CK-12 FlexBook

MCPS C2.0 Geometry Unit 1 Topic 2 FlexBook

MCPS

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

AUTHOR

MCPS

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook®**, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform®**.

Printed: May 20, 2014

Contents

CONCEPT 1 SLT 7 Define and identify examples and non-examples of rotations, reflections, and translations.

In one of the rooms of the museum, this tile completely covered two of the walls. The students walked inside and began moving automatically because of the pattern on the wall.

"This tile makes me dizzy," Greg stated sitting on a nearby bench.

"Yes, it seems to move," Lane commented.

"I think it's cool. Look at all of the transformations are everywhere," Emma said smiling.

"Are you sure? You are beginning to sound like Mrs. Gilman," Lane said jokingly.

"I am sure, and thank you for the compliment," Emma said nudging her best friend.

Do you think Emma can see the transformations? Well, they are there. While sometimes a pattern like this one can make you dizzy, there are many different transformations in pattern. Your task is to find them. Use what you learn in this lesson to revisit the problem and find the examples of transformations in the tile pattern.

Guidance

In this Concept, we will examine different kinds of transformations.

A *transformation* is the movement of a geometric figure.

There are three different kinds of transformations.

In a ***translation***, also called a slide, the figure moves left, right, up, or down.

In a ***reflection***, the figure flips. A reflection is like a mirror image of the original figure.

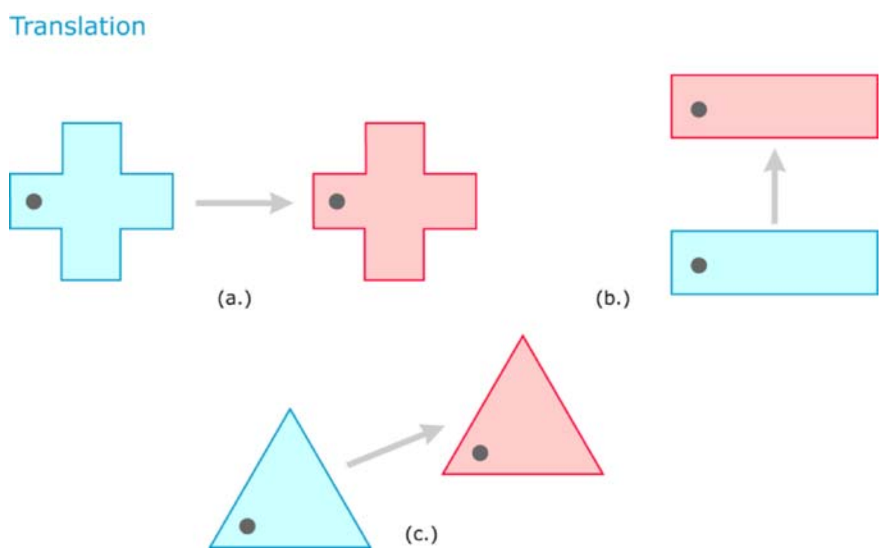
Finally, in a ***rotation***, the figure turns.

The key thing to remember is that in any transformation, the size and shape of the figure stay exactly the same, only its location changes or shifts.

Translation

The first type of transformation is called a ***translation.*** It is also known as a ***slide*** because the figure in question does exactly that. It moves up, down, to the left or to the right. **Nothing about the figure changes except its location.**

Here are some translations.

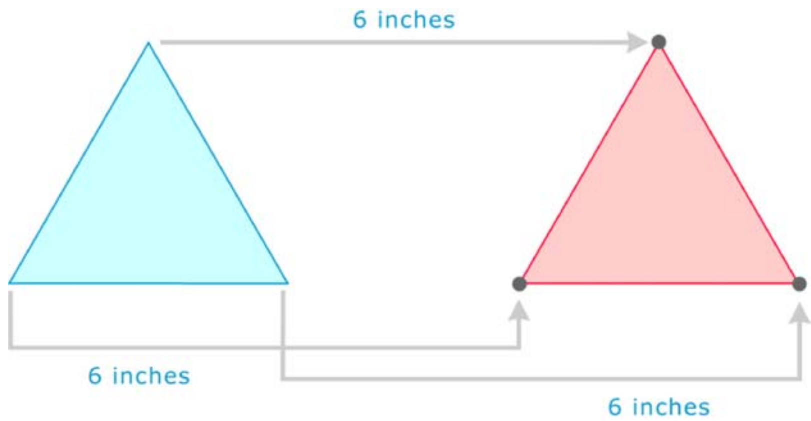


Notice when you look at each picture that all that changed for each figure was its location. There are different colors used to show you the actual translation, but other than the location, each looks exactly the same.

This is how you always know that you are working with a translation or a slide.

How do we perform a translation?

To perform a translation, we measure a distance and then redraw the figure. For example, let's move this triangle 6 inches.



We measure 6 inches from each point of the triangle and make a new point. This way, every part of the triangle moves 6 inches.

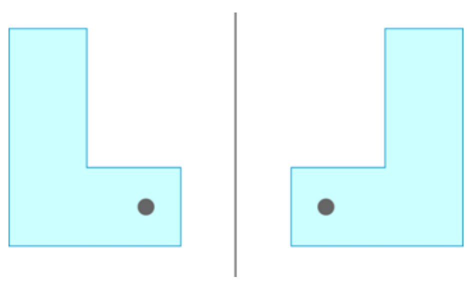
In this way, we can translate any figure in any direction for any distance.

Reflection

You have heard the word "reflection" all the time. From the reflection in a mirror to the reflection in a pond, reflections are all around us.

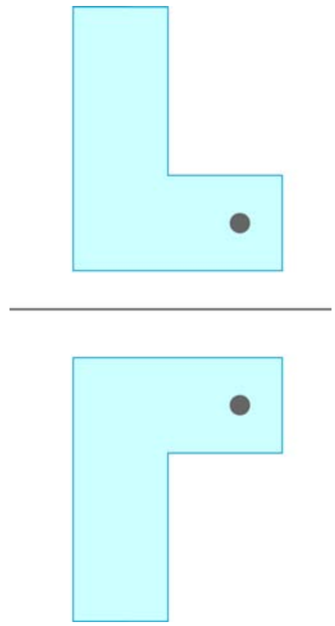
How do we apply the term reflection to geometry?

A ***reflection*** **is a different kind of transformation. In a reflection, the figure flips to make a mirror image of itself.** Take a look at the reflection below.



The line in the middle acts like a mirror. We call this ***the line of symmetry.*** This is a vertical line of symmetry. Imagine standing in front of a mirror and holding up your left hand. Where is your hand in the mirror's reflection? A reflected figure works the same way: when we flip it over the line, all of its points are reversed. When reflected, the figure above looks like a backwards L . Notice that, on both sides of the line, the dot is closest to the line.

We can also reflect figures across a horizontal line of symmetry. Then our reflection would look like this.

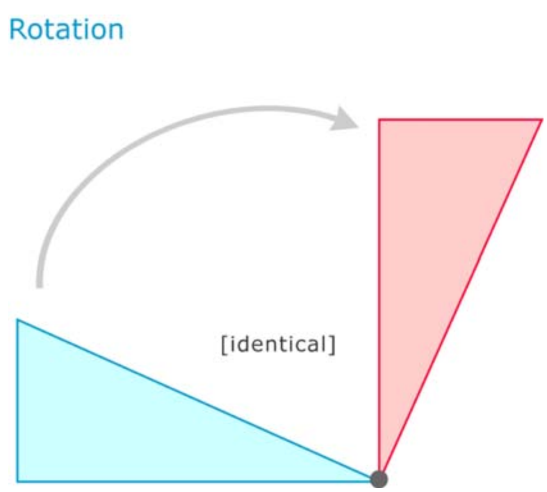


In this case, the "top" of the figure becomes the "bottom" in the reflection! Notice, however, that in both cases the figures are symmetrical.

Rotation

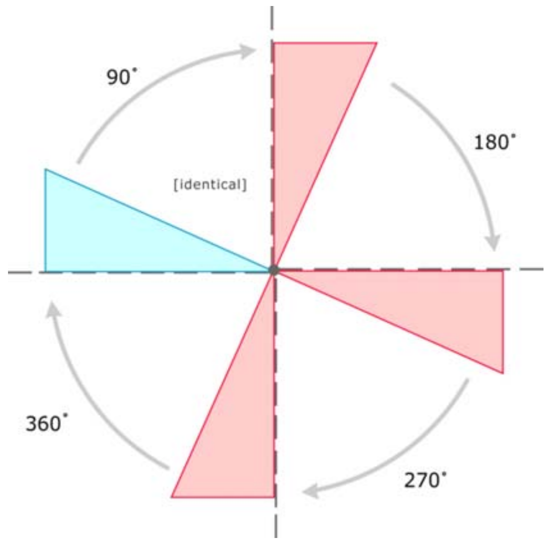
Now let's learn about the third kind of transformation. **A** ***rotation*** **is a transformation that turns the figure in either a clockwise or counterclockwise direction.**

How does the figure below change as it is rotated?



Imagine you could spin the figure around in a circle. It would not change, but might turn upside down.

Figures can rotate as much as 360° , a full circle. Let's see how that might look.



When we rotate the figure a full 360° , it ends up in the same place it began, unchanged!

Let's identify some transformations.

Example A

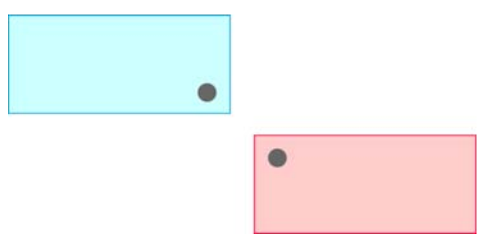
Solution: Reflection

Example B

A transformation with a line of symmetry must be ______________?

Solution: Reflection

Example C



Solution: Reflection

Here is the original problem once again.

In one of the rooms of the museum, this tile completely covered two of the walls. The students walked inside and began moving automatically because of the pattern on the wall.

"This tile makes me dizzy," Greg stated sitting on a nearby bench.

"Yes, it seems to move," Lane commented.

"I think it's cool. Look at all of the transformations are everywhere," Emma said smiling.

"Are you sure? You are beginning to sound like Mrs. Gilman," Lane said jokingly.

"I am sure, and thank you for the compliment," Emma said nudging her best friend.

In this tile pattern, make a note of a reflection. Make a note of a translation. Make a note of a figure that rotates or has rotational symmetry. Make a note of the line of symmetry.

When finished, compare your answers with a partner.

While you won't find the exact answer here, there are many ways to explore transformations and symmetry by using this pattern. Work with a partner or with your whole class to figure out the math in this tile pattern.

Vocabulary

Transformation
: when a figure moved on a plane. The figure doesn't change but it's position does.

Translation
: a slide. The figure moved up, down, to the right, or to the left or diagonally.

Reflection
: a flip. The figure flips over a line of symmetry, much like a reflection in a mirror.

Rotation
: a turn. The figure turns either clockwise or counterclockwise.

Line of Symmetry

the line that a figure reflects over. Also the line that divided a figure in half showing line symmetry.

Guided Practice

Here is one for you to try on your own.

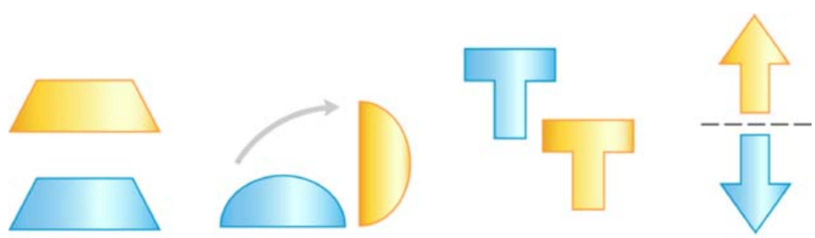
Could this figure be a rotation? Why or why not?

Answer

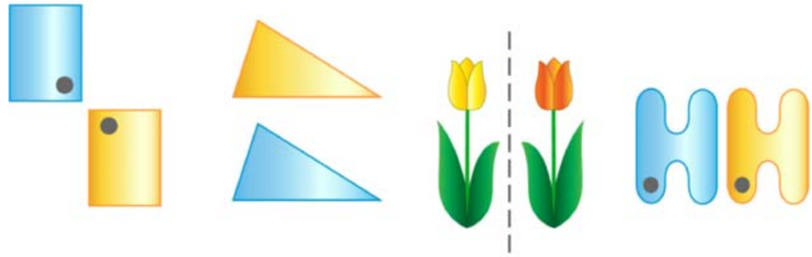
This figure is congruent in each and every way. This figure could be a rotation.

Practice

Directions: Identify the transformations shown below as translations, reflections, or rotations.

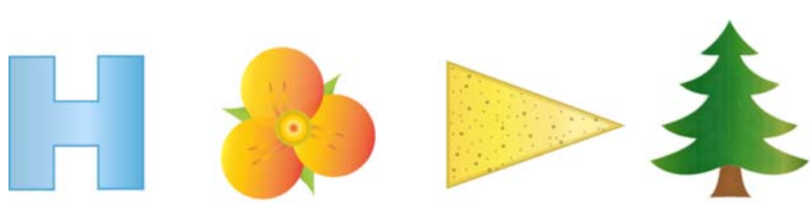


(1.) (2.) (3.) (4.)



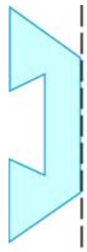
(5.) (6.) (7.) (8.)

Directions: Tell whether the figures below have line symmetry, rotational symmetry, both, or neither.

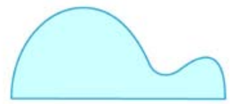


(9.) (10.) (11.) (12.)

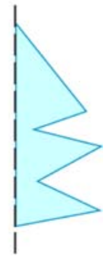
Directions: Draw the second half of each figure.



(13.)



(14.)



(15.)

Concept 2 SLT 8 Describe and draw horizontal and vertical translations.

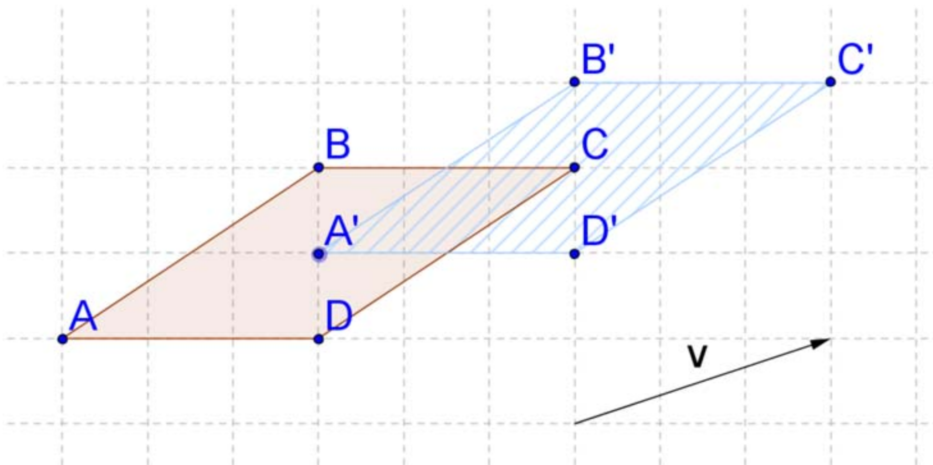
Translations are often informally called “slides”. Why is this?

Watch This

http://learnzillion.com/lessons/2574-explore-translations-by-investigating-their-effects-on-line-segments-and-angles LearnZillion: Explore translations by investigating their effects on line segments and angles

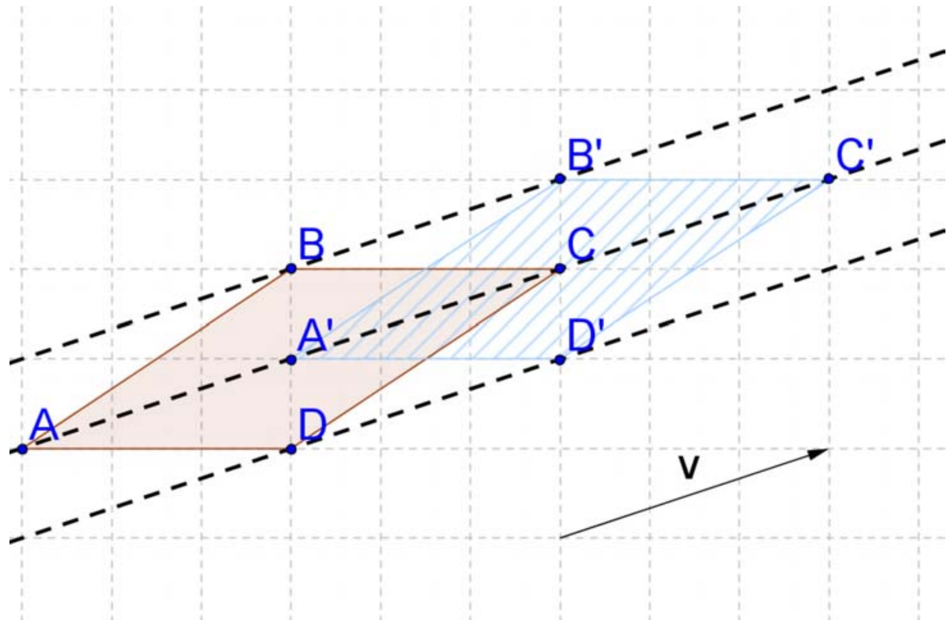
Guidance

A **translation** is one example of a **rigid transformation** . A translation moves each point in a shape a specified distance in a specified direction. Below, the parallelogram has been translated along $\vec{v}$ to create a new parallelogram (the image).



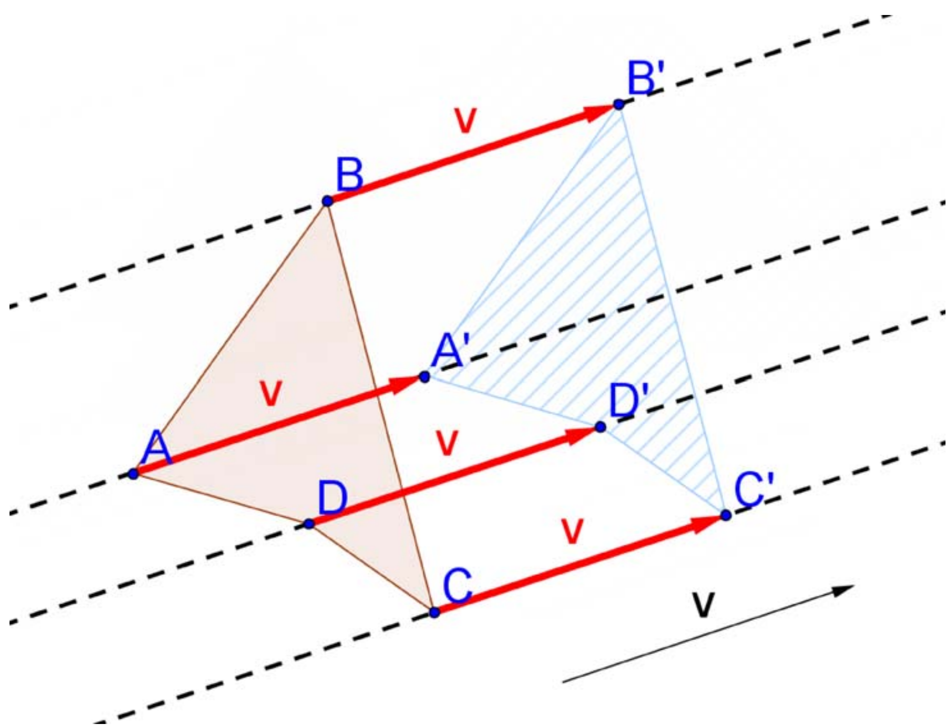
Keep in mind that the location of $\vec{v}$ **does not matter** . $\vec{v}$ essentially tells you that **all points move three units to the right and one unit up** .

The lines that connect corresponding points will all be parallel to $\vec{v}$.



Each point in the original parallelogram was moved 3 units to the right and 1 unit up to create its corresponding point in the image.

