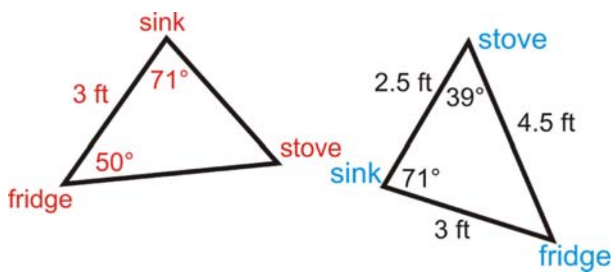
CONCEPT 5 SLT 27 Explore and apply Angle Angle Side (AAS) criteria to prove triangle congruence.

What if your parents changed their minds at the last second about their kitchen layout? Now, they have decided they to have the distance between the sink and the fridge be 3 ft, the angle at the sink 71° and the angle at the fridge is 50° . You used your protractor to measure the angle at the stove and sink at your neighbor's house. Are the kitchen triangles congruent now? After completing this Concept, you'll be able to use a congruence shortcut to help you answer this question.



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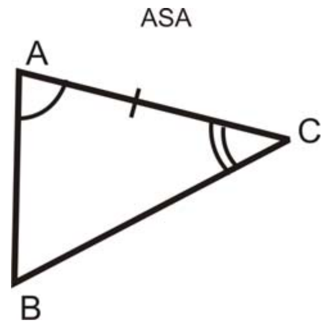
CK-12 Foundation: Chapter4ASAandAASTriangleCongruenceA

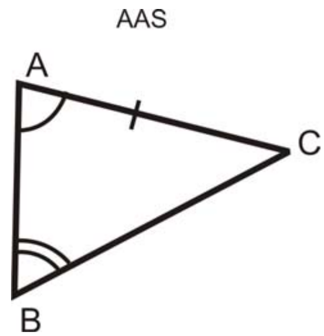
Guidance

"Angle-angle-side" or "AAS" is another criterion for triangle congruence that directly follows from ASA.

The AAS criterion for triangle congruence states that if two triangles have two pairs of congruent angles and a non- common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.

A variation on ASA is AAS, which is Angle-Angle-Side. Recall that for ASA you need two angles and the side between them. But, if you know two pairs of angles are congruent, then the third pair will also be congruent by the Third Angle Theorem. Therefore, you can prove a triangle is congruent whenever you have any two angles and a side.





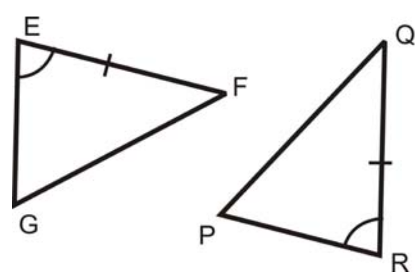
Be careful to note the placement of the side for ASA and AAS. As shown in the pictures above, the side is ***between*** the two angles for ASA and it is not for AAS.

Example A

What information do you need to prove that these two triangles are congruent using:

a) ASA?

b) AAS?



a) For ASA, we need the angles on the other side of $\overline{EF}$ and $\overline{QR}$. Therefore, we would need $\angle F \cong \angle Q$.

b) For AAS, we would need the angle on the other side of $\angle E$ and $\angle R$. $\angle G \cong \angle P$.

Concept Problem Revisited

Even though we do not know all of the angle measures in the two triangles, we can find the missing angles by using the Third Angle Theorem. In your parents' kitchen, the missing angle is 39° . The missing angle in your neighbor's kitchen is 50° . From this, we can conclude that the two kitchens are now congruent, either by ASA or AAS.

Vocabulary

Two figures are ***congruent*** if they have exactly the same size and shape. By definition, two triangles are ***congruent*** if the three corresponding angles and sides are congruent. The symbol $\cong$ means congruent. There are shortcuts for proving that triangles are congruent.

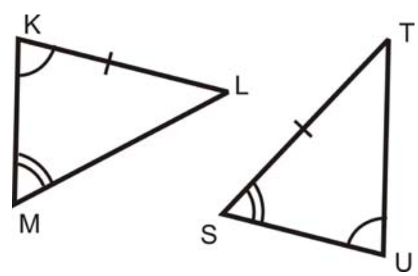
The ***ASA Triangle Congruence Postulate*** states that if two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the two triangles are congruent.

The ***AAS Triangle Congruence Theorem*** states that if two angles and a non-included side in one triangle are congruent to two corresponding angles and a non-included side in another triangle, then the triangles are congruent.

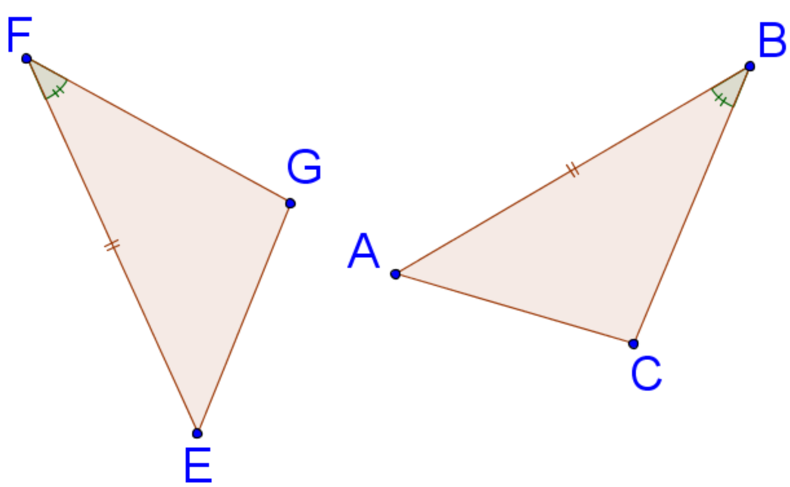
CPCTC refers to ***Corresponding Parts of Congruent Triangles are Congruent*** . It is used to show two sides or two angles in triangles are congruent after having proved that the triangles are congruent.

Guided Practice

1. Can you prove that the following triangles are congruent? Why or why not?



2. What additional information would you need in order to be able to state that the triangles below are congruent by AAS?



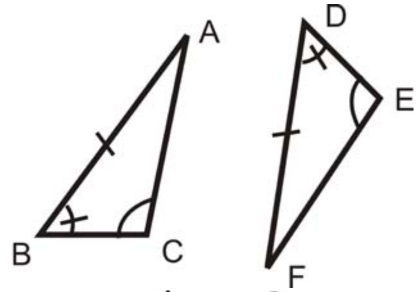
Answers:

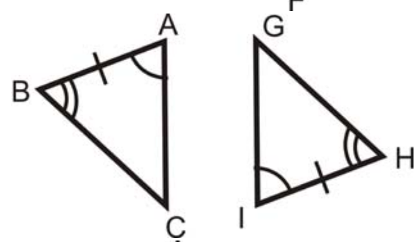
1. Even though $\overline{KL} \cong \overline{ST}$, they are not corresponding. Look at the angles around $\overline{KL}$, $\angle K$ and $\angle L$. $\angle K$ has **one** arc and $\angle L$ is unmarked. The angles around $\overline{ST}$ are $\angle S$ and $\angle T$. $\angle S$ has **two** arcs and $\angle T$ is unmarked. In order to use AAS, $\angle S$ needs to be congruent to $\angle K$. They are not congruent because the arcs marks are different. Therefore, we cannot conclude that these two triangles are congruent.

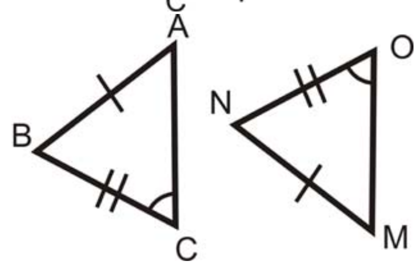
2. You would need to know that $\angle G \cong \angle C$.

Practice

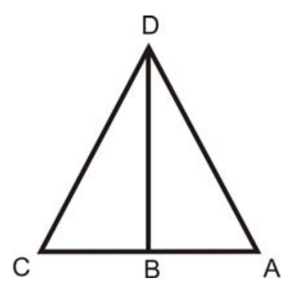
For questions 1-3, determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.

1. 

2. 

3. 

For questions 4-6, use the picture and the given information below.