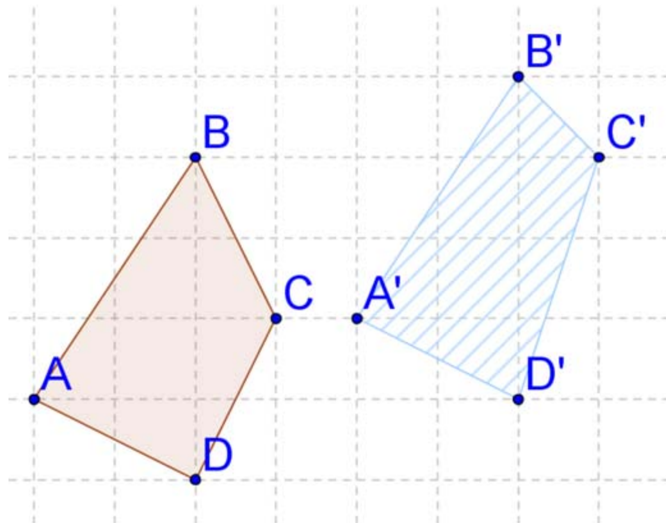
If you are performing a translation of a shape **not** on a grid, the direction of $\vec{v}$ becomes crucial. You can no longer say "move one unit up and three units to the right" because without a grid there are no units. Below is a translation of another quadrilateral **without** a grid in the background.



Notice that lines parallel to $\vec{v}$ have been drawn through each of the original points. $\vec{v}$ has been copied onto each of those lines at the points that define the original quadrilateral. The ends of $\vec{v}$ define the points on the image.

Example A

Is the following transformation a translation?



Solution: One way to check if a transformation is a translation is to look at how each point moves to create its image. If all points move in the same way, then it is a translation.

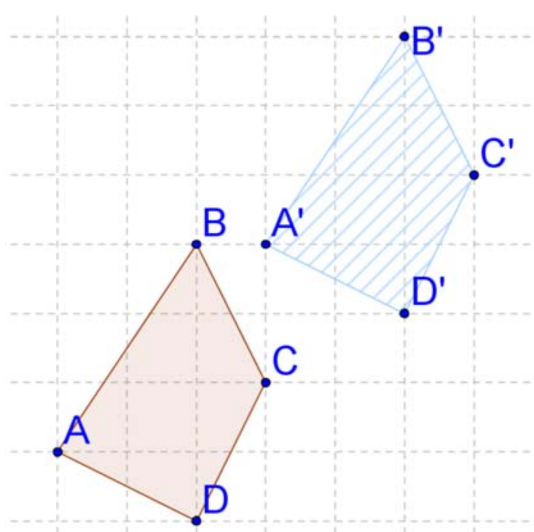
TABLE 2.1:

Point to Image Point	Description of Motion
A to A'	4 to the right and 1 up
B to B'	4 to the right and 1 up
C to C'	**4 to the right and 2 up**
D to D'	4 to the right and 1 up

Because C to C' is different, this is not a translation.

Example B

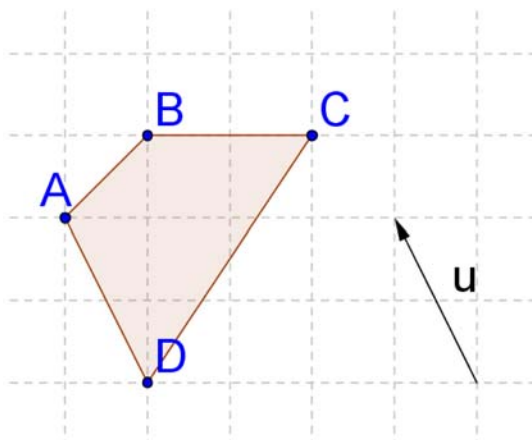
Describe the vector that defined the translation below.



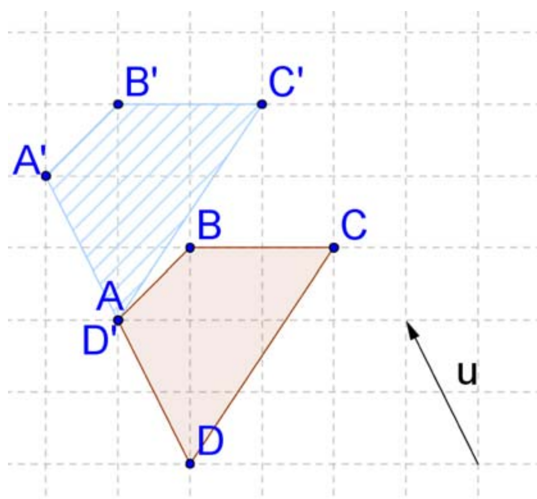
Solution: The vector moved each point three units to the right and three units up.

Example C

Perform the translation defined by $\vec{u}$ on the quadrilateral below.



Solution: With the grid in the background, you can see that $\vec{u}$ tells you to move each point 2 units up and 1 unit to the left. Here is the translation:



Concept Problem Revisited

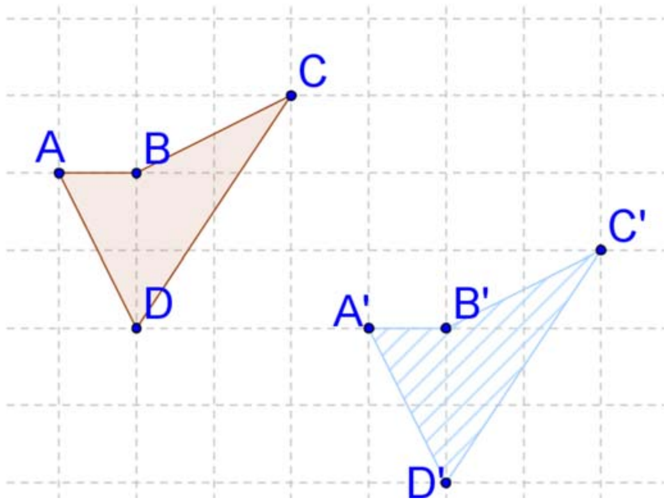
A translation is informally called a slide, because it essentially slides a shape to a new position. The orientation of the points does not change.

Vocabulary

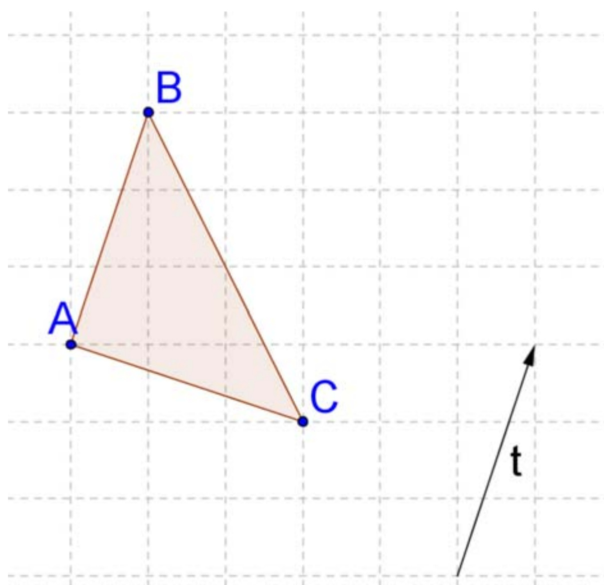
A ***translation*** is a rigid transformation that moves each point in a shape a specified distance in a specified direction as defined by a vector.

Guided Practice

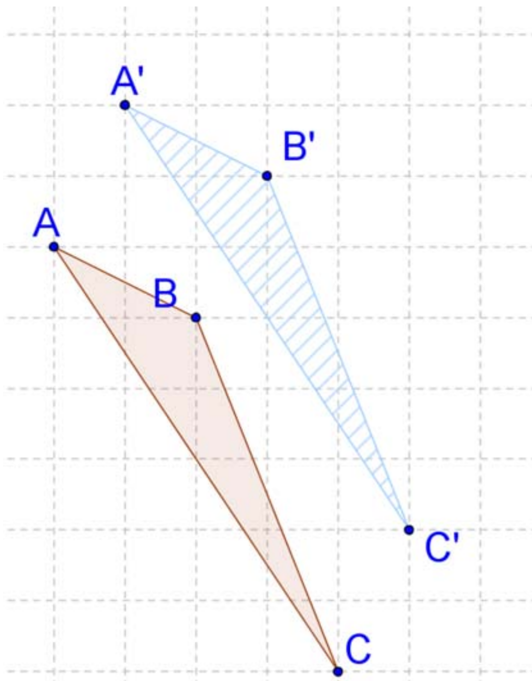
1. Describe the translation of quadrilateral ABCD below.



2. Perform the translation defined by $\vec{t}$ on the triangle below.



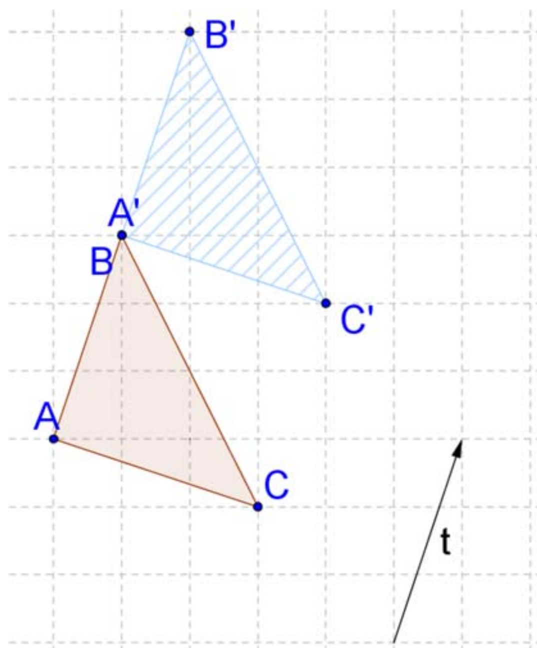
3. Is the following transformation a translation?



Answers:

1. The vector moved each point 4 units to the right and 2 units down.

2.



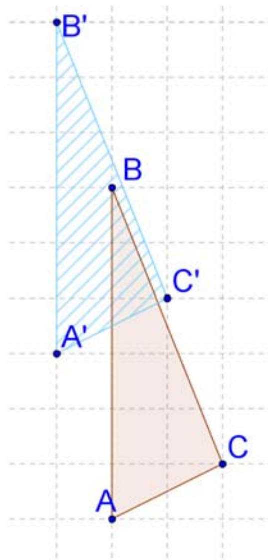
3. Yes, each point moves one unit to the right and two units up.

Practice

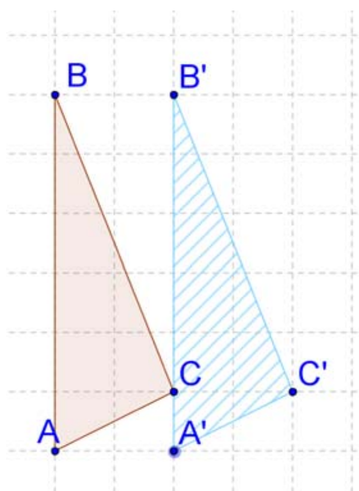
1. Is a translation a rigid transformation? Explain.
2. What role does direction play in a translation?
3. How are parallel lines relevant to translations?
4. How can you tell if a transformation is a translation?

Describe the following translations.

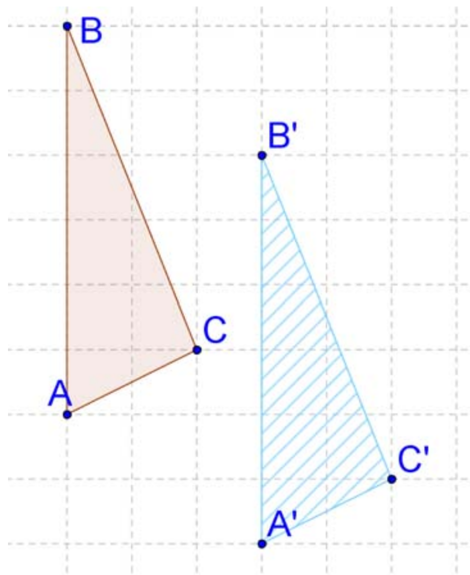
5.



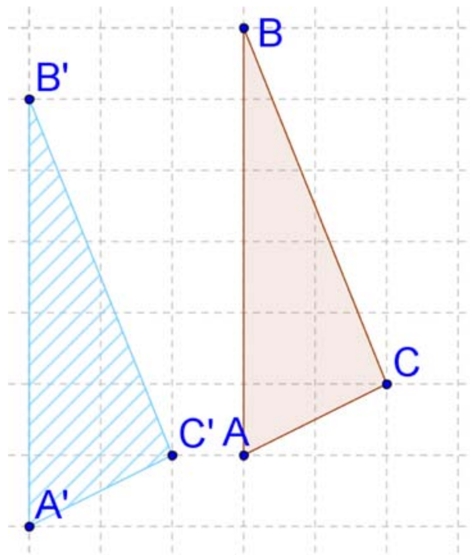
6.



7.

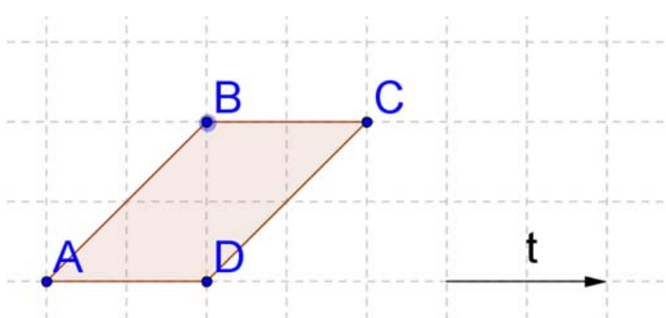


8.

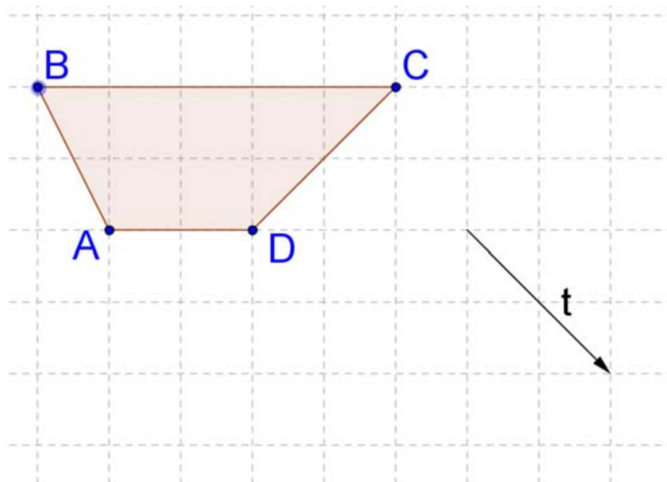


Perform the translation defined by $\vec{t}$ on the polygons below.

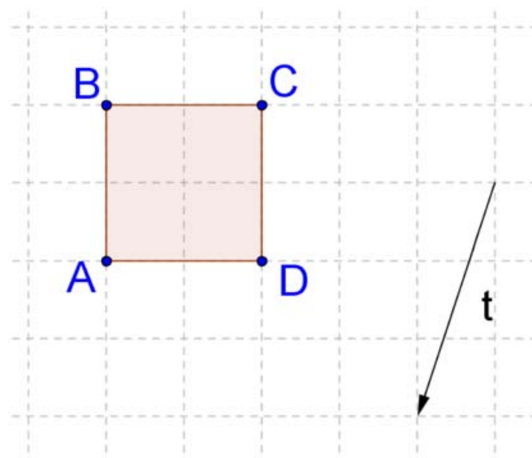
9.



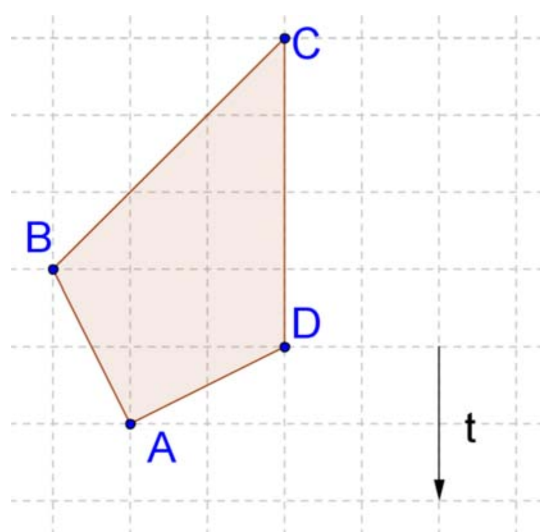
10.



11.

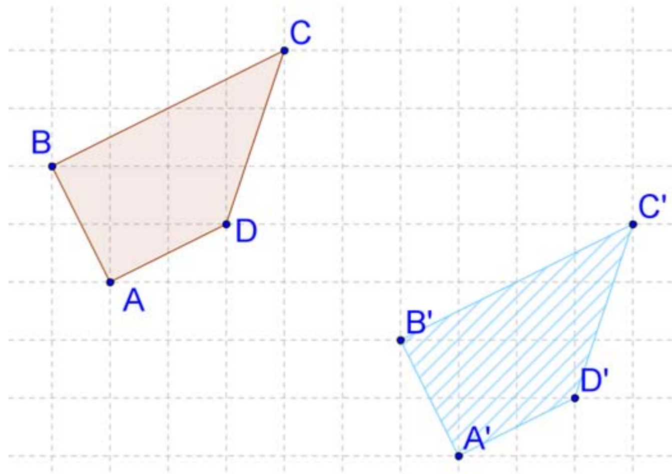


12.

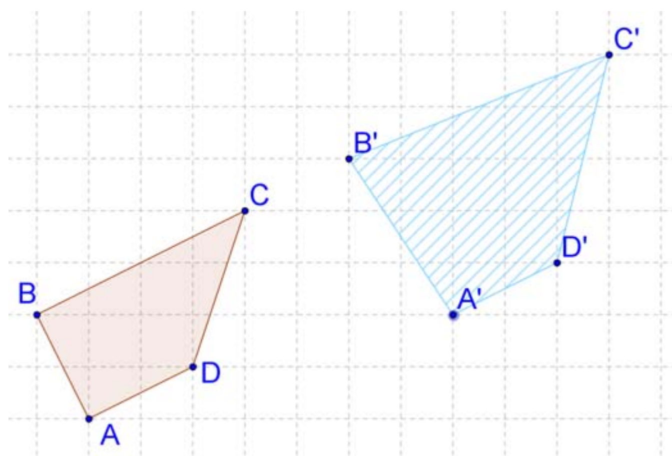


Are the following transformations translations? Explain.

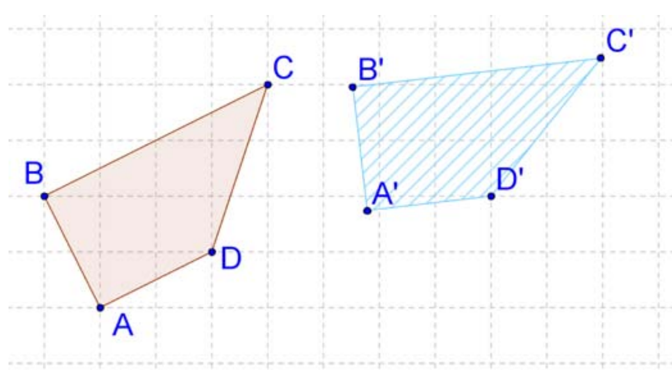
13.



14.



15.

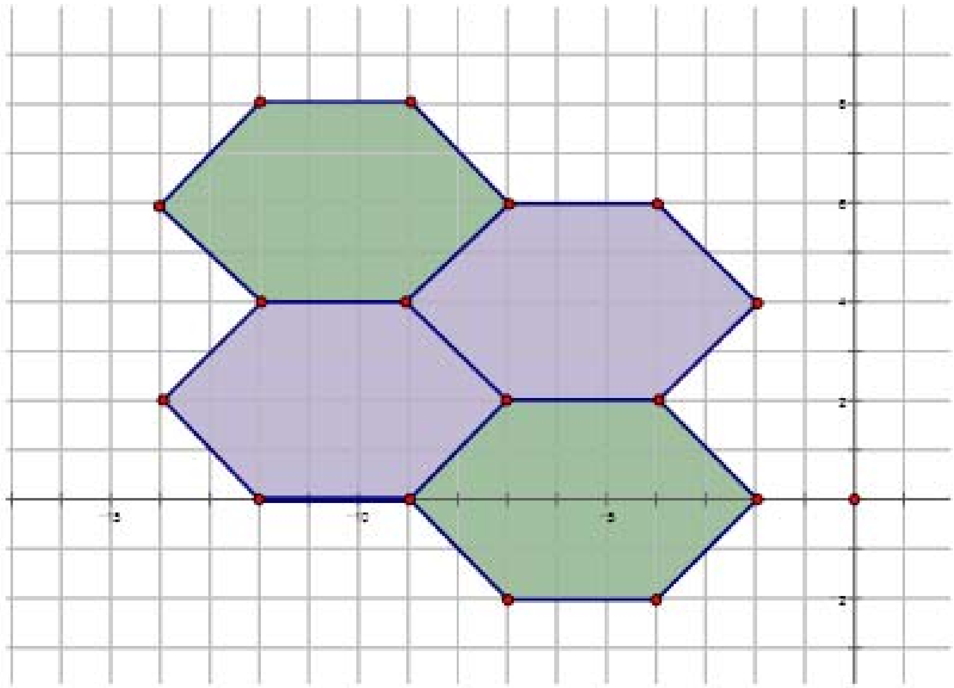


References

1. . . CC BY-NC-SA
2. . . CC BY-NC-SA
3. . . CC BY-NC-SA
4. . . CC BY-NC-SA
5. . . CC BY-NC-SA
6. . . CC BY-NC-SA
7. . . CC BY-NC-SA
8. . . CC BY-NC-SA
9. . . CC BY-NC-SA
10. . . CC BY-NC-SA
11. . . CC BY-NC-SA
12. . . CC BY-NC-SA
13. . . CC BY-NC-SA
14. . . CC BY-NC-SA
15. . . CC BY-NC-SA
16. . . CC BY-NC-SA
17. . . CC BY-NC-SA
18. . . CC BY-NC-SA
19. . . CC BY-NC-SA
20. . . CC BY-NC-SA
21. . . CC BY-NC-SA
22. . . CC BY-NC-SA

CONCEPT 3 SLT 9 Describe and draw horizontal and vertical translations and write the associated function.