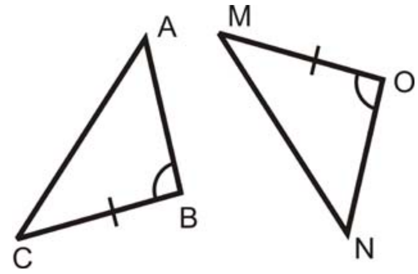


Given : $\overline{DB} \perp \overline{AC}$, $\overline{DB}$ is the angle bisector of $\angle CDA$

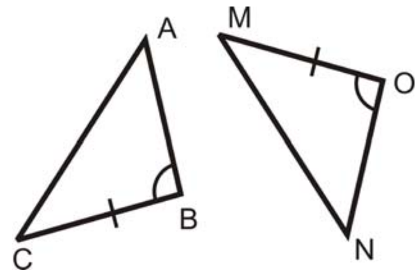
4. From $\overline{DB} \perp \overline{AC}$, which angles are congruent and why?
5. Because $\overline{DB}$ is the angle bisector of $\angle CDA$, what two angles are congruent?
6. From looking at the picture, what additional piece of information are you given? Is this enough to prove the two triangles are congruent?

Determine the additional piece of information needed to show the two triangles are congruent by the given postulate.

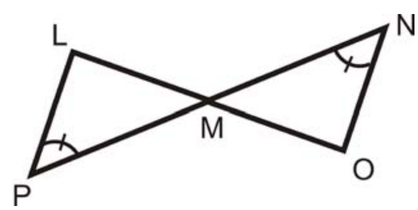
7. AAS



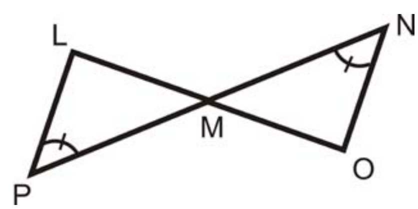
8. ASA



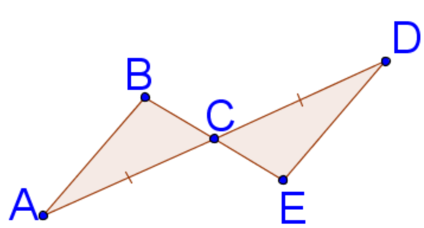
9. ASA



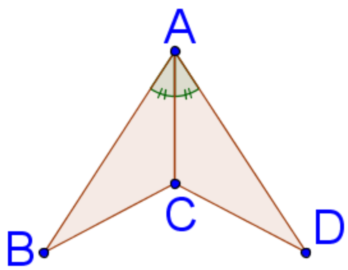
10. AAS



11. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by AAS? Assume that points B, C, and E are collinear.



12. What is the minimum additional information you would need in order to be able to state that the triangles below are congruent by AAS?



13. If you can show that two triangles are congruent with AAS, can you also show that they are congruent with ASA?

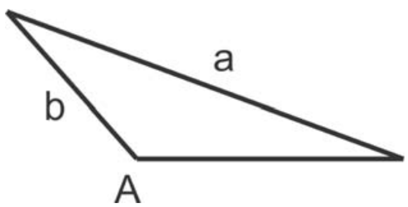
CONCEPT 6 SLT 28 SSA and AA are not sufficient criteria to prove triangles congruent.

Side Side Angle (SSA) Criteria

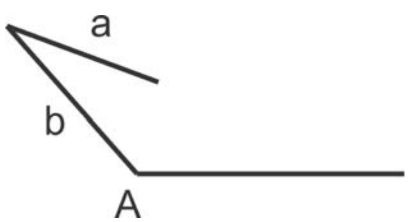
Side Side Angle criteria is not sufficient to prove triangle congruence. Depending on the lengths of two sides and the measure of the angle, one, two, or no triangle may be formed. Consider the given angle measures and relationships between the given sides to see if any triangle can be formed.

First, consider when A is **obtuse** :

If $a > b$, then **one** triangle can be formed.

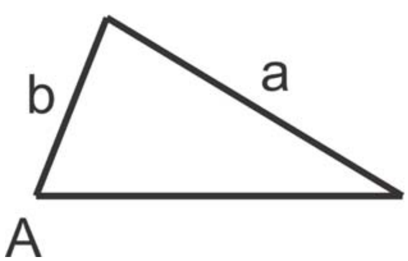


If $a \leq b$, then **no** triangle can be formed.



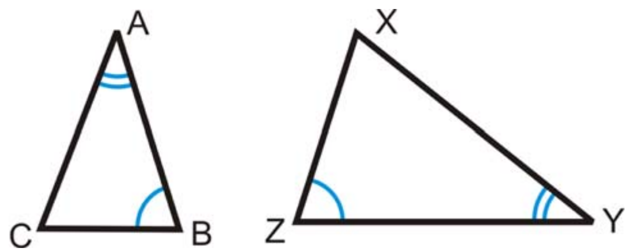
Now, consider the possible scenarios when A is **acute** .

If $a > b$, the **one** triangle can be formed.



Angle Angle (AA) Criteria

Angle Angle (AA) criteria is not sufficient to prove triangle congruence. Notice that in each pair the figures look the same, but one is smaller than the other. Since they are not the same size, they are not congruent. They do have corresponding features, but only their corresponding angles are congruent; the corresponding sides are not.



Hypotenuse-Leg (HL) criteria

What if you were given two right triangles and provided with only the measure of their hypotenuses and one of their legs? How could you determine if the two right triangles were congruent?

Watch This

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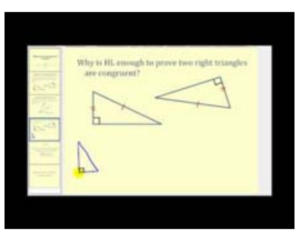
CK-12 HL Triangle Congruence



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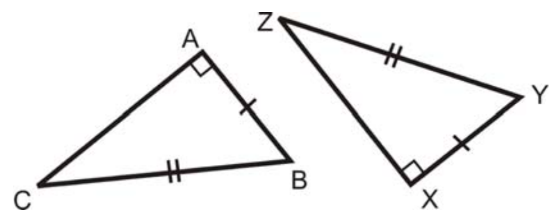
James Sousa: Hypotenuse-Leg Congruence Theorem

Guidance

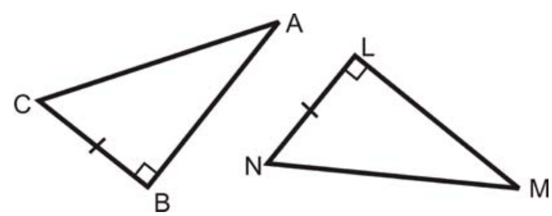
If the hypotenuse and leg in one ***right*** triangle are congruent to the hypotenuse and leg in another ***right*** triangle, then the two triangles are congruent. This is called the **Hypotenuse-Leg (HL) Congruence Theorem** . Note that it will only work for **right** triangles.

If $\triangle ABC$ and $\triangle XYZ$ are both right triangles and $\overline{AB} \cong \overline{XY}$ and $\overline{BC} \cong \overline{YZ}$ then $\triangle ABC \cong \triangle XYZ$.



Example A

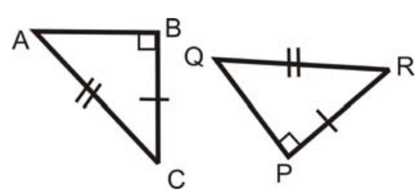
What additional information would you need to prove that these two triangles were congruent using the HL Theorem?



For HL, you need the hypotenuses to be congruent. $\overline{AC} \cong \overline{MN}$.

Example B

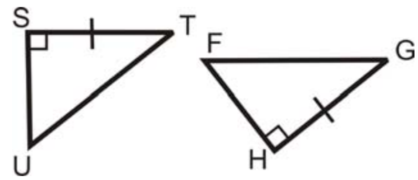
Determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.



We know the two triangles are right triangles. The have one pair of legs that is congruent and their hypotenuses are congruent. This means that $\triangle ABC \cong \triangle RQP$ by HL.

Example C

Determine the additional piece of information needed to show the two triangles are congruent by HL.



We already know one pair of legs is congruent and that they are right triangles. The additional piece of information we need is that the two hypotenuses are congruent, $\overline{UT} \cong \overline{FG}$.

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CK-12 HL Triangle Congruence

Guided Practice

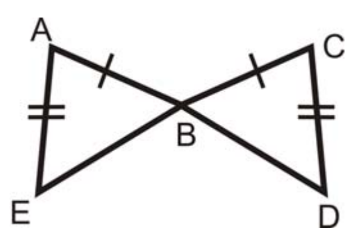
Use the given side lengths and angle measure to determine whether zero, one or two triangles exists.

1. $m\angle A = 100^\circ, a = 3, b = 4$.

2. $m\angle A = 50^\circ, a = 8, b = 10$.

3. $m\angle A = 72^\circ, a = 7, b = 6$.

4. Determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.



5. Explain why the HL Congruence shortcut works.

Answers:

1. Since A is obtuse and $a \leq b$, no triangle can be formed.

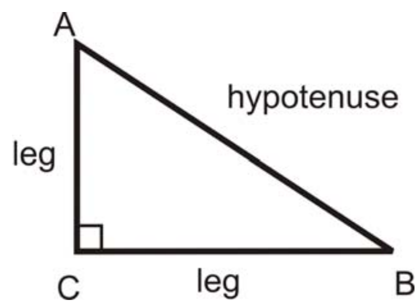
2. Since A is acute, a and $b \sin A$, two triangles can be formed.

3. Since A is acute and $a > b$, there is one possible triangle.

4. All we know is that two pairs of sides are congruent. Since we do not know if these are right triangles, we cannot use HL. We do not know if these triangles are congruent.

5. The Pythagorean Theorem, which says, for any ***right*** triangle, this equation is true:

$$(leg)^2 + (leg)^2 = (hypotenuse)^2$$



What this means is that if you are given two sides of a right triangle, you can always find the third. Therefore, if you know two sides of a right triangle are congruent to two sides of another right triangle, then you can conclude that the third sides are also congruent. If three pairs of sides are congruent, then we know the triangles are congruent by SSS.

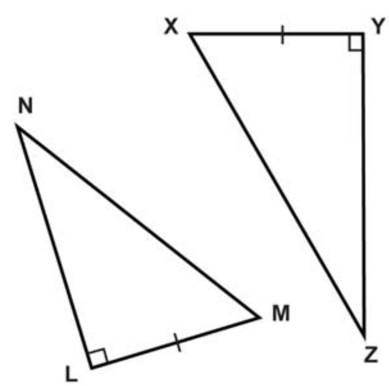
Practice

For problems 1-5, use the rules to determine if there will be one, two or no possible triangle with the given measurements.

1. $m\angle A = 65^\circ, a = 10, b = 11$
2. $m\angle A = 25^\circ, a = 8, b = 15$
3. $m\angle A = 100^\circ, a = 6, b = 4$
4. $m\angle A = 75^\circ, a = 25, b = 30$
5. $m\angle A = 48^\circ, a = 41, b = 50$

Using the HL Theorem, what information do you need to prove the two triangles are congruent?

6.



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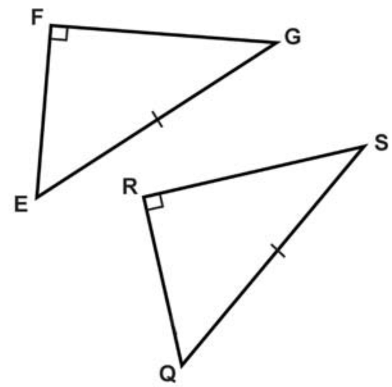

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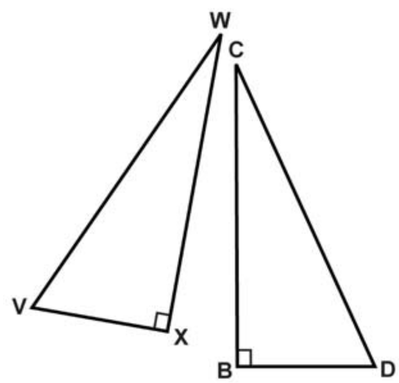


Based on the following details, are the two right triangles definitively congruent?

9. The hypotenuses of two right triangles are congruent.

10. Both sets of legs in the two right triangles are congruent.

11. One set of legs are congruent in the two right triangles.

12. The hypotenuses and one pair of legs are congruent in the two right triangles.

13. One of the non right angles of the two right triangles is congruent.

14. All of the angles of the two right triangles are congruent.

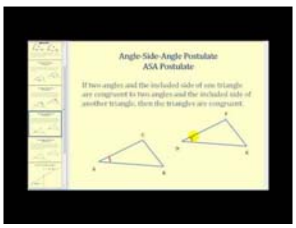
15. All of the sides of the two right triangles are congruent.

16. Both triangles have one leg that is twice the length of the other.

CONCEPT 7 SLT 29 & 30 Use triangle congruence to evaluate if two triangles are congruent.

Max constructs a triangle in *Geogebra* . He tells Alicia that his triangle has a 42° angle, a side of length 12, and a side of length 8. With only this information, will Alicia be able to construct a triangle that must be congruent to Max's triangle?