The figure below shows a pattern of a floor tile. Write the mapping rule for the translation of the two blue floor tiles.



Watch This

First watch this video to learn about writing rules for translations.

MEDIA
Click image to the left for more content.

CK-12 Foundation Chapter10RulesforTranslationsA

Then watch this video to see some examples.

MEDIA
Click image to the left for more content.

CK-12 Foundation Chapter10RulesforTranslationsB

Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape (called the preimage) to create a new shape (called the image). A translation is a type of transformation that moves each point in a figure the same distance in the same direction. Translations are often referred to as slides. You can describe a translation using words like "moved up 3 and over 5 to the left" or with notation. There are two types of notation to know.

1. One notation looks like $T_{(3,\ 5)}$. This notation tells you to add 3 to the x values and add 5 to the y values.
2. The second notation is a mapping rule of the form $(x,y) \rightarrow (x-7,y+5)$. This notation tells you that the x and y coordinates are translated to $x-7$ and $y+5$.

The mapping rule notation is the most common.

Example A

Sarah describes a translation as point P moving from $P(-2,2)$ to $P'(1,-1)$. Write the mapping rule to describe this translation for Sarah.

Solution: In general , $P(x,y) \rightarrow P'(x+a,y+b)$.

In this case, $P(-2,2) \rightarrow P'(-2+a,2+b)$ or $P(-2,2) \rightarrow P'(1,-1)$

Therefore:

$$\begin{aligned} -2+a &= 1 \quad \textit{and} \quad & 2+b &= -1 \\ a &= 3 & b &= -3 \end{aligned}$$

The rule is:

$$(x,y) \rightarrow (x+3,y-3)$$

Example B

Mikah describes a translation as point D in a diagram moving from $D(1,-5)$ to $D'(-3,1)$. Write the mapping rule to describe this translation for Mikah.

Solution: In general , $P(x,y) \rightarrow P'(x+a,y+b)$.

In this case, $D(1,-5) \rightarrow D'(1+a,-5+b)$ or $D(1,-5) \rightarrow D'(-3,1)$

Therefore:

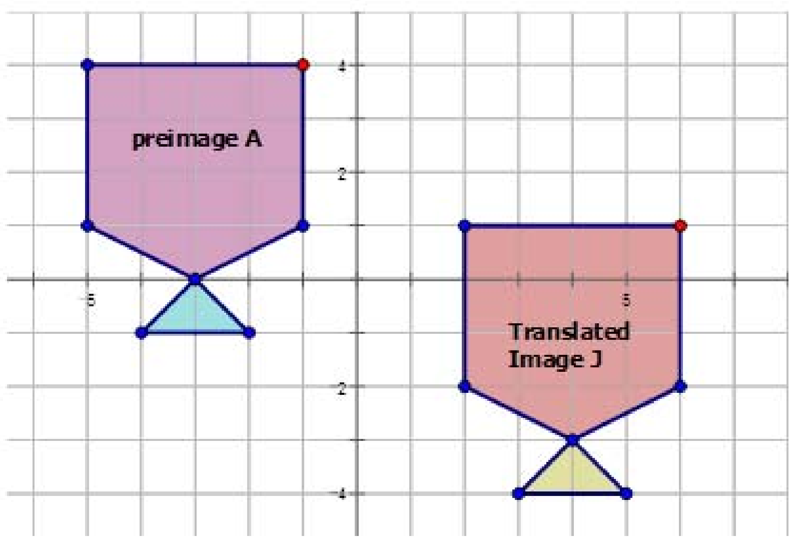
$$\begin{aligned} 1+a &= -3 \quad \textit{and} \quad & -5+b &= 1 \\ a &= -4 & b &= 6 \end{aligned}$$

The rule is:

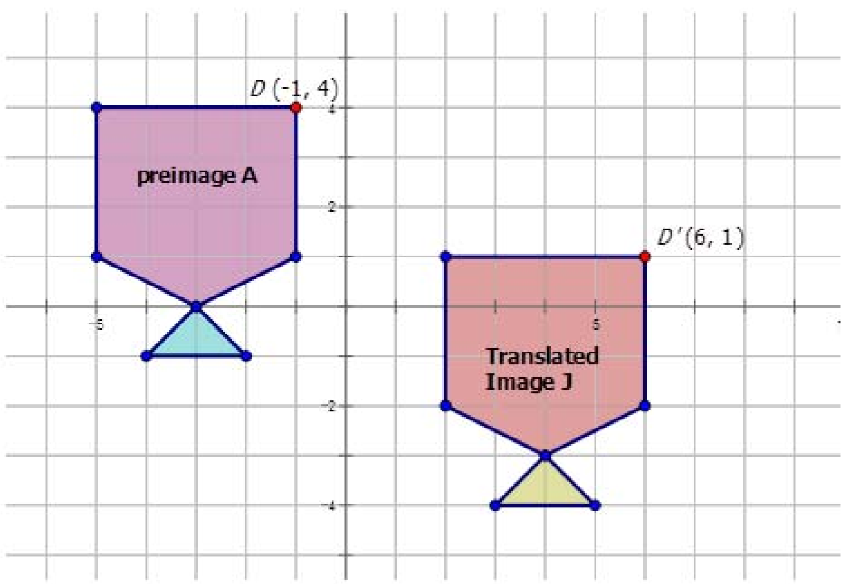
$$(x,y) \rightarrow (x-4,y+6)$$

Example C

Write the mapping rule that represents the translation of the preimage A to the translated image J in the diagram below.



Solution: First, pick a point in the diagram to use to see how it is translated.



$D:(-1,4)\ D':(6,1)$

$D(x,y) \rightarrow D'(x+a,y+b)$

So: $D(-1,4) \rightarrow D'(-1+a,4+b)$ or $D(-1,4) \rightarrow D'(6,1)$

Therefore:

$$\begin{aligned} -1+a &= 6 \qquad and \qquad 4+b = 1 \\ a &= 7 \qquad\qquad\qquad b = -3 \end{aligned}$$

The rule is:

$$(x,y) \rightarrow (x+7, y-3)$$

Vocabulary

Mapping Rule

A ***mapping rule*** has the following form $(x,y) \rightarrow (x-7, y+5)$ and tells you that the x and y coordinates are translated to $x-7$ and $y+5$.

Translation

A ***translation*** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides** .

Image

In a transformation, the final figure is called the ***image*** .

Preimage

In a transformation, the original figure is called the ***preimage.***

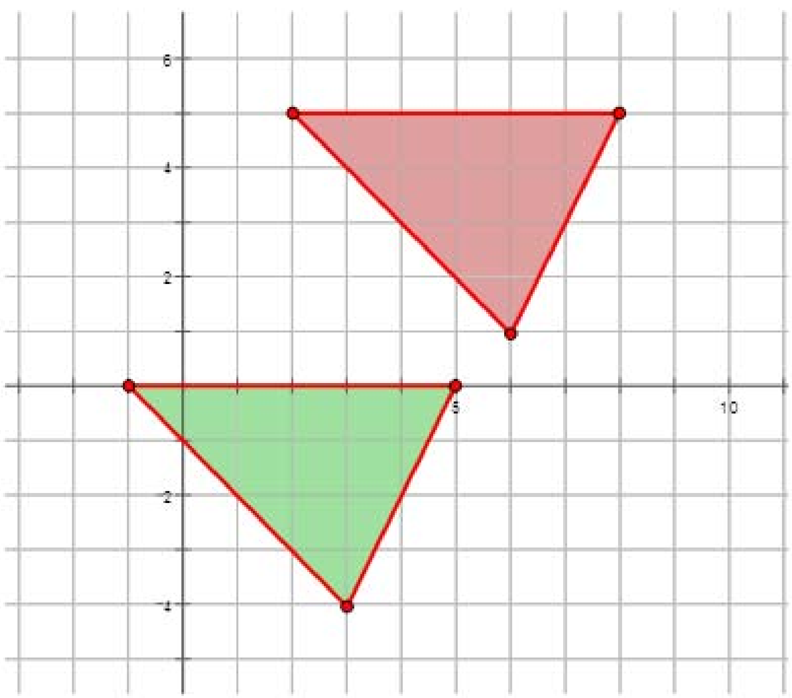
Transformation

A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

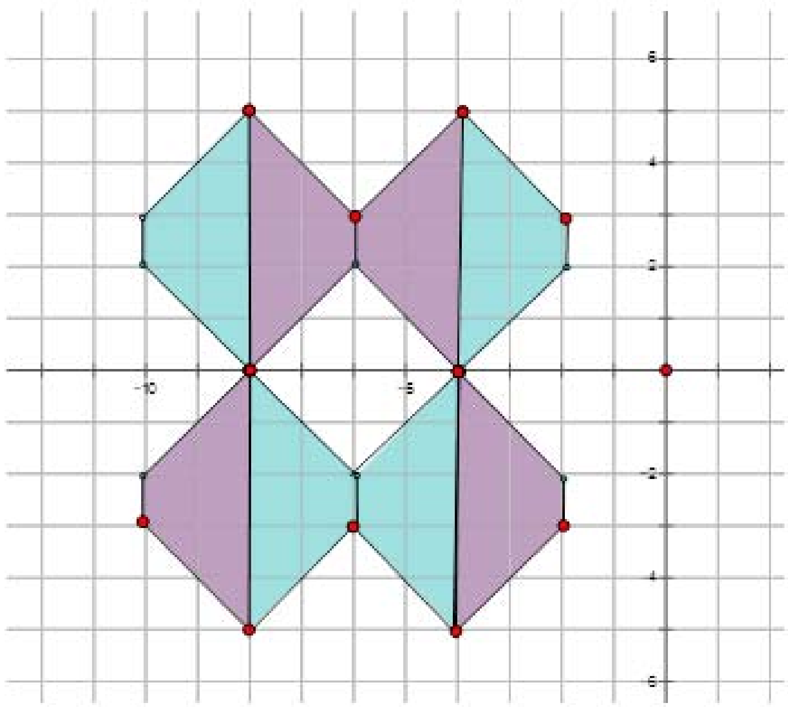
Guided Practice

1. Jack describes a translation as point J moving from $J(-2,6)$ to $J'(4,9)$. Write the mapping rule to describe this translation for Jack.

2. Write the mapping rule that represents the translation of the red triangle to the translated green triangle in the diagram below.



3. The following pattern is part of wallpaper found in a hotel lobby. Write the mapping rule that represents the translation of one blue trapezoid to a translated blue trapezoid shown in the diagram below.



Answers:

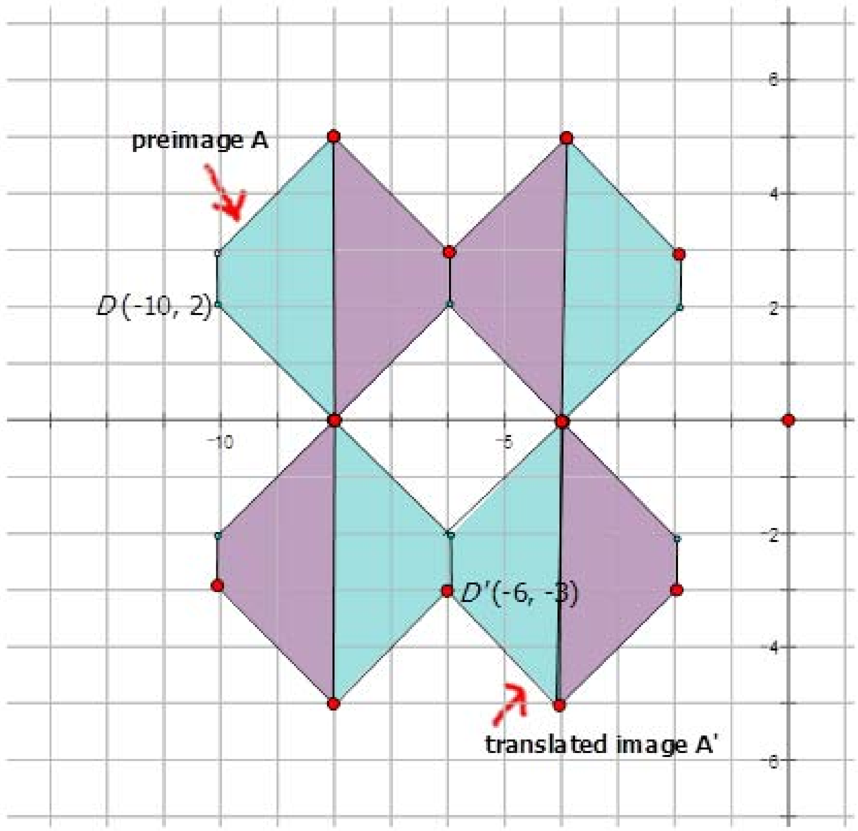
1.

$$(x,y) \rightarrow (x+6, y+3)$$

2.

$$(x,y) \rightarrow (x-3, y-5)$$

3. If you look closely at the diagram below, there two pairs of trapezoids that are translations of each other. Therefore you can choose one blue trapezoid that is a translation of the other and pick a point to find out how much the shape has moved to get to the translated position.



For those two trapezoids:

$$(x,y) \rightarrow (x+4, y-5)$$

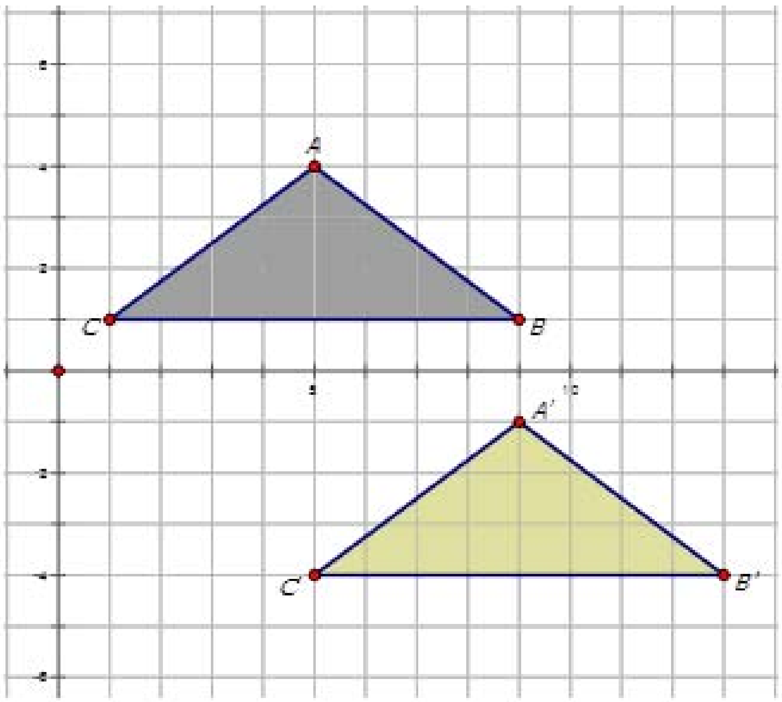
Practice

Write the mapping rule to describe the movement of the points in each of the translations below.

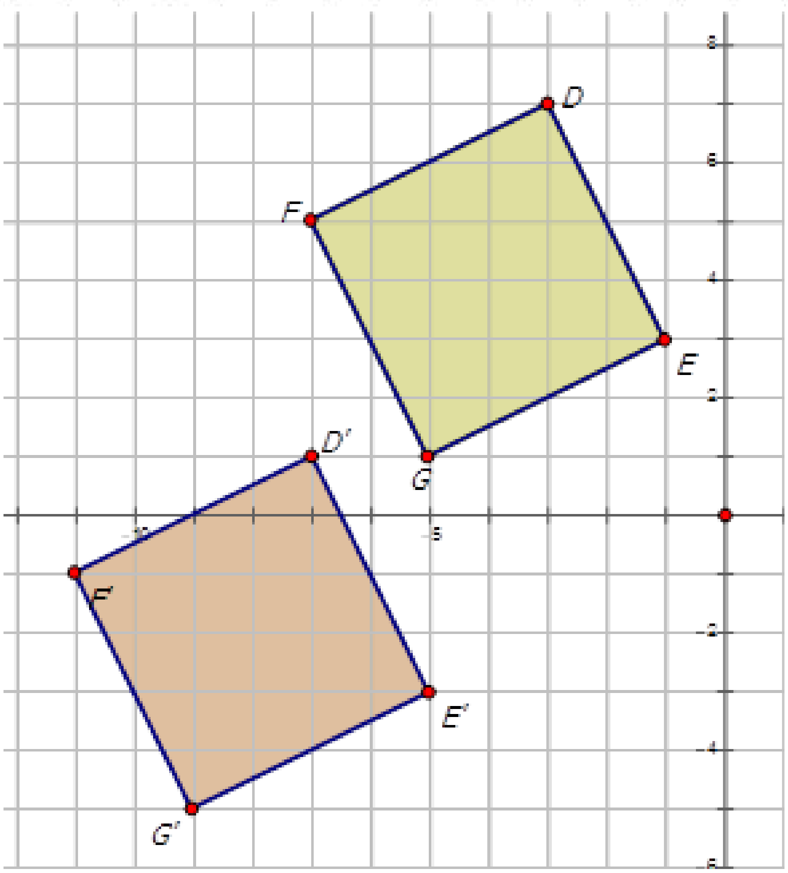
1. $S(1,5) \rightarrow S'(2,7)$
2. $W(-5,-1) \rightarrow W'(-3,1)$
3. $Q(2,-5) \rightarrow Q'(-6,3)$
4. $M(4,3) \rightarrow M'(-2,9)$
5. $B(-4,-2) \rightarrow B'(2,-2)$
6. $A(2,4) \rightarrow A'(2,6)$
7. $C(-5,-3) \rightarrow C'(-3,4)$
8. $D(4,-1) \rightarrow D'(-4,2)$
9. $Z(7,2) \rightarrow Z'(-3,6)$
10. $L(-3,-2) \rightarrow L'(4,-1)$

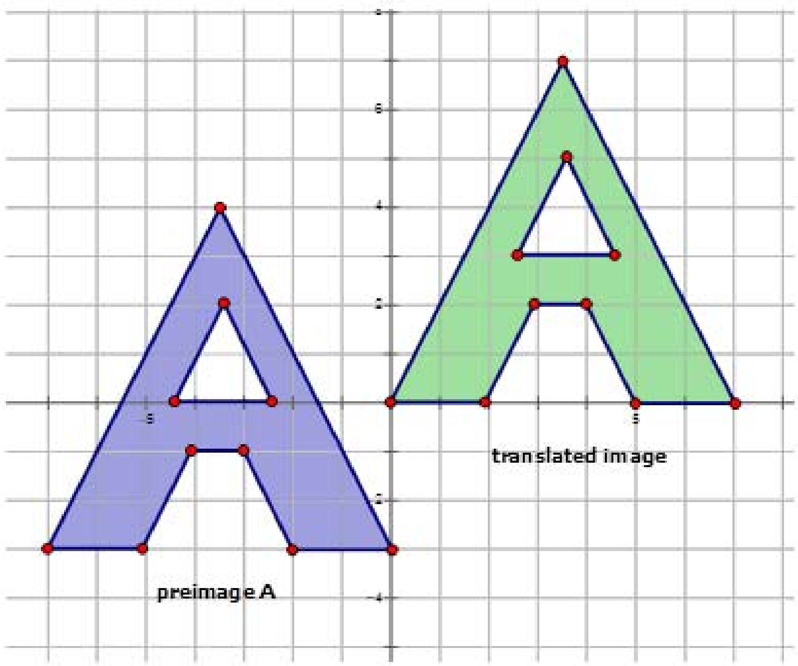
Write the mapping rule that represents the translation of the preimage to the image for each diagram below.