Watch This



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Click image to the left for more content.

http://www.youtube.com/watch?v=CA1TvVRApkQ James Sousa: Introduction to Congruent Triangles

Guidance

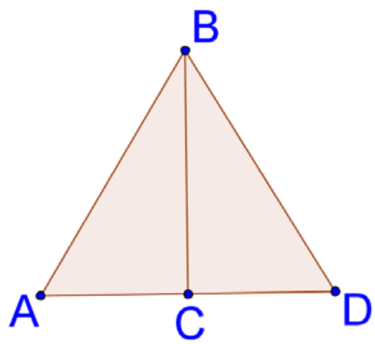
Two triangles are **congruent** if and only if corresponding pairs of sides and corresponding pairs are congruent. While one way to show that two triangles are congruent is to verify that all side and angle pairs are congruent, there are five "shortcuts". The following list summarizes the different criteria that can be used to show triangle congruence.

- **AAS (Angle-Angle-Side):** If two triangles have two pairs of congruent angles and a non-common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.
- **ASA (Angle-Side-Angle):** If two triangles have two pairs of congruent angles and the common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.
- **SAS (Side-Angle-Side):** If two triangles have two pairs of congruent sides and the included angle in one triangle is congruent to the included angle in the other triangle, then the triangles are congruent.
- **SSS (Side-Side-Side):** If two triangles have three pairs of congruent sides, then the triangles are congruent.
- *[FOR RIGHT TRIANGLES]* **HL (Hypotenuse-Leg):** If two ***right*** triangles have one pair of legs congruent and hypotenuses congruent, then the triangles are congruent.

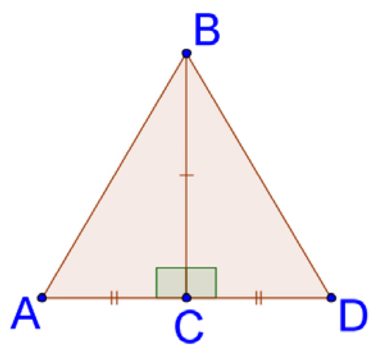
If two triangles don't satisfy at least one of the criteria above, you cannot be confident that they are congruent.

Example A

$\overline{BC}$ is the perpendicular bisector of $\overline{AD}$. Is $\triangle ABC \cong \triangle ADC$?



Solution: If $\overline{BC}$ is the perpendicular bisector of $\overline{AD}$, then $\overline{AC} \cong \overline{CD}$. Also, $m\angle ACB = 90^\circ$ and $m\angle DCB = 90^\circ$, so $\angle ACB \cong \angle DCB$. You also know that $\overline{BC}$ is a side of both triangles, and is clearly congruent to itself (this is called the **reflexive property**).



The triangles are congruent by SAS. *Note that even though these are right triangles, you would not use HL to show triangle congruence in this case since you are not given that the hypotenuses are congruent.*

Example B

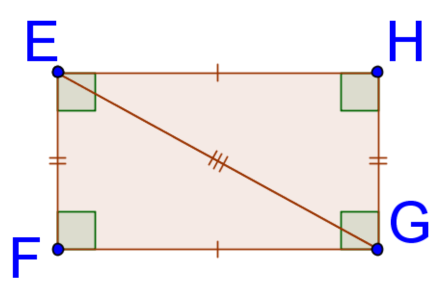
Using the information from Example A, if $m\angle A = 50^\circ$, what is $m\angle D$?

Solution: $m\angle D = 50^\circ$. Since the triangles are congruent, all of their corresponding angles and sides must be congruent. $\angle A$ and $\angle D$ are corresponding angles, so $\angle A \cong \angle D$.

Example C

Does one diagonal of a rectangle divide the rectangle into congruent triangles?

Solution: Recall that a rectangle is a quadrilateral with four right angles. The opposite sides of a rectangle are congruent.



There is more than enough information to show that $\triangle EFG \cong \triangle GHE$.

- **Method #1:** The triangles have three pairs of congruent sides, so they are congruent by SSS.
- **Method #2:** The triangles have two pairs of congruent sides and congruent included angles, so they are congruent by SAS.
- **Method #3:** The triangles are right triangles with congruent hypotenuses and a pair of congruent legs, so they are congruent by HL.

Concept Problem Revisited

Max constructs a triangle in Geogebra. He tells Alicia that his triangle has a 42° angle, a side of length 12, and a side of length 8. If Max also told Alicia that the angle was in between the two sides, then she would be able to construct a triangle that must be congruent due to SAS. If the angle is not between the two sides, she cannot be confident that her triangle is congruent because SSA is not a criterion for triangle congruence. Because Max did not state where the angle was in relation to the sides, Alicia cannot create a triangle that must be congruent to Max's triangle.