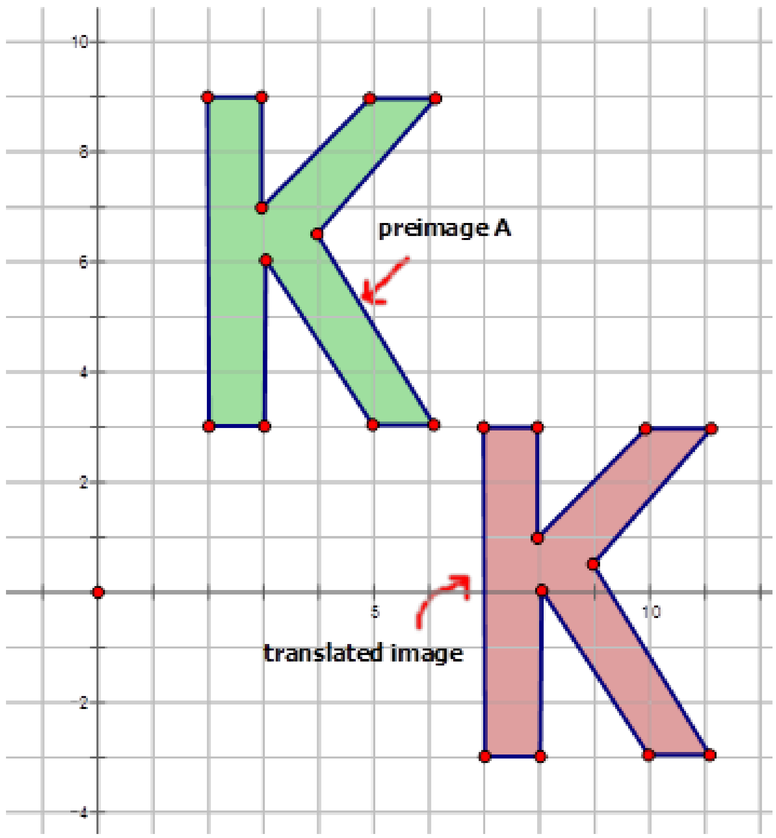
11.

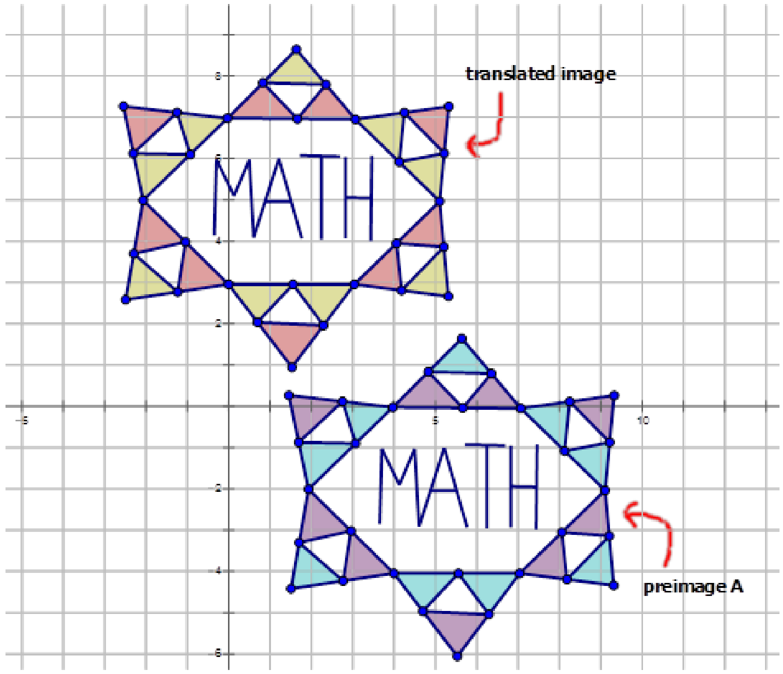


12.



13. 

14. 

15. 

CONCEPT 4 SLT 10 Describe and draw reflections.

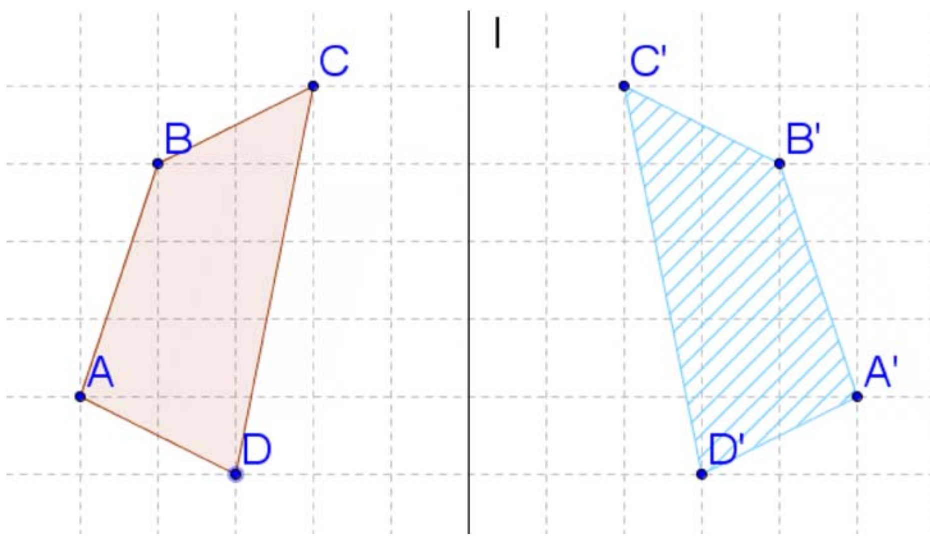
Reflections are often informally called "flips". Why is this?

Watch This

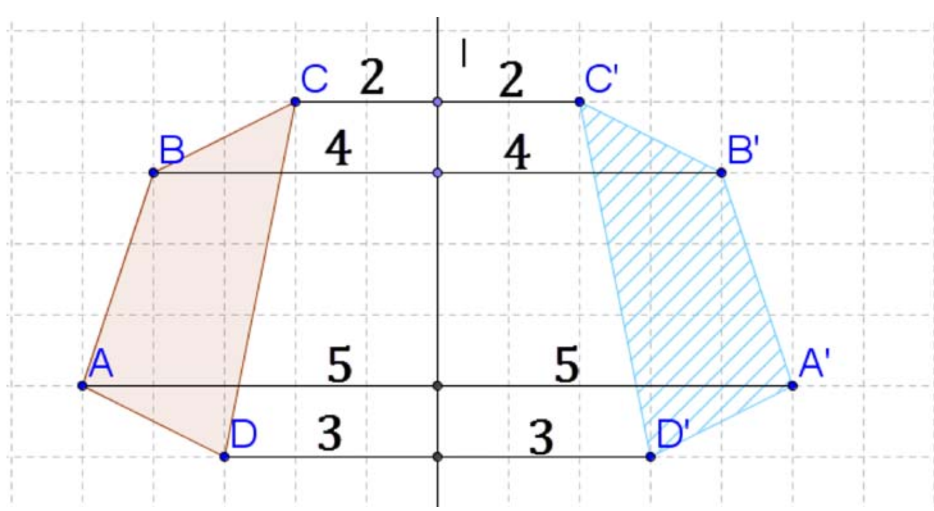
http://learnzillion.com/lessons/2733-explore-reflections-by-investigating-their-effects-on-line-segments-and-angles
LearnZillion: Explore reflections by investigating their effects on line segments and angles

Guidance

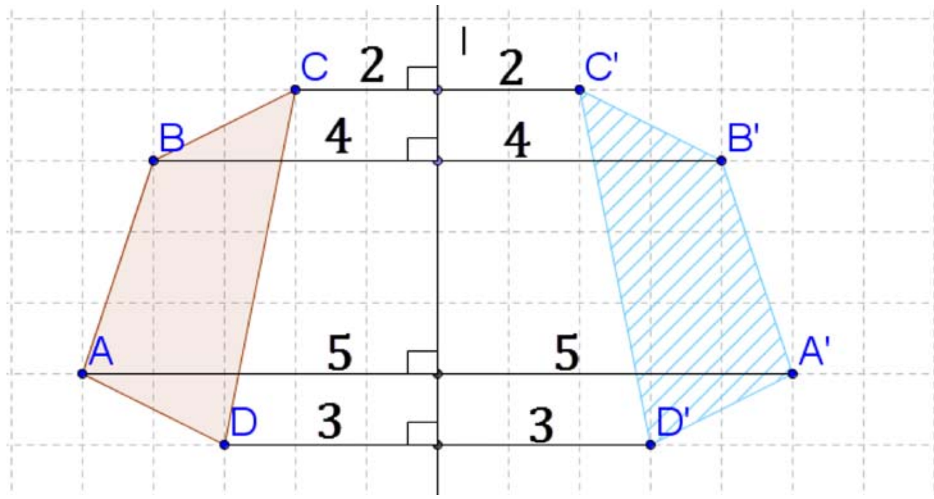
A **reflection** is one example of a **rigid transformation**. A reflection across line l moves each point P to P' such that line l is the perpendicular bisector of the segment connecting P and P'. Below, the quadrilateral has been reflected across line l to create a new quadrilateral (the image).



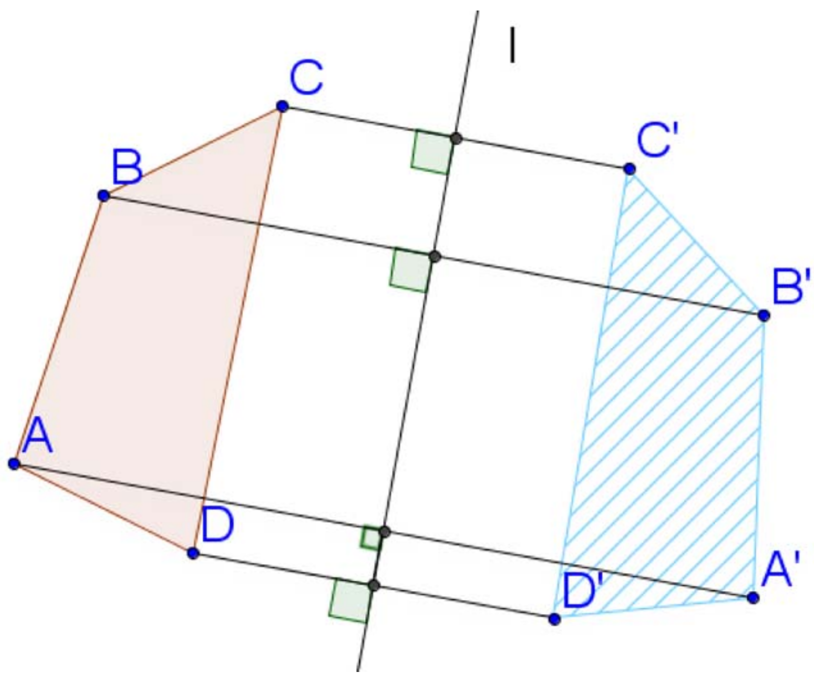
Reflections move all points of a shape across a line called the line of reflection. A point and its corresponding point in the image are each the same distance from the line.



Notice that the segments connecting each point with its corresponding point are all perpendicular to line *l* . Because each point and its corresponding point are the same distance from the line, line *l* bisects each of these segments. This is why line *l* is called the **perpendicular bisector** for each of these segments.

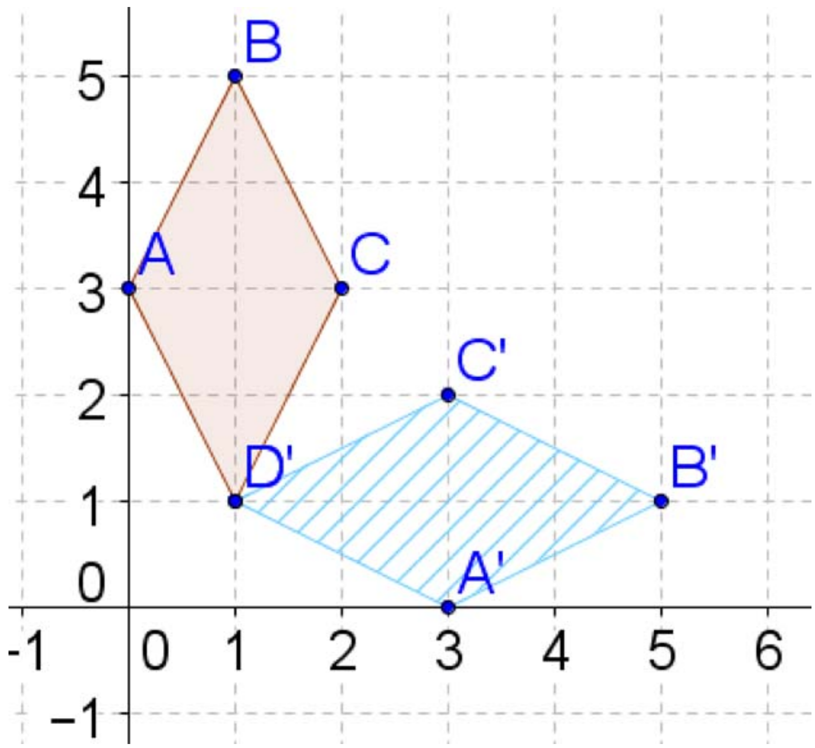


Keep in mind that you can perform reflections even when the line of reflection is "slanted" or the grid is not visible; however, it is much harder to do by hand.

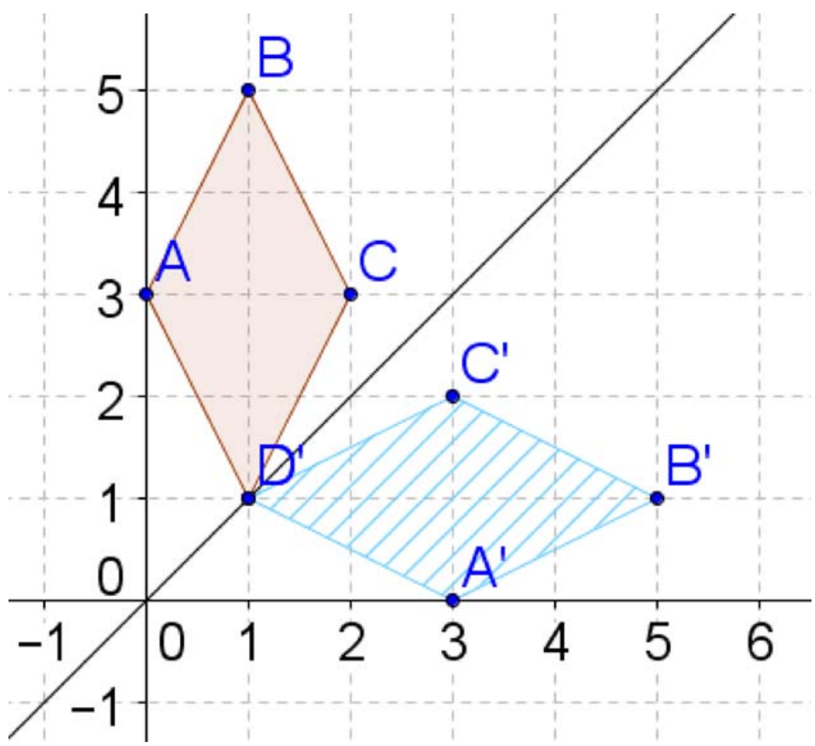


Example A

Describe the line of reflection that created the reflected image below.



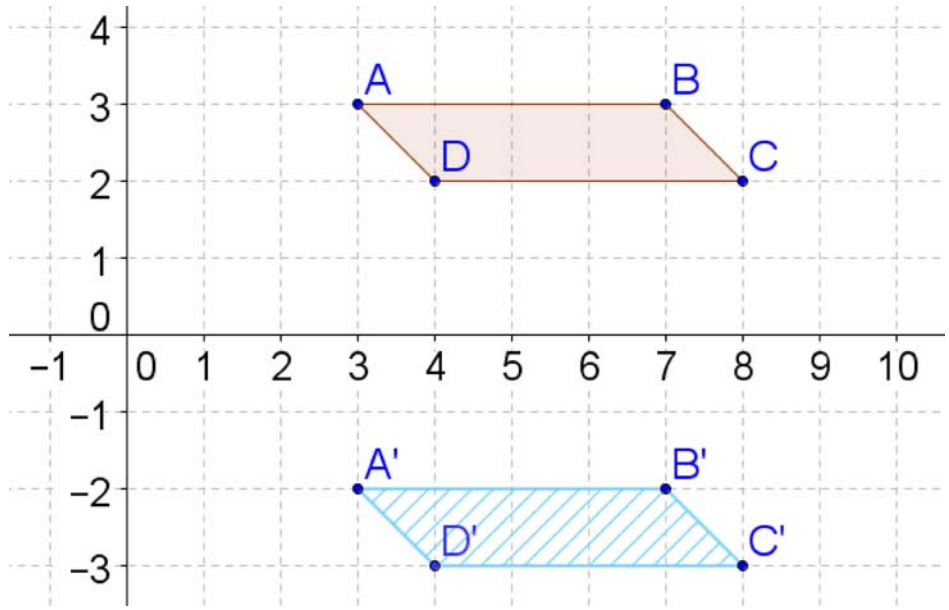
Solution: The fact that point D is in the same location as point D' tells you that the line of reflection passes through point D . Imagine folding the graph so that each point on the original parallelogram matched its point on the image. Where would the fold be?



The line of reflection is the line $y = x$. When reflections are performed on graph paper with axes, you can define the lines of reflections with their equations.

Example B

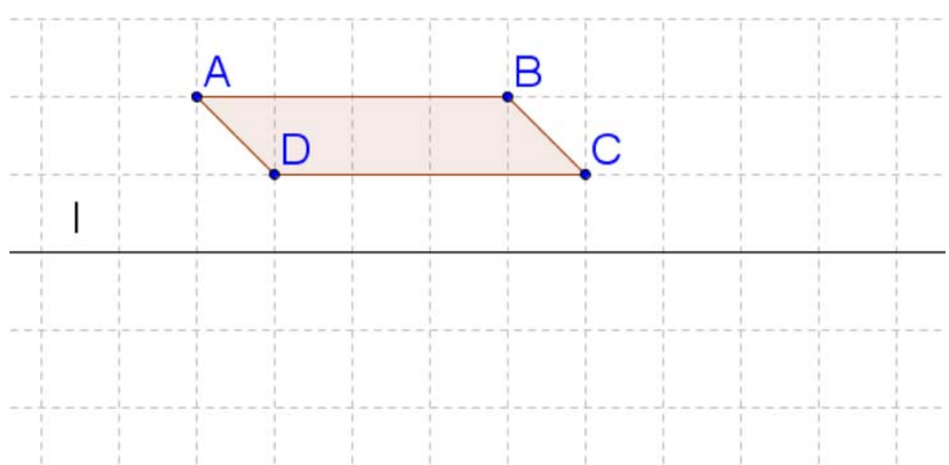
Is the following transformation a reflection?



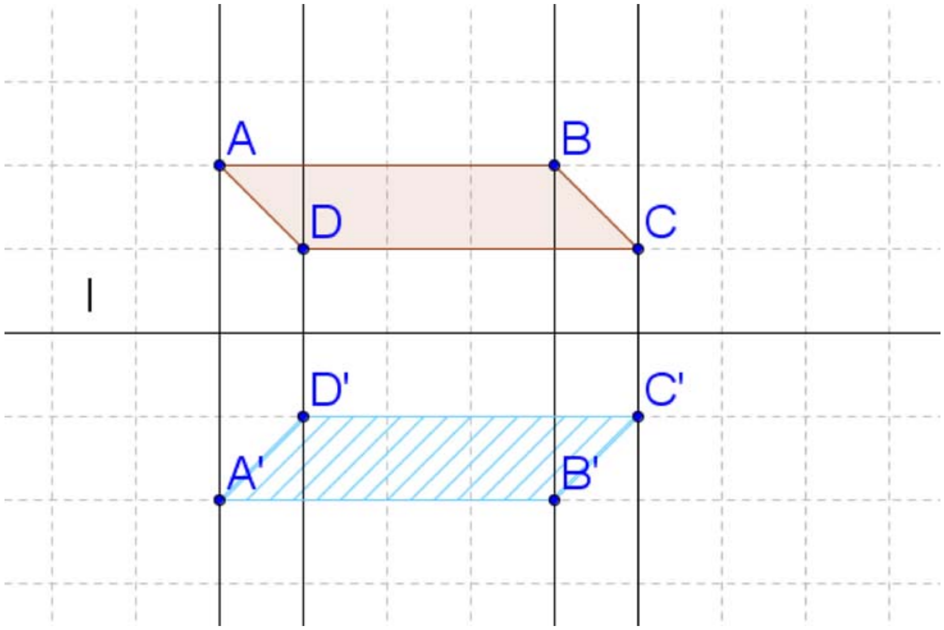
Solution: Even though overall both the parallelogram and its image are 2 units from the x -axis, each individual point and its image are not the same distance from the x -axis. For example, point A is 3 units from the x -axis and point A' is 2 units from the x -axis. The line of reflection for points A and A' would be the line $y = \frac{1}{2}$, which is not the same line of reflection for points D and D' . **This is not a reflection (it's a translation).**

Example C

Perform the reflection across line l .



Solution: Draw a perpendicular line from each point that defines the parallelogram to line l . Count how many units there are between each point and line l along the perpendicular lines. Count the same number of units on the other side of line l along the perpendicular lines to create the image.



Concept Problem Revisited

A reflection is informally called a "flip" because it's as if you are flipping the shape over the line of reflection.

Vocabulary

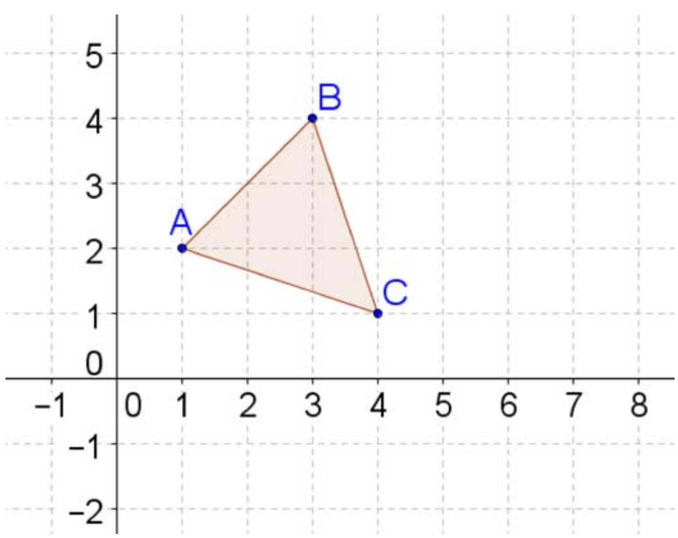
A ***reflection*** across line l moves each point P to P' such that line l is the perpendicular bisector of the segment connecting P and P' . An informal way to think about a reflection is as a "flip".

A ***rigid transformation*** is a transformation that preserves distance and angles.

A ***perpendicular bisector*** is a line that bisects a segment (cuts it in half) and is perpendicular to the segment.

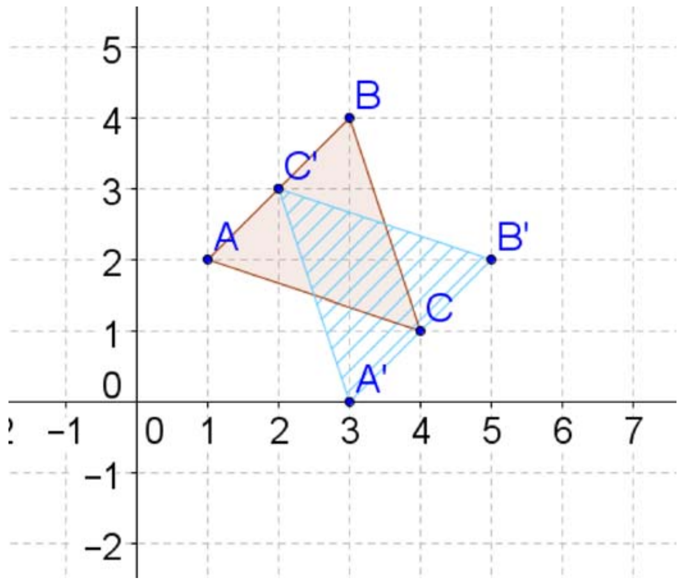
Guided Practice

1. Reflect the triangle across the x -axis.



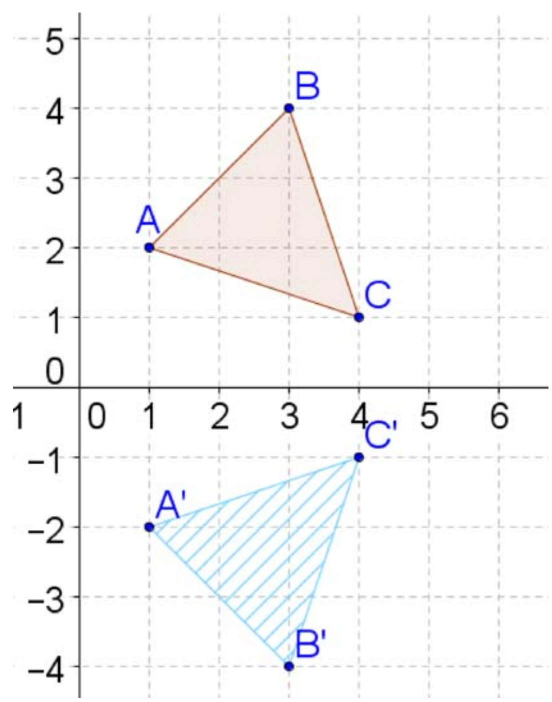
2. From #1, what do you notice about each point and its image when a reflection is done across the x -axis?

3. Describe the line of reflection that created the reflected image below.



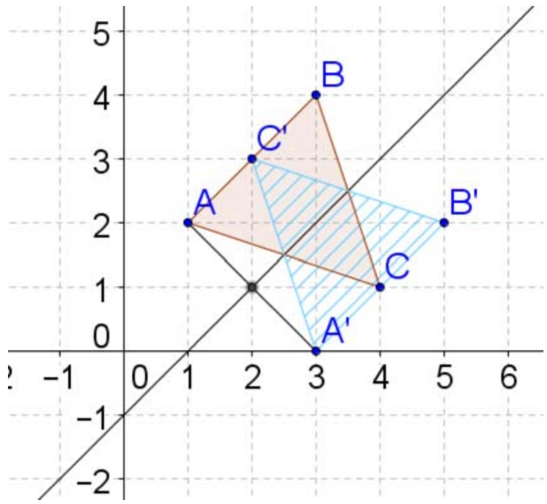
Answers:

1.



2. The x -coordinate of each point and its image are the same, but the y -coordinate has changed sign. You could describe this as $(x,y) \rightarrow (x,-y)$.

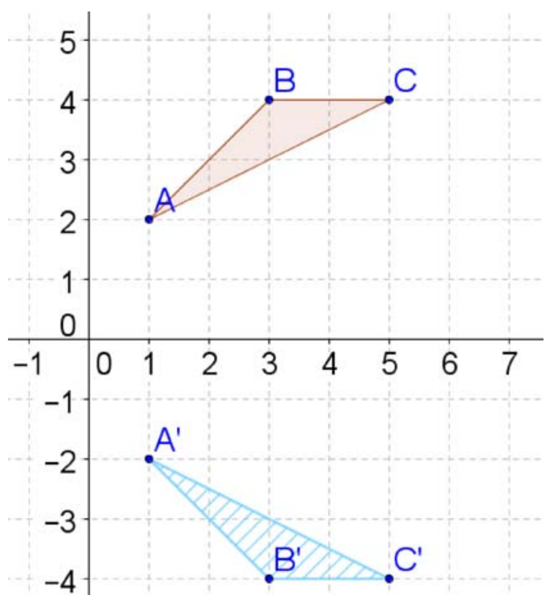
3. Connect A with A' . This line segment has a slope of -1 and a midpoint at (2, 1). The line of reflection is the perpendicular bisector of this segment. This means it passes through its midpoint and has an opposite reciprocal slope $\left(-\frac{1}{1} \rightarrow +\frac{1}{1}\right)$. The line of reflection is the line $y = x - 1$.



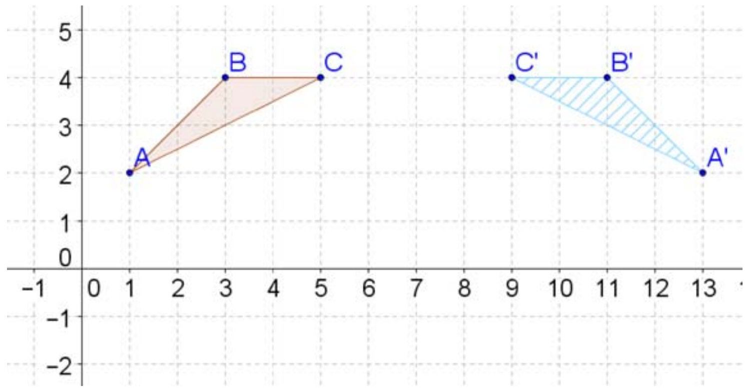
Practice

Describe the line of reflection that created each of the reflected images below.

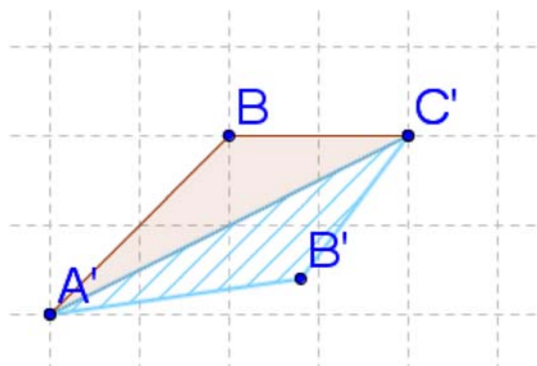
1.



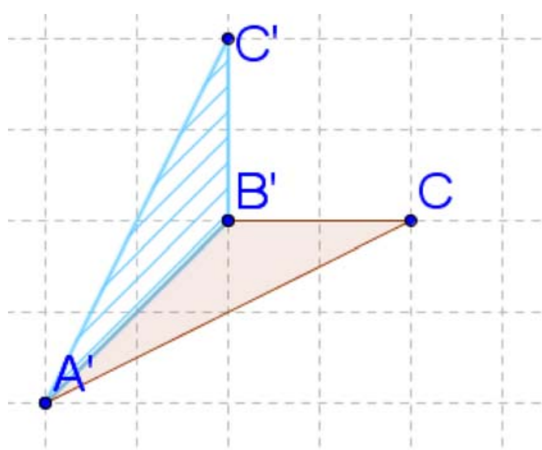
2.



3 .

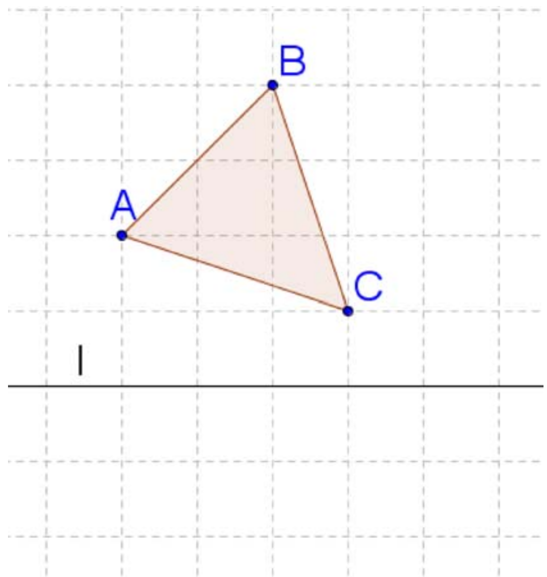


4.

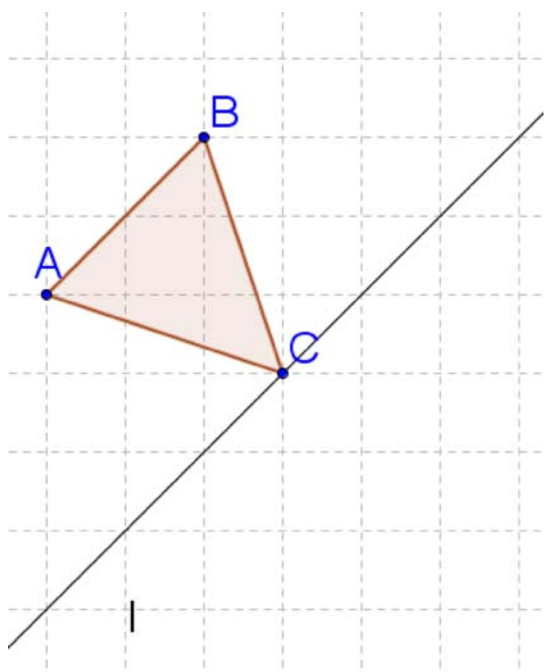


Reflect each shape across line l .

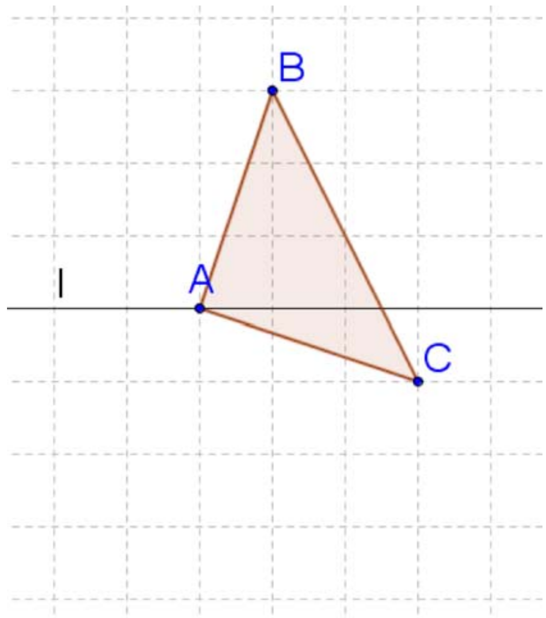
5.



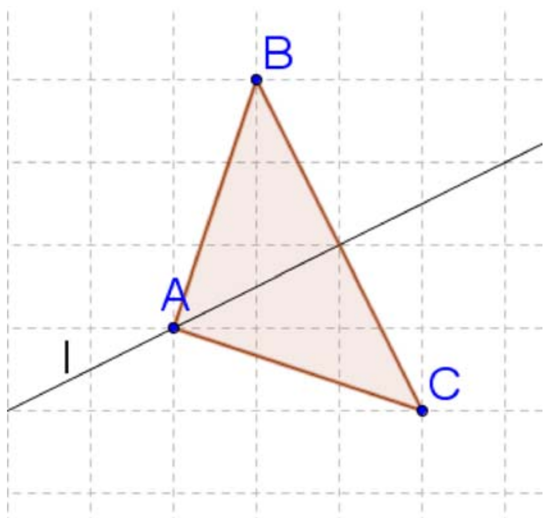
6.



7.

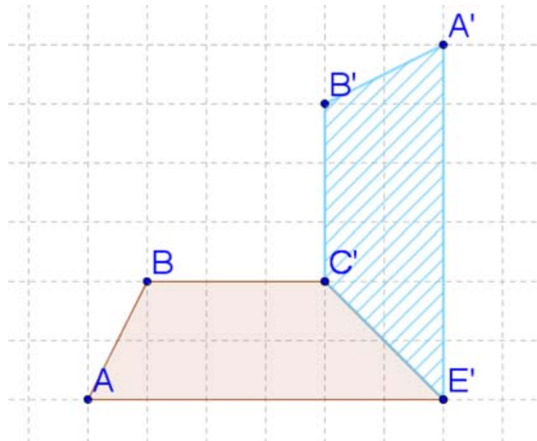


8.

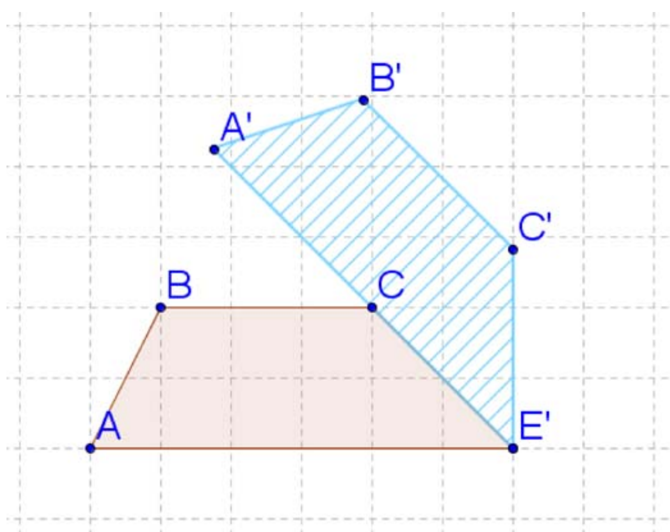


Is the transformation a reflection? Explain.

9.



10.



11. Reflect a shape across the y -axis. How are the points of the original shape related to the points of the image?

12. The point (7, 2) is reflected across the y -axis. Can you find the coordinates of the image point using the relationship you found in #11?

13. Reflect a shape across the line $y = x$. How are the points of the original shape related to the points of the image?

14. The point (7, 2) is reflected across the line $y = x$. Can you find the coordinates of the image point using the relationship you found in #13?

15. Reflect a shape across the line $y = -x$. How are the points of the original shape related to the points of the image?

16. The point (7, 2) is reflected across the line $y = -x$. Can you find the coordinates of the image point using the relationship you found in #15?

References

1. . . CC BY-NC-SA
2. . . CC BY-NC-SA
3. . . CC BY-NC-SA
4. . . CC BY-NC-SA
5. . . CC BY-NC-SA
6. . . CC BY-NC-SA
7. . . CC BY-NC-SA
8. . . CC BY-NC-SA
9. . . CC BY-NC-SA
10. . . CC BY-NC-SA
11. . . CC BY-NC-SA
12. . . CC BY-NC-SA
13. . . CC BY-NC-SA
14. . . CC BY-NC-SA
15. . . CC BY-NC-SA
16. . . CC BY-NC-SA
17. . . CC BY-NC-SA
18. . . CC BY-NC-SA
19. . . CC BY-NC-SA
20. . . CC BY-NC-SA
21. . . CC BY-NC-SA
22. . . CC BY-NC-SA

CONCEPT 5 SLT 11 Draw a reflection when given a rule and write a rule given a reflection.

What if you noticed that a lake can act like a mirror in nature? Describe the line of reflection in the photo below. If this image were on the coordinate plane, what could the equation of the line of reflection be? (There could be more than one correct answer, depending on where you place the origin.) After completing this Concept, you'll be able to answer this question.

Watch This

MEDIA
Click image to the left for more content.

CK-12 Foundation: Chapter12ReflectionsA

Watch more about transformations and isometries by watching the last part of this video.

Guidance

A ***transformation*** *is an operation that moves, flips, or changes a figure to create a new figure. A* ***rigid transformation*** *is a transformation that preserves size and shape. The rigid transformations are: translations, reflections (discussed here), and rotations. The new figure created by a transformation is called the* ***image*** *. The original figure is called the* ***preimage*** *. Another word for a rigid transformation is an* ***isometry*** *. Rigid transformations are also called* ***congruence transformations*** *. If the preimage is A, then the image would be labeled A', said "a prime." If there is an image of A', that would be labeled A'', said "a double prime."*

A **reflection** is a transformation that turns a figure into its mirror image by flipping it over a line. Another way to describe a reflection is a "flip." The **line of reflection** is the line that a figure is reflected over. If a point is on the line of reflection then the image is the same as the original point.

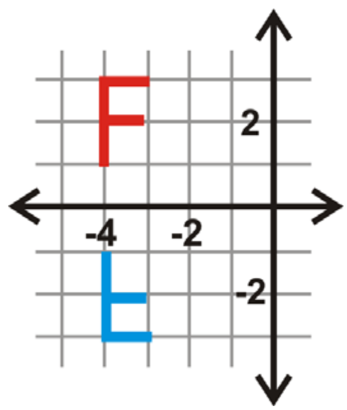
Common Reflections

- **Reflection over the** $y-$ **axis:** If (x,y) is reflected over the $y-$ axis, then the image is $(-x,y)$.
- **Reflection over the** $x-$ **axis:** If (x,y) is reflected over the $x-$ axis, then the image is $(x,-y)$.
- **Reflection over** $x=a$: If (x,y) is reflected over the vertical line $x=a$, then the image is $(2a-x,y)$.
- **Reflection over** $y=b$: If (x,y) is reflected over the horizontal line $y=b$, then the image is $(x,2b-y)$.
- **Reflection over** $y=x$ **:** If (x,y) is reflected over the line $y=x$, then the image is (y,x) .
- **Reflection over** $y=-x$ **:** If (x,y) is reflected over the line $y=-x$, then the image is $(-y,-x)$.