Vocabulary

AAS (Angle-Angle-Side): If two triangles have two pairs of congruent angles and a non-common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.

ASA (Angle-Side-Angle): If two triangles have two pairs of congruent angles and the common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.

SAS (Side-Angle-Side): If two triangles have two pairs of congruent sides and the included angle in one triangle is congruent to the included angle in the other triangle, then the triangles are congruent.

SSS (Side-Side-Side) : If two triangles have three pairs of congruent sides, then the triangles are congruent.

HL (Hypotenuse-Leg): If two ***right*** triangles have one pair of legs congruent and hypotenuses congruent, then the triangles are congruent.

Rigid transformations are transformations that preserve distance and angles. The rigid transformations are reflections, rotations, and translations.

Two figures are ***congruent*** if a sequence of rigid transformations will carry one figure to the other. ***Congruent figures*** will always have corresponding angles and sides that are congruent as well.

The ***perpendicular bisector*** of a segment is a line that intersects the segment at its midpoint and meets the segment at a right angle.

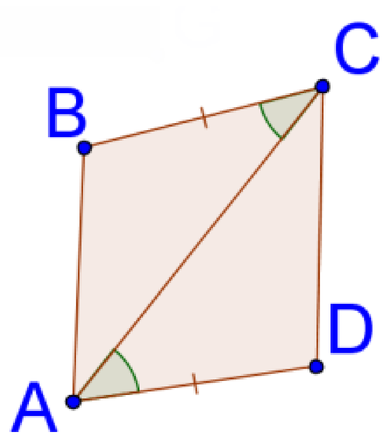
The ***midpoint*** is a point directly in between the two endpoints of the segment. It divides the segment into two congruent segments.

The ***reflexive property*** states that an object or quantity is equal to itself.

Guided Practice

For each pair of triangles, tell whether the given information is enough to show that the triangles are congruent. If the triangles are congruent, state the criterion that you used to determine the congruence and write a congruency statement. *Note that the figures are not necessarily drawn to scale!*

1.



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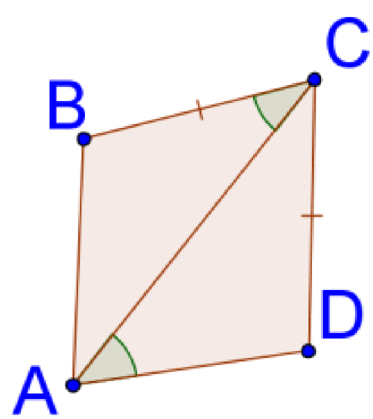
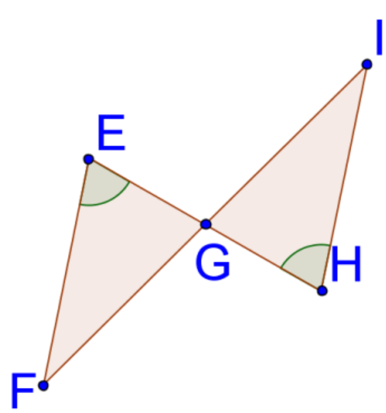


3. G is the midpoint of $\overline{EH}$.



Answers:

1. Notice that besides the one pair of congruent sides and the one pair of congruent angles, $\overline{AC} \cong \overline{CA}$. $\triangle ACB \cong \triangle CAD$ by SAS.
2. The congruent sides are not corresponding in the same way that the congruent angles are corresponding. The given information for $\triangle ACB$ is SAS while the given information for $\triangle CAD$ is SSA. The triangles are not necessarily congruent.
3. Because G is the midpoint of $\overline{EH}$, $\overline{EG} \cong \overline{GH}$. You also know that $\angle EGF \cong \angle HGI$ because they are vertical angles. $\triangle EGF \cong \triangle HGI$ by ASA.

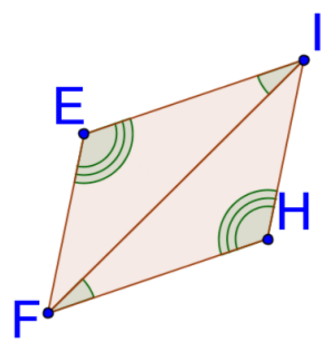
Practice

1. List the five criteria for triangle congruence and draw a picture that demonstrates each.

2. Given two triangles, do you always need at least three pieces of information about each triangle in order to be able to state that the triangles are congruent?