Example A

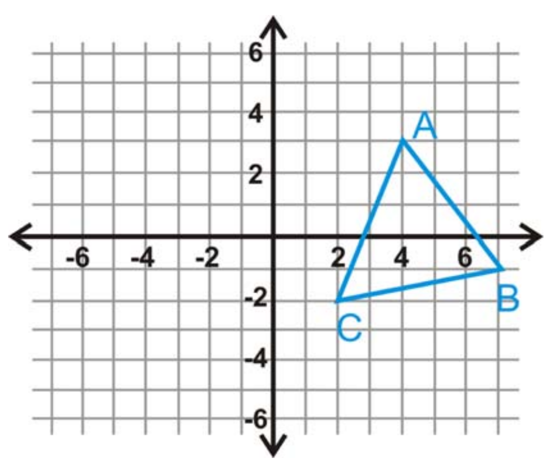
Reflect the letter $''F''$ over the $x-$ axis.

To reflect the letter F over the $x-$ axis, now the $x-$ coordinates will remain the same and the $y-$ coordinates will be the same distance away from the $x-$ axis on the other side.

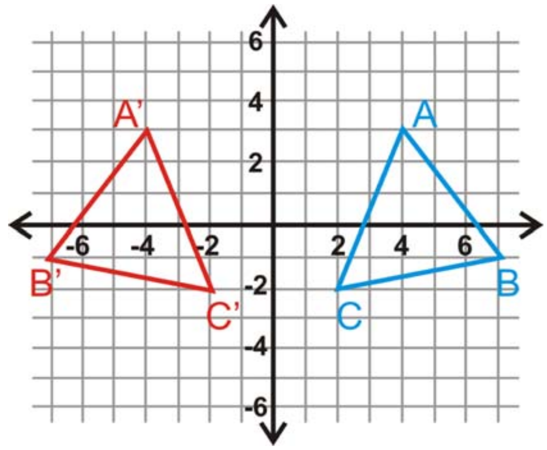


Example B

Reflect $\triangle ABC$ over the $y-$ axis. Find the coordinates of the image.



To reflect $\triangle ABC$ over the $y-$ axis the $y-$ coordinates will remain the same. The $x-$ coordinates will be the same distance away from the $y-$ axis, but on the other side of the $y-$ axis.

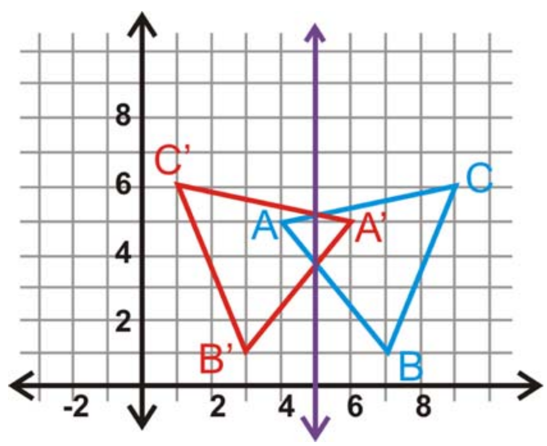


$$A(4,3) \rightarrow A'(-4,3)$$
$$B(7,-1) \rightarrow B'(-7,-1)$$
$$C(2,-2) \rightarrow C'(-2,-2)$$

Example C

Reflect the triangle $\triangle ABC$ with vertices $A(4,5), B(7,1)$ and $C(9,6)$ over the line $x = 5$.

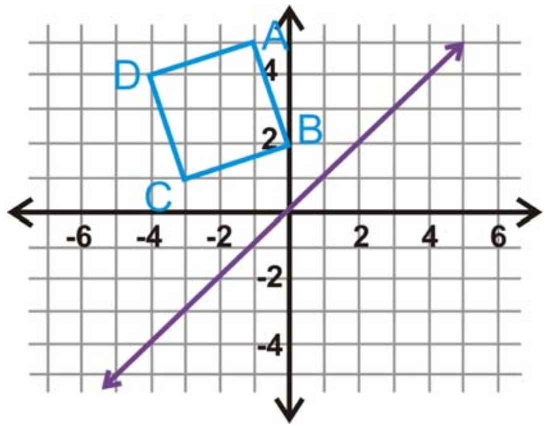
Notice that this vertical line is through our preimage. Therefore, the image's vertices are the same distance away from $x = 5$ as the preimage. As with reflecting over the $y-$ axis (or $x = 0$), the $y-$ coordinates will stay the same.



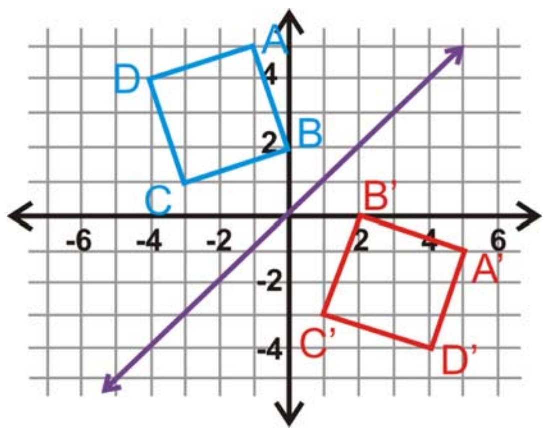
$$A(4,5) \rightarrow A'(6,5)$$
$$B(7,1) \rightarrow B'(3,1)$$
$$C(9,6) \rightarrow C'(1,6)$$

Example D

Reflect square $ABCD$ over the line $y = x$.



The purple line is $y = x$. To reflect an image over a line that is not vertical or horizontal, you can fold the graph on the line of reflection.



$$A(-1,5) \rightarrow A'(5,-1)$$
$$B(0,2) \rightarrow B'(2,0)$$
$$C(-3,1) \rightarrow C'(1,-3)$$
$$D(-4,4) \rightarrow D'(4,-4)$$

Watch this video for help with the Examples above.

MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter12ReflectionsB

Concept Problem Revisited

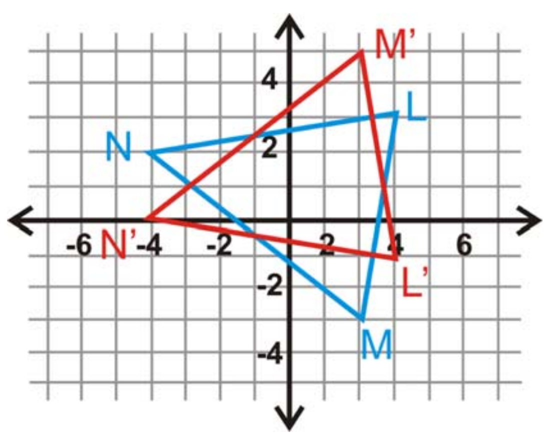
The white line in the picture is the line of reflection. This line coincides with the water's edge. If we were to place this picture on the coordinate plane, the line of reflection would be any horizontal line. One example could be the $x-$ axis.

Vocabulary

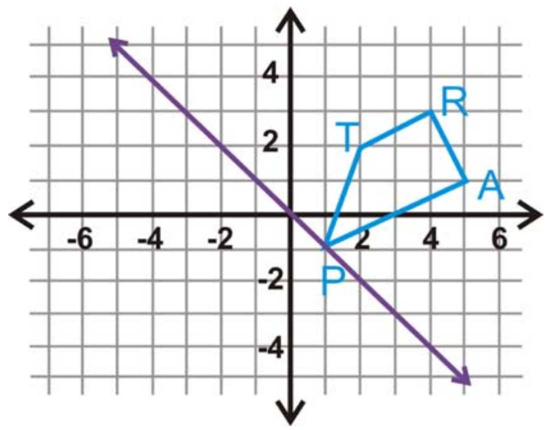
A ***transformation*** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A ***rigid transformation*** (also known as an ***isometry*** or ***congruence transformation***) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the ***image***. The original figure is called the ***preimage***. A ***reflection*** is a transformation that turns a figure into its mirror image by flipping it over a line. The ***line of reflection*** is the line that a figure is reflected over.

Guided Practice

1. Reflect the line segment $\overline{PQ}$ with endpoints $P(-1,5)$ and $Q(7,8)$ over the line $y=5$.
2. A triangle $\triangle LMN$ and its reflection, $\triangle L'M'N'$ are to the left. What is the line of reflection?



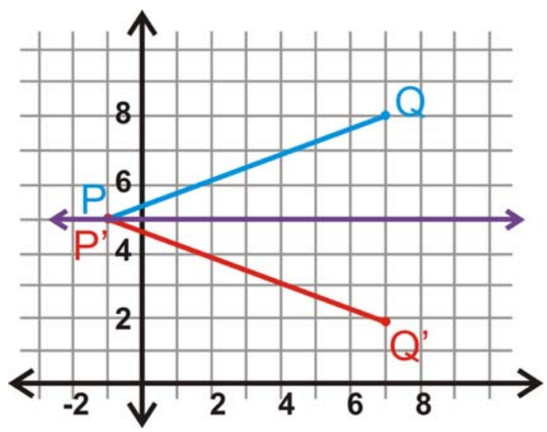
3. Reflect the trapezoid $TRAP$ over the line $y = -x$.



Answers:

1. Here, the line of reflection is on P , which means P' has the same coordinates. Q' has the same $x-$ coordinate as Q and is the same distance away from $y = 5$, but on the other side.

$$P(-1,5) \rightarrow P'(-1,5)$$
$$Q(7,8) \rightarrow Q'(7,2)$$



2. Looking at the graph, we see that the preimage and image intersect when $y = 1$. Therefore, this is the line of reflection.

3. The purple line is $y = -x$. You can reflect the trapezoid over this line just like we did in Example D.

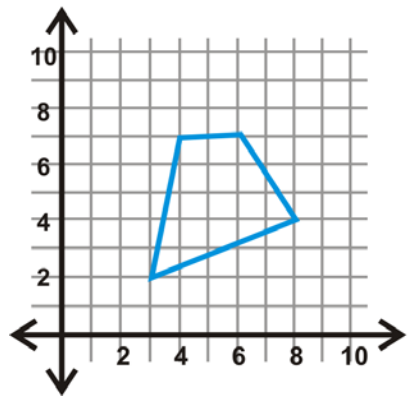
$$T(2,2) \rightarrow T'(-2,-2)$$
$$R(4,3) \rightarrow R'(-3,-4)$$
$$A(5,1) \rightarrow A'(-1,-5)$$
$$P(1,-1) \rightarrow P'(1,-1)$$

Practice

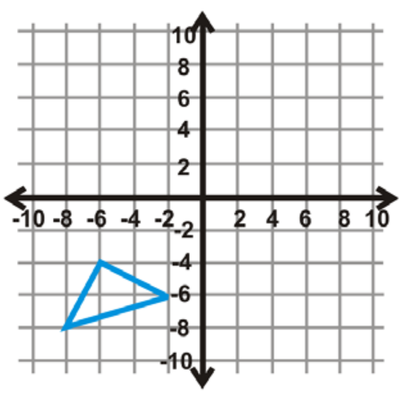
1. Which letter is a reflection over a vertical line of the letter *b* ?
2. Which letter is a reflection over a horizontal line of the letter *b* ?

Reflect each shape over the given line.

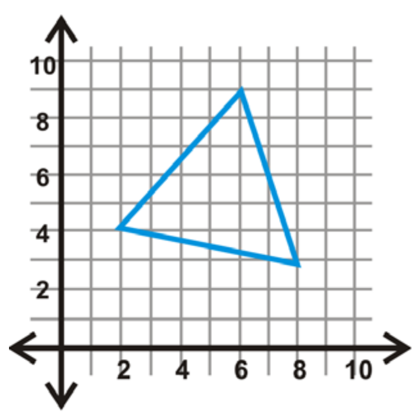
3. $y-$ axis



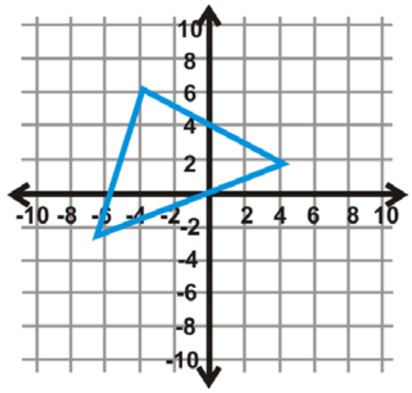
4. $x-$ axis



5. $y = 3$

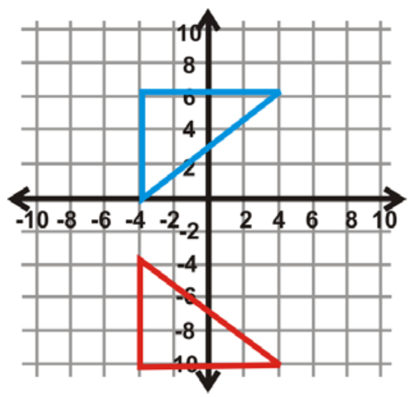


6. $x = -1$

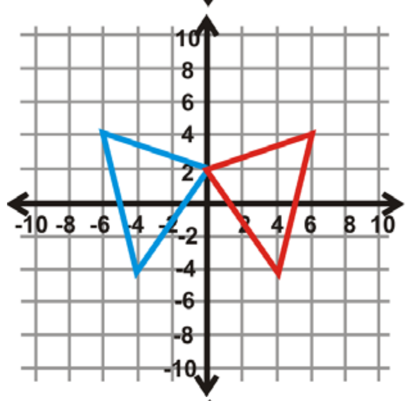


Find the line of reflection of the blue triangle (preimage) and the red triangle (image).

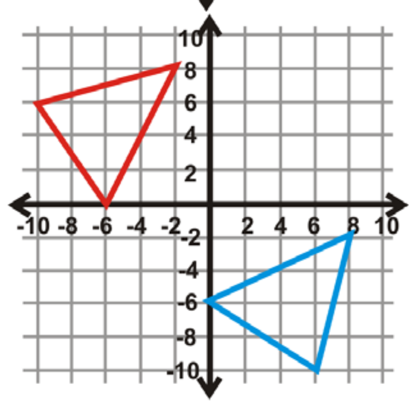
7.



8.



9.



Two Reflections The vertices of $\triangle ABC$ are $A(-5,1), B(-3,6)$, and $C(2,3)$. Use this information to answer questions 10-13.

10. Plot $\triangle ABC$ on the coordinate plane.
11. Reflect $\triangle ABC$ over $y=1$. Find the coordinates of $\triangle A'B'C'$.
12. Reflect $\triangle A'B'C'$ over $y=-3$. Find the coordinates of $\triangle A''B''C''$.
13. What ***one*** transformation would be the same as this double reflection?

Two Reflections The vertices of $\triangle DEF$ are $D(6,-2), E(8,-4)$, and $F(3,-7)$. Use this information to answer questions 14-17.

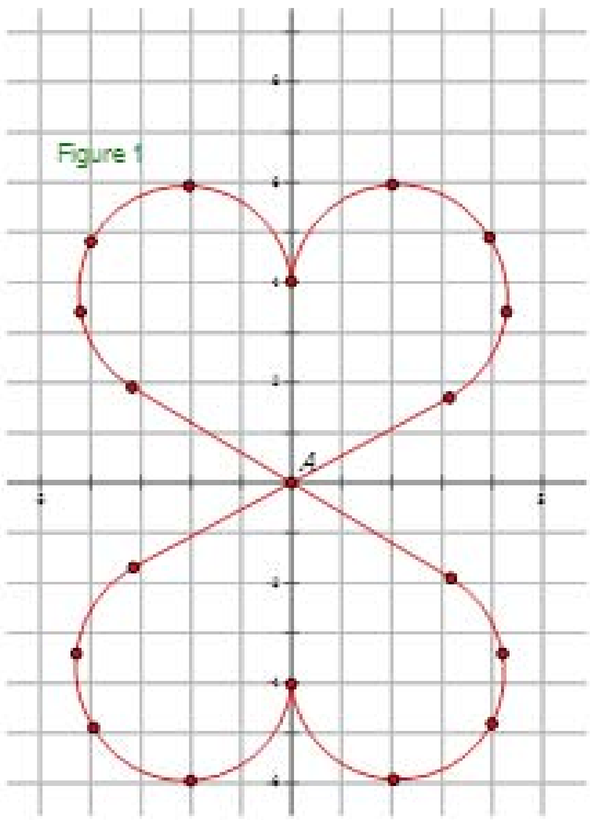
14. Plot $\triangle DEF$ on the coordinate plane.
15. Reflect $\triangle DEF$ over $x=2$. Find the coordinates of $\triangle D'E'F'$.
16. Reflect $\triangle D'E'F'$ over $x=-4$. Find the coordinates of $\triangle D''E''F''$.
17. What ***one*** transformation would be the same as this double reflection?

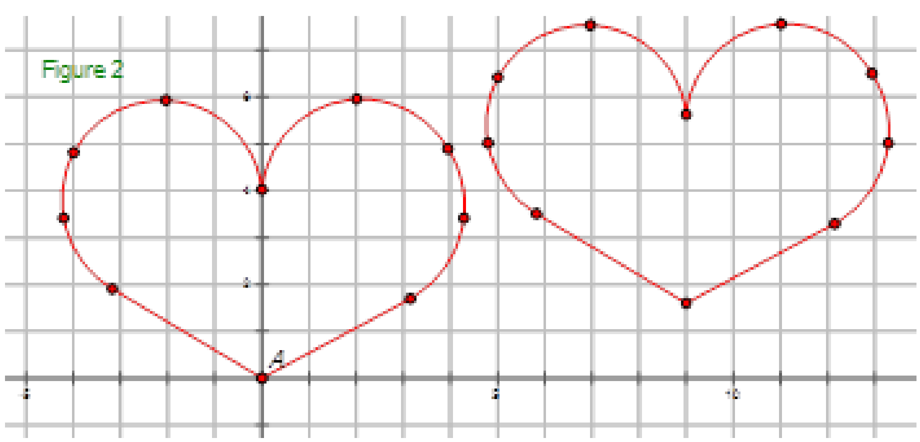
Two Reflections The vertices of $\triangle GHI$ are $G(1,1), H(5,1)$, and $I(5,4)$. Use this information to answer questions 18-21.

18. Plot $\triangle GHI$ on the coordinate plane.
19. Reflect $\triangle GHI$ over the $x-$ axis. Find the coordinates of $\triangle G'H'I'$.
20. Reflect $\triangle G'H'I'$ over the $y-$ axis. Find the coordinates of $\triangle G''H''I''$.
21. What ***one*** transformation would be the same as this double reflection?

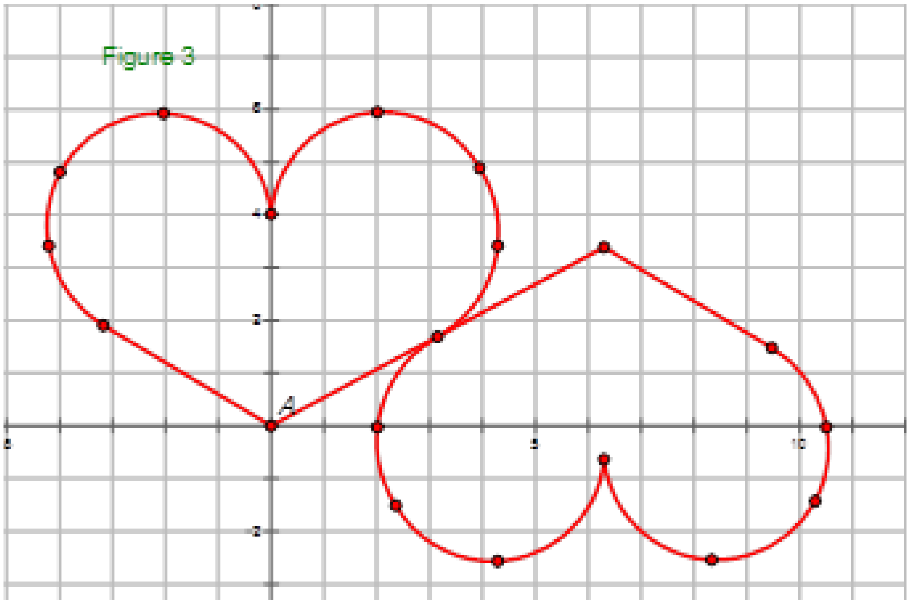
CONCEPT 6 SLT 12 & 13 Draw a rotation when given a rule with inputs and outputs & write a rule for a rotation.

Which one of the following figures represents a rotation? Explain.





Concept 6 SLT 12 & 13 Draw a rotation when given a rule with inputs and outputs & write a rule for a rotation.



Watch This

First watch this video to learn about rotations.

MEDIA

Click image to the left for more content.

CK-12 Foundation Chapter10RotationsA

Then watch this video to see some examples.

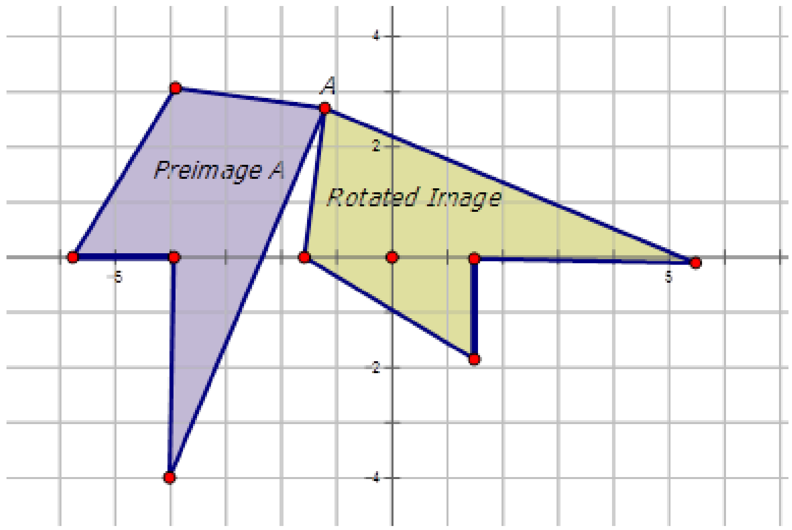
MEDIA

Click image to the left for more content.

CK-12 Foundation Chapter10RotationsB

Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A rotation is an example of a transformation where a figure is rotated about a specific point (called the center of rotation), a certain number of degrees. The figure below shows that the Preimage A has been rotated 90° about point A to form the rotated image. Point A is the center of rotation.



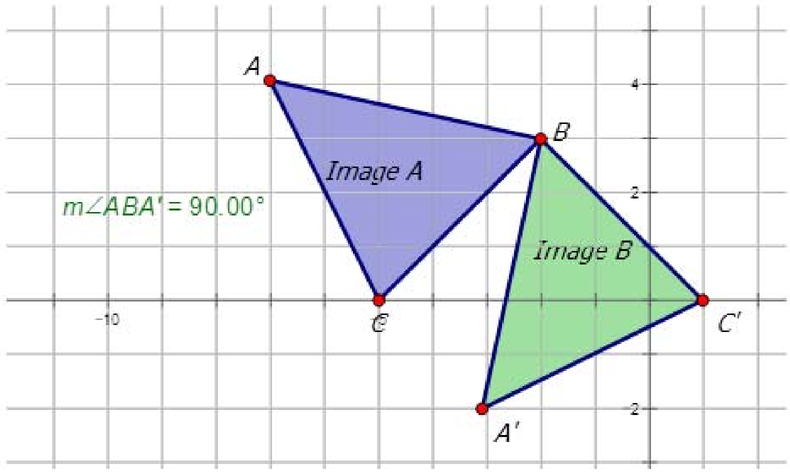
In order to describe a rotation, you need to state how many degrees the preimage rotated, the center of rotation, and the direction of the rotation (clockwise or counterclockwise). The most common center of rotation is the origin. The table below shows what happens to points when they have undergone a rotation about the origin. The angles are given as counterclockwise.

TABLE 6.1:

Center of Rotation	Angle of Rotation	Preimage (Point P)	Rotated Image (Point P')
(0, 0)	90° (or -270°)	(x, y)	$(-y, x)$
(0, 0)	180° (or -180°)	(x, y)	$(-x, -y)$
(0, 0)	270° (or -90°)	(x, y)	$(y, -x)$

Example A

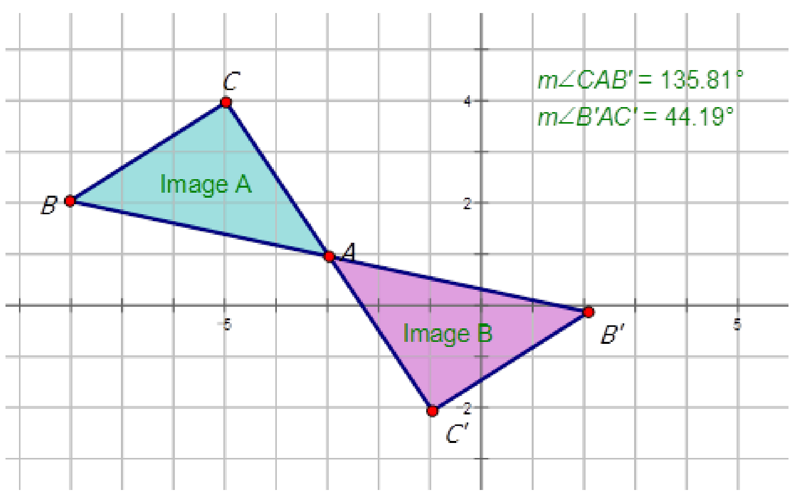
Describe the rotation of the blue triangle in the diagram below.



Solution: Looking at the angle measures, $\angle ABA' = 90^\circ$. Therefore the preimage, Image A, has been rotated 90° counterclockwise about the point B .

Example B

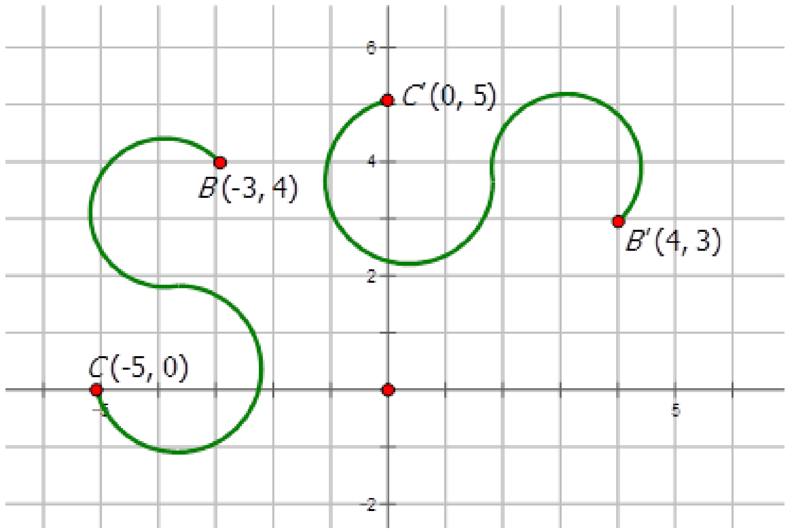
Describe the rotation of the triangles in the diagram below.



Solution: Looking at the angle measures, $\angle CAB' + \angle B'AC' = 180^\circ$. The triangle ABC has been rotated 180° CCW about the center of rotation Point A .

Example C

Describe the rotation in the diagram below.



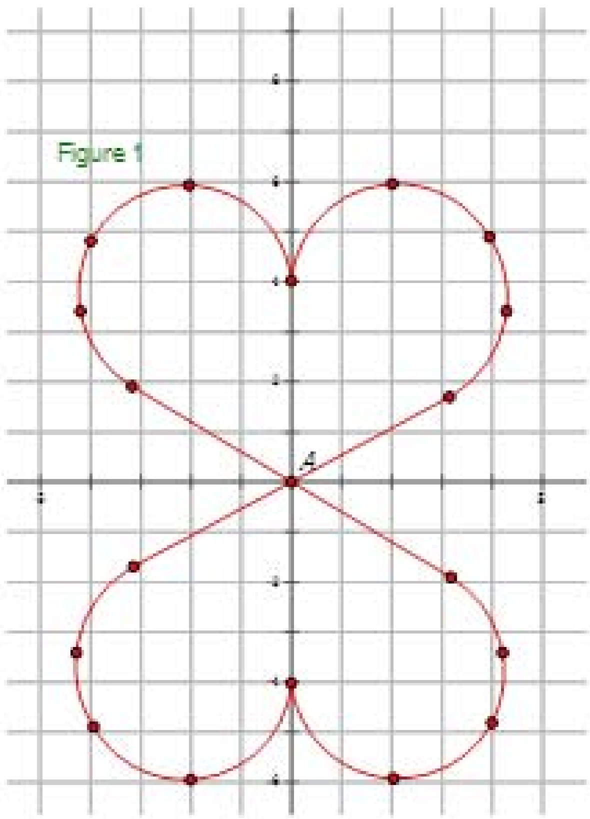
Solution: To describe the rotation in this diagram, look at the points indicated on the S shape.

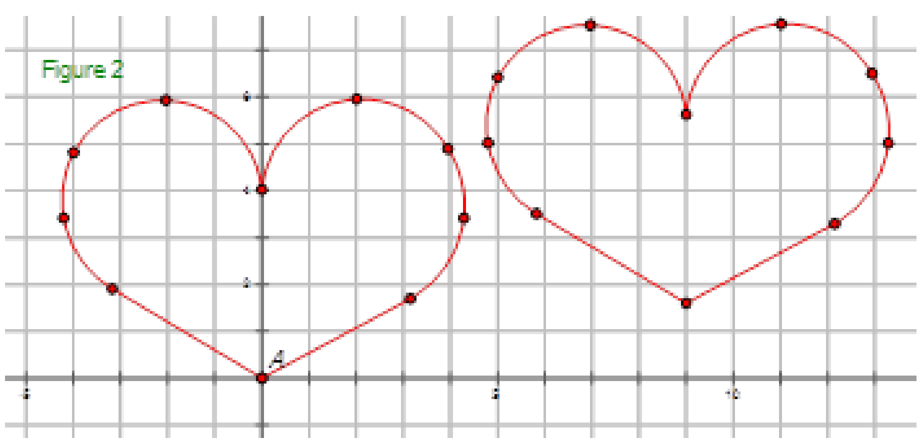
- Points BC : $B(-3,4)C(-5,0)$
- Points $B'C'$: $B'(4,3)C'(0,5)$

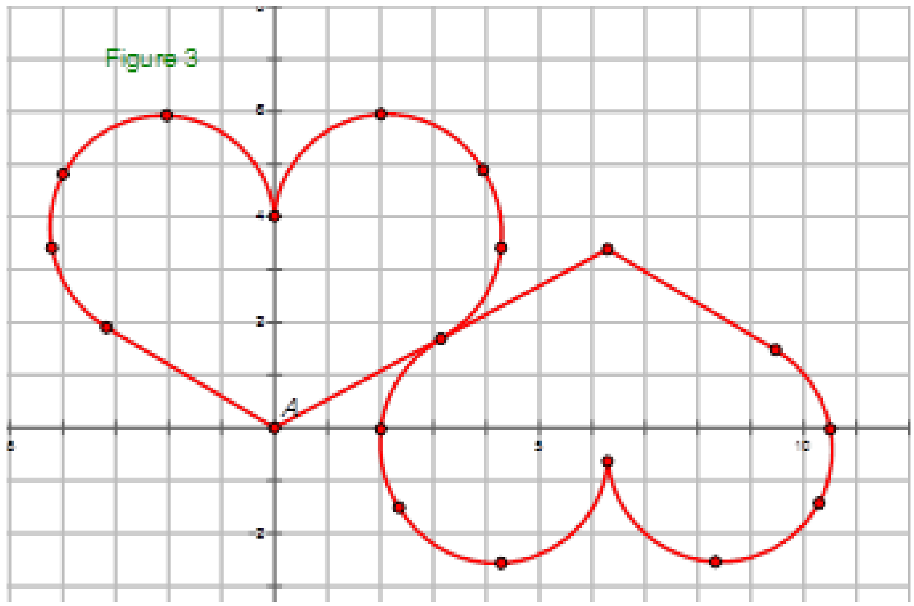
These points represent a rotation of 90° clockwise about the origin. Each coordinate point (x,y) has become the point $(y,-x)$.

Concept Problem Revisited

Which one of the following figures represents a rotation? Explain.







You know that a rotation is a transformation that turns a figure about a fixed point. This fixed point is the turn center or the center of rotation. In the figures above, Figure 1 and Figure 3 involve turning the heart about a fixed point. Figure 1 rotates the heart about the point A . Figure 3 rotates the heart about the point directly to the right of A . Figure 2 does a translation, not a rotation.

Vocabulary

Center of rotation
A ***center of rotation*** is the fixed point that a figure rotates about when undergoing a rotation.

Rotation
A ***rotation*** is a transformation that rotates (turns) an image a certain amount about a certain point.

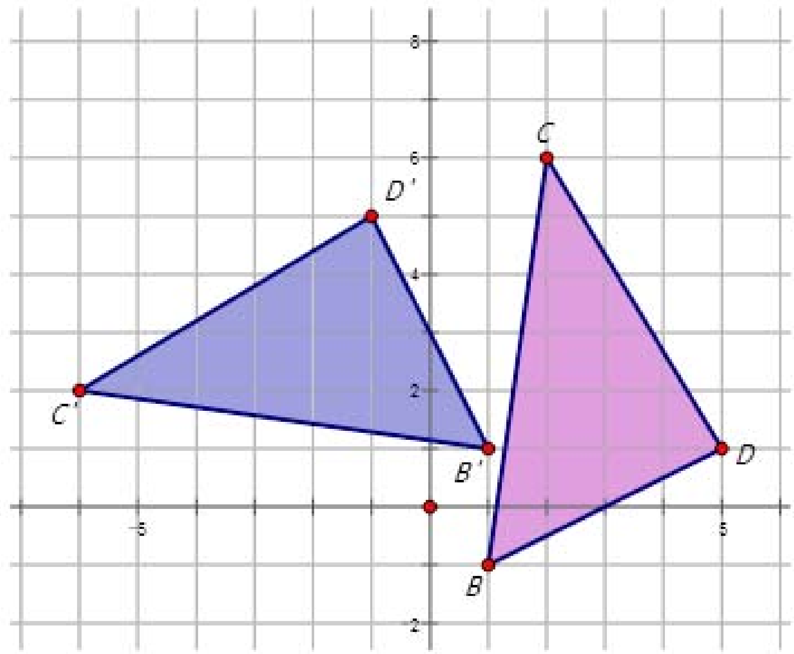
Image
In a transformation, the final figure is called the ***image*** .

Preimage
In a transformation, the original figure is called the ***preimage.***

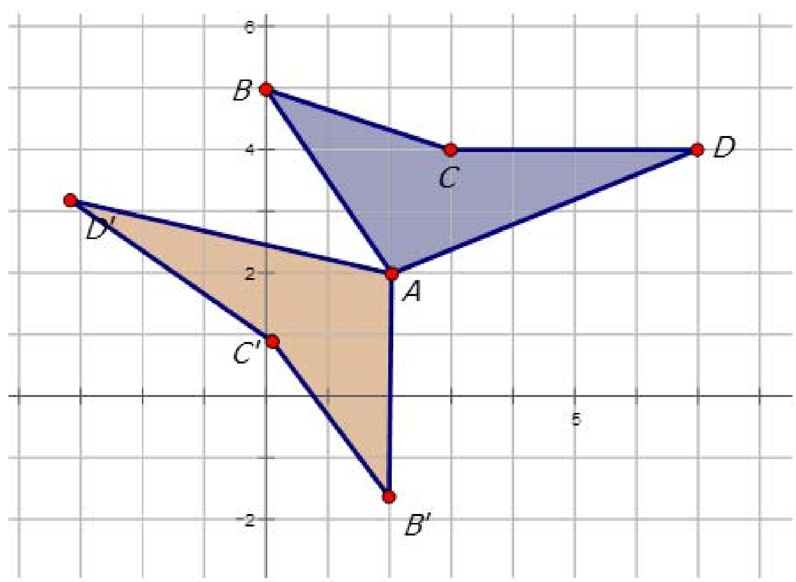
Transformation
A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

Guided Practice

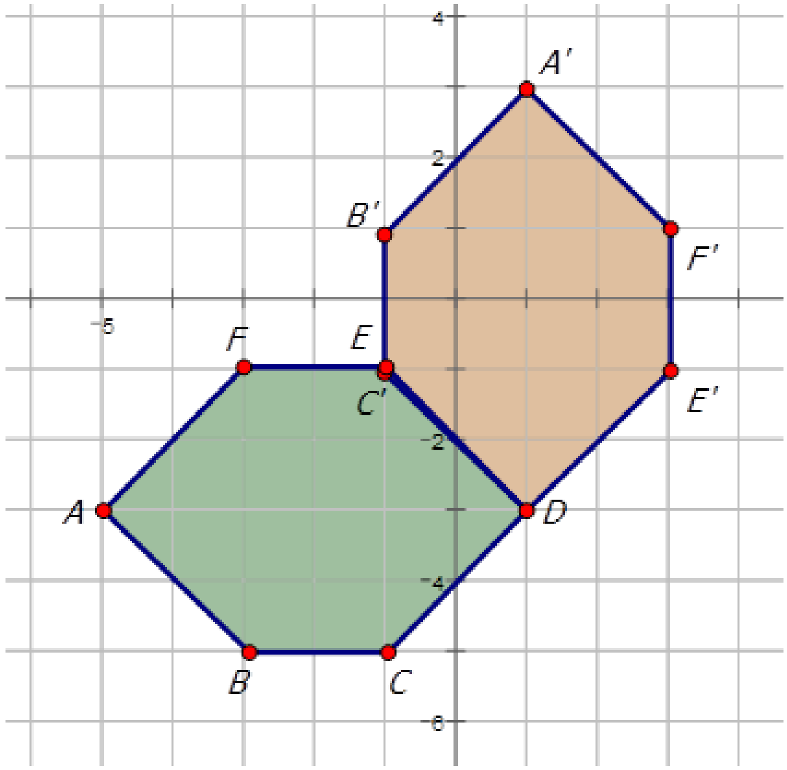
1. Describe the rotation of the pink triangle in the diagram below.



2. Describe the rotation of the blue polygon in the diagram below.



3. Describe the rotation of the green hexagon in the diagram below.



Answers:

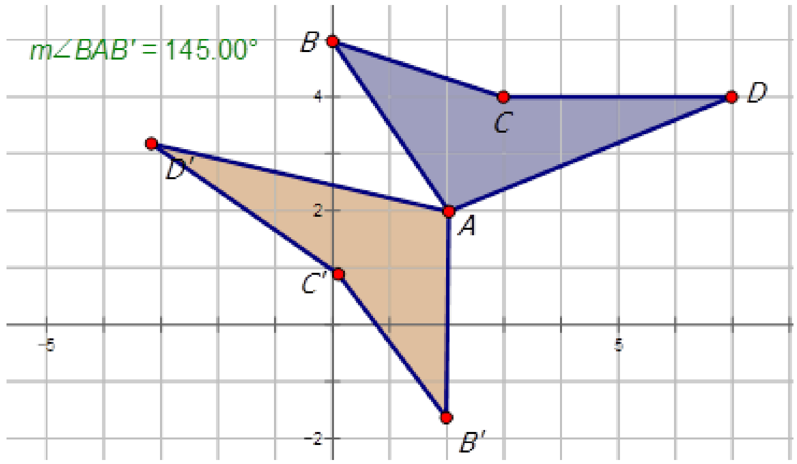
1. Examine the points of the preimage and the rotated image (the blue triangle).

TABLE 6.2:

Points on BCD	$B(1,-1)$	$C(2,6)$	$D(5,1)$
Points on $B'C'D'$	$B'(1,1)$	$C'(-6,2)$	$D'(-1,5)$

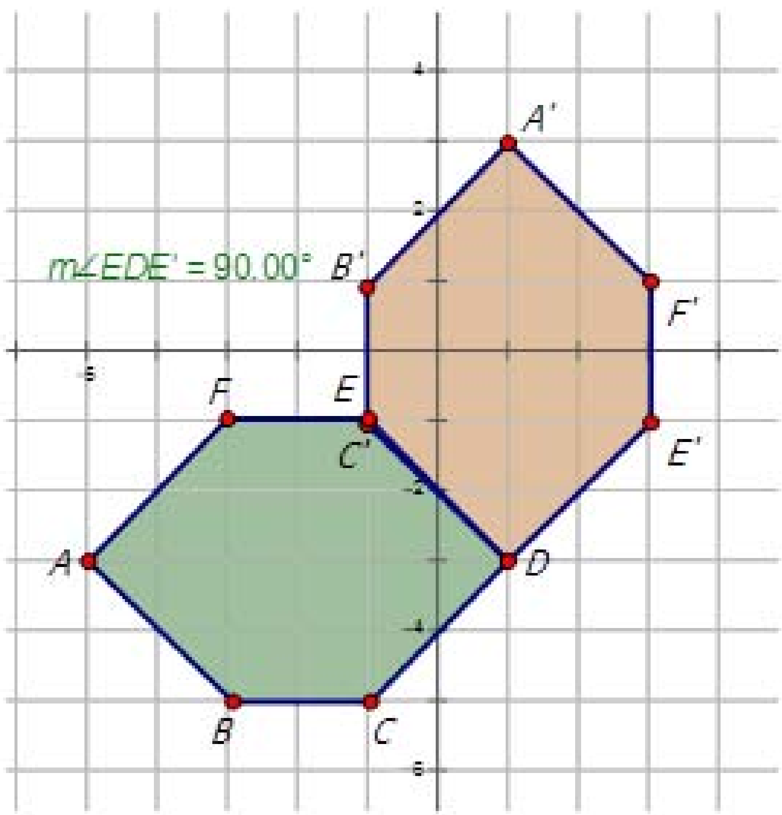
These points represent a rotation of 90° CW about the origin. Each coordinate point (x,y) has become the point $(-y,x)$.