For each pair of triangles, tell whether the given information is enough to show that the triangles are congruent. If the triangles are congruent, state the criterion that you used to determine the congruence and write a congruency statement.

3.



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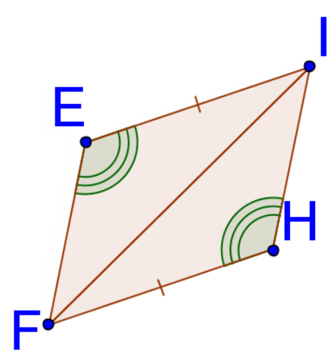

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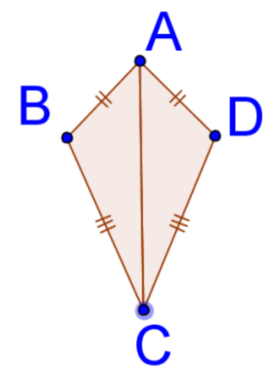

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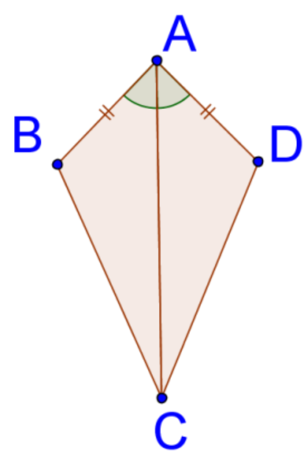

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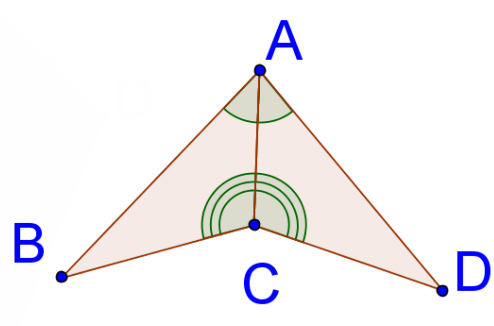
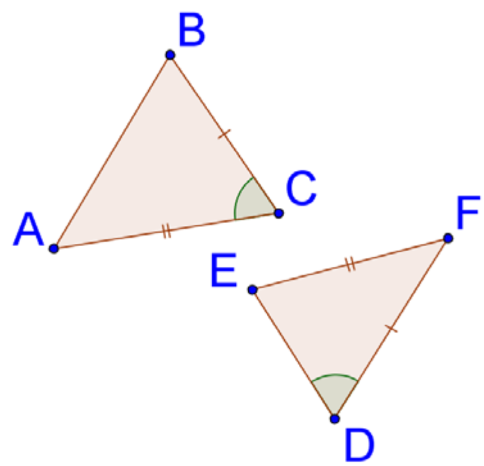


8.



For 9-11, state whether the given information about a hidden triangle would be enough for you to construct a triangle that must be congruent to the hidden triangle. Explain your answer.

9. $\triangle ABC$ with $m\angle A = 72^\circ$, $AB = 6\ cm$, $BC = 8\ cm$.

10. $\triangle ABC$ with $m\angle A = 90^\circ$, $AB = 4\ cm$, $BC = 5\ cm$.

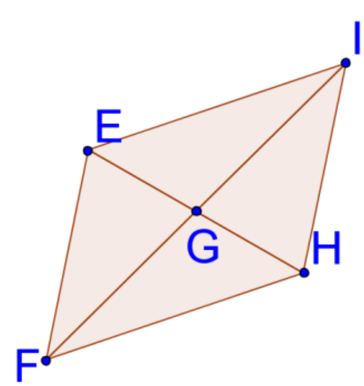
11. $\triangle ABC$ with $m\angle A = 72^\circ$, $AB = 6\ cm$, $AC = 8\ cm$.

12. Recall that a square is a quadrilateral with four right angles and four congruent sides. Show and explain why a diagonal of a square divides the square into two congruent triangles.

13. Show and explain using a different criterion for triangle congruence why a diagonal of a square divides the square into two congruent triangles.

14. Recall that a kite is a quadrilateral with two pairs of adjacent, congruent sides. Will one of the diagonals of a kite divide the kite into two congruent triangles? Show and explain your answer.

15. In the picture below, G is the midpoint of both $\overline{EH}$ and $\overline{FI}$. Explain why $\overline{FH} \cong \overline{IE}$ and $\overline{FE} \cong \overline{HI}$.



16. Explain why AAA is not a criterion for triangle congruence.

References

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