2. For this image, look at the rotation. It is not rotated about the origin but rather about the point A . We can measure the angle of rotation:



The blue polygon is being rotated about the point A 145° clockwise. You would say that the blue polygon is rotated 145° CW to form the orange polygon.

3. For this image, look at the rotation. It is not rotated about the origin but rather about the point A . We can measure the angle of rotation:



The green polygon is being rotated about the point D 90° clockwise. You would say that the green hexagon is rotated 90° CW to form the orange hexagon.

Practice

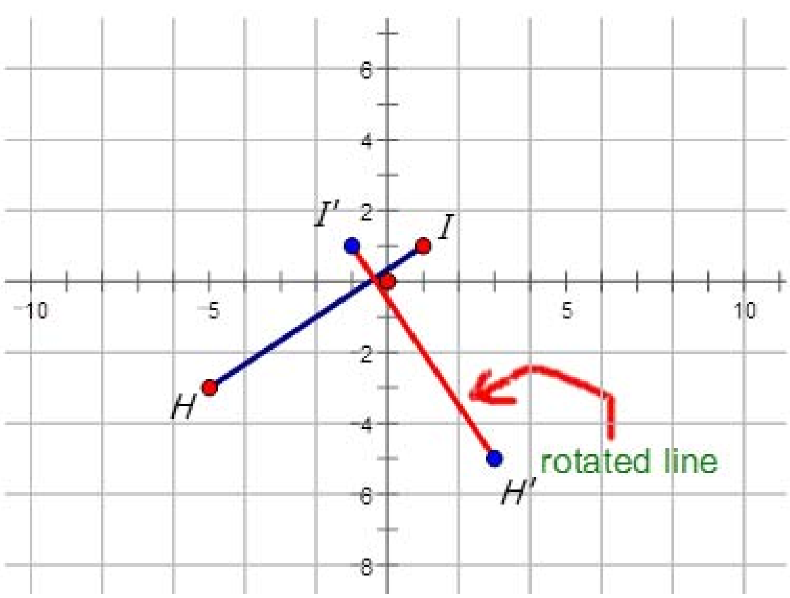
If the following points were rotated about the origin with a 180° CCW rotation, what would be the coordinates of the rotated points?

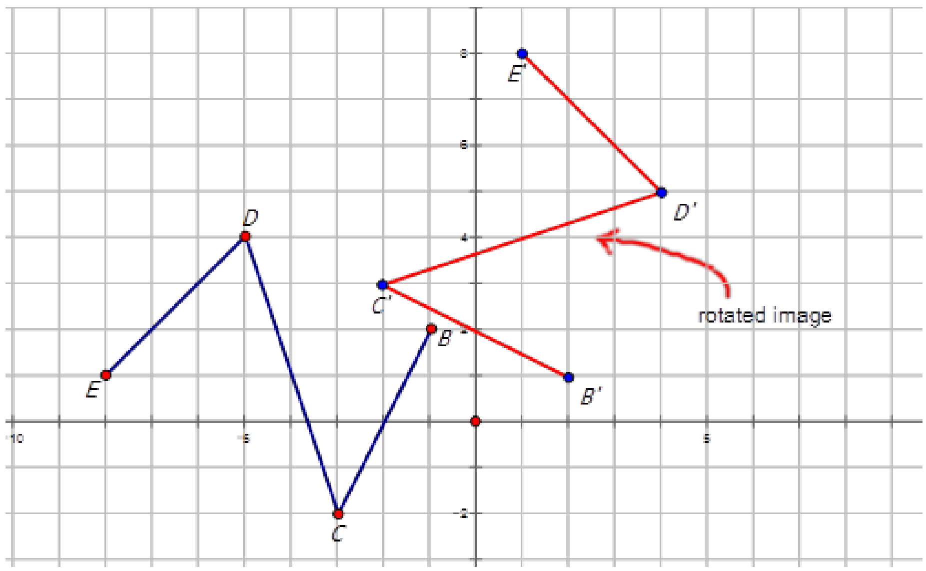
1. (3, 1)
2. (4, -2)
3. (-5, 3)
4. (-6, 4)
5. (-3, -3)

If the following points were rotated about the origin with a 90° CW rotation, what would be the coordinates of the rotated points?

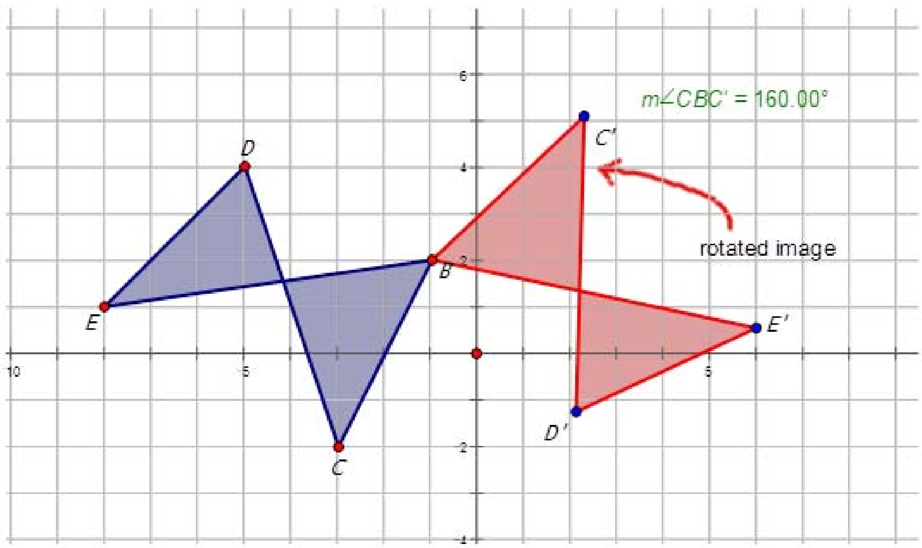
6. (-4, 3)
7. (5, -4)
8. (-5, -4)
9. (3, 3)
10. (-8, -9)

Describe the following rotations:

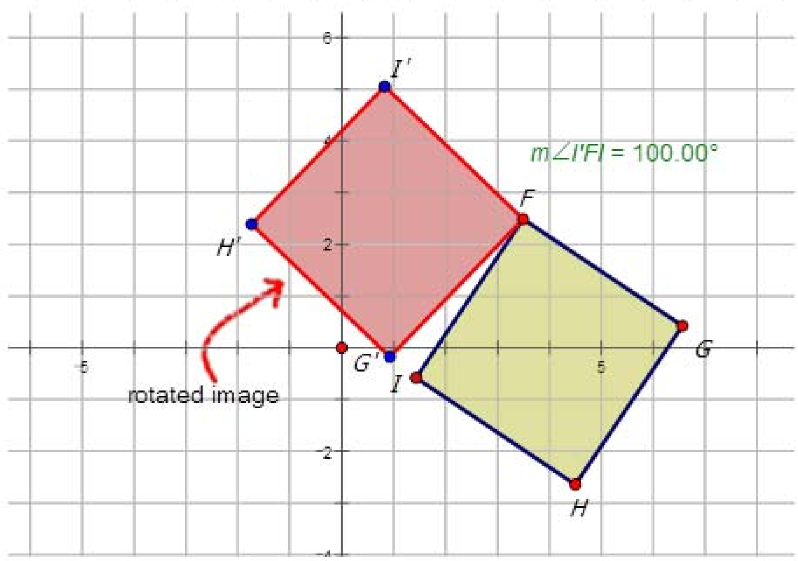
11. 



12.



13.



14.

15. Why is it not necessary to specify the direction when rotating 180° ?

CONCEPT 7 SLT 14 Identify the degree of rotational symmetry and the number of lines of reflectional symmetry

What if you you had a six-pointed star and you rotated that star less than 360° ? If the rotated star looked exactly the same as the original star, what would that say about the star? After completing this Concept, you'll be able to determine if a figure like this one has rotational symmetry.

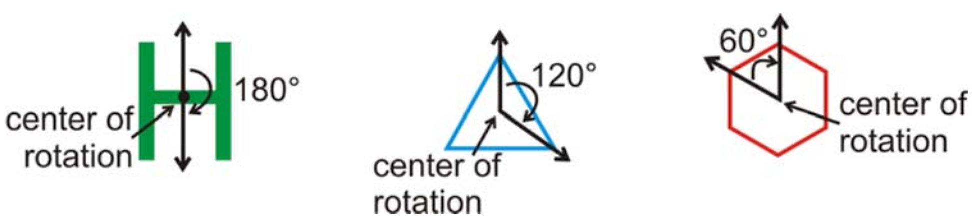
Watch This

MEDIA
Click image to the left for more content.

Rotation Symmetry CK-12

Guidance

Rotational symmetry is present when a figure can be rotated (less than 360°) such that it looks like it did before the rotation. The **center of rotation** is the point a figure is rotated around such that the rotational symmetry holds.

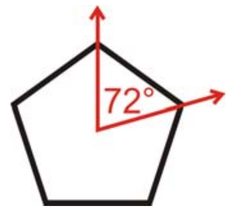


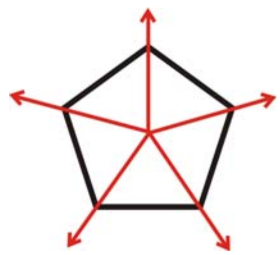
For the *H* , we can rotate it twice, the triangle can be rotated 3 times and still look the same and the hexagon can be rotated 6 times.

Example A

Determine if the figure below has rotational symmetry. Find the angle and how many times it can be rotated.

The pentagon can be rotated 5 times. Because there are 5 lines of rotational symmetry, the angle would be $\frac{360^\circ}{5} = 72^\circ$.





Example B

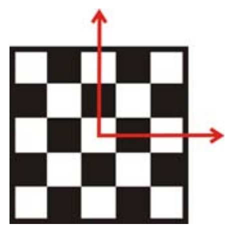
Determine if the figure below has rotational symmetry. Find the angle and how many times it can be rotated.

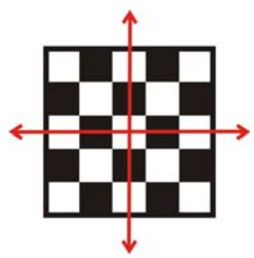
The *N* can be rotated twice. This means the angle of rotation is 180° .

Example C

Determine if the figure below has rotational symmetry. Find the angle and how many times it can be rotated.

The checkerboard can be rotated 4 times. There are 4 lines of rotational symmetry, so the angle of rotation is $\frac{360^\circ}{4} = 90^\circ$.





MEDIA

Click image to the left for more content.

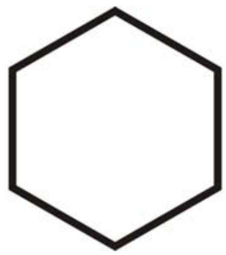
Rotation Symmetry CK-12

Guided Practice

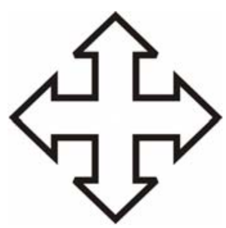
Find the angle of rotation and the number of times each figure can rotate.

1.

2.



3.



Answers:

1. The parallelogram can be rotated twice. The angle of rotation is 180° .
2. The hexagon can be rotated six times. The angle of rotation is 60° .
3. This figure can be rotated four times. The angle of rotation is 90° .

Practice

1. If a figure has 3 lines of rotational symmetry, it can be rotated _______ times.
2. If a figure can be rotated 6 times, it has _______ lines of rotational symmetry.
3. If a figure can be rotated *n* times, it has _______ lines of rotational symmetry.
4. To find the angle of rotation, divide 360° by the total number of _____________.
5. Every square has an angle of rotation of _________.

Determine whether each statement is true or false.

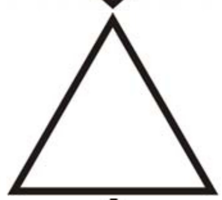
6. Every parallelogram has rotational symmetry.
7. Every figure that has line symmetry also has rotational symmetry.

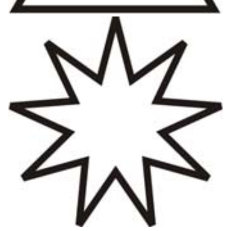
Determine whether the words below have rotation symmetry.

8. **OHIO**
9. **MOW**
10. **WOW**
11. **KICK**
12. **pod**

Find the angle of rotation and the number of times each figure can rotate.

13. 

14. 

15. 

Determine if the figures below have rotation symmetry. Identify the angle of rotation.

16. 

17. 

18. 

Concept 8 SLT 15 Specify a sequence of transformations that will carry a given figure onto another.

What if you were given the coordinates of a quadrilateral and you were asked to reflect the quadrilateral and then translate it? What would its new coordinates be? After completing this Concept, you'll be able to perform a series of transformations on a figure like this one in the coordinate plane.

Watch This

Multimedia

MEDIA

Click image to the left for more content.

Composing Transformations CK-12

Guidance

Transformations Summary

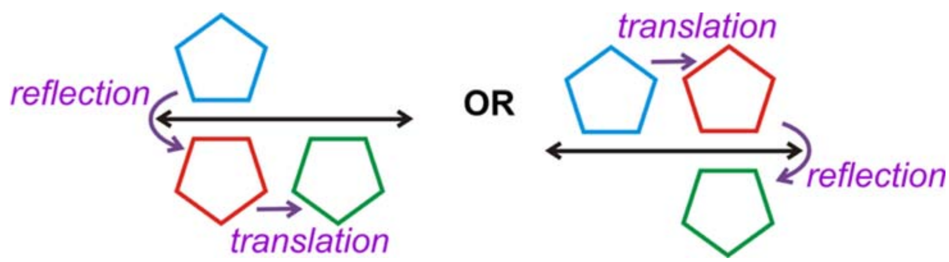
A **transformation** *is an operation that moves, flips, or otherwise changes a figure to create a new figure. A* **rigid transformation** *(also known as an* **isometry** *or* **congruence transformation** *) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the* **image** *. The original figure is called the* **preimage** *.*

There are three rigid transformations: translations, rotations and reflections. A **translation** *is a transformation that moves every point in a figure the same distance in the same direction. A* **rotation** *is a transformation where a figure is turned around a fixed point to create an image. A* **reflection** *is a transformation that turns a figure into its mirror image by flipping it over a line.*

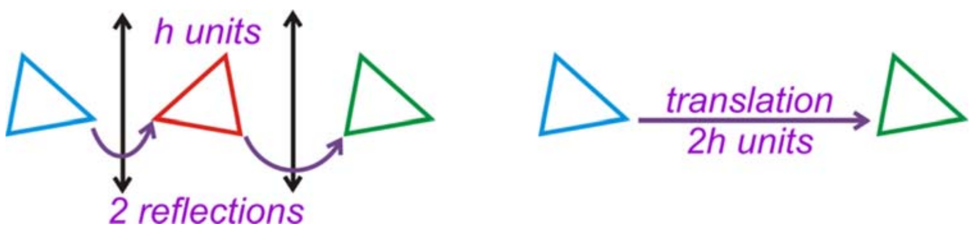
Composition of Transformations

A **composition (of transformations)** is when more than one transformation is performed on a figure. Compositions can always be written as one rule. You can compose any transformations, but here are some of the most common compositions:

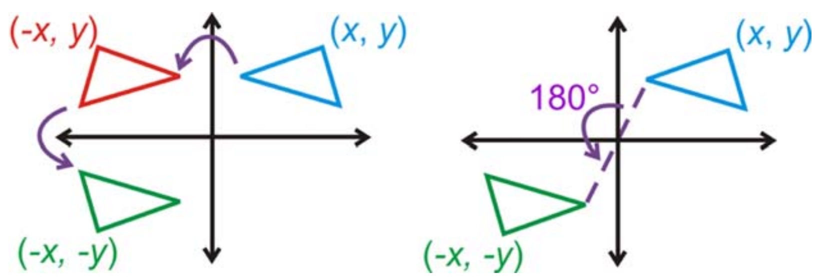
1) A **glide reflection** is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.



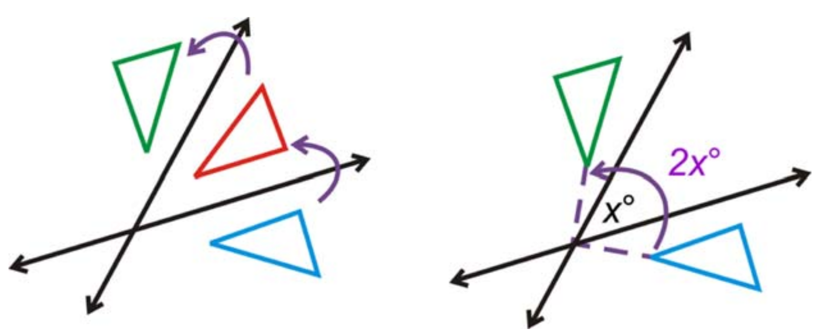
2) The composition of two reflections over parallel lines that are h units apart is the same as a translation of $2h$ units (**Reflections over Parallel Lines Theorem**).



3) If you compose two reflections over each axis, then the final image is a rotation of 180° around the origin of the original (**Reflection over the Axes Theorem**).

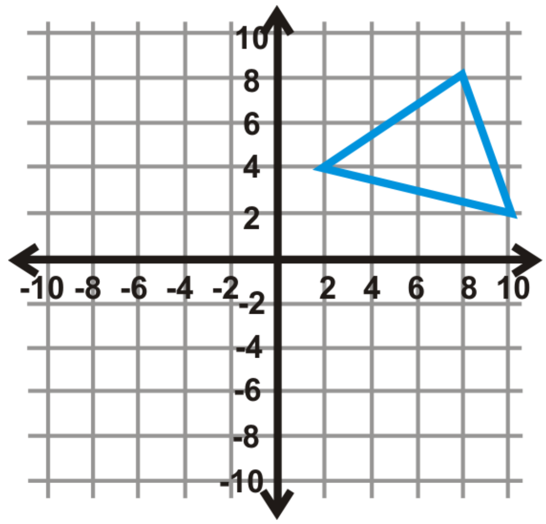


4) A composition of two reflections over lines that intersect at $x°$ is the same as a rotation of $2x°$. The center of rotation is the point of intersection of the two lines of reflection (**Reflection over Intersecting Lines Theorem**).

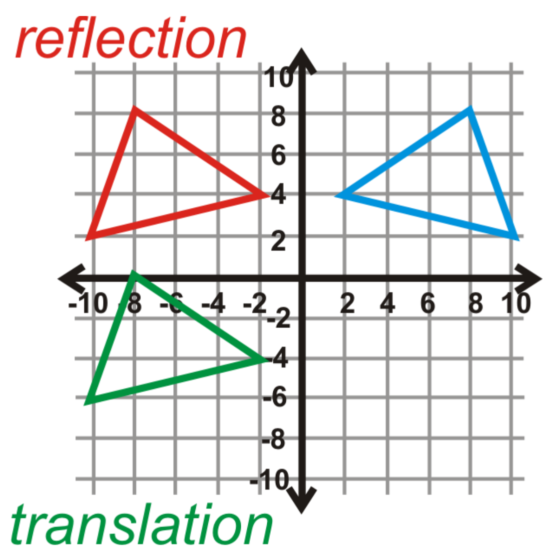


Example A

Reflect $\triangle ABC$ over the $y-$ axis and then translate the image 8 units down.



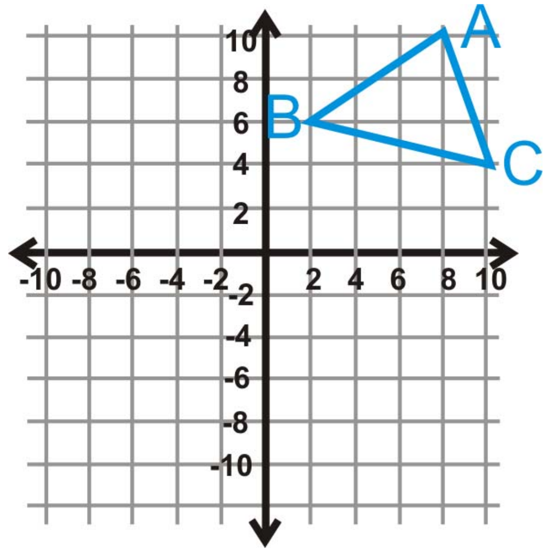
The green image to the right is the final answer.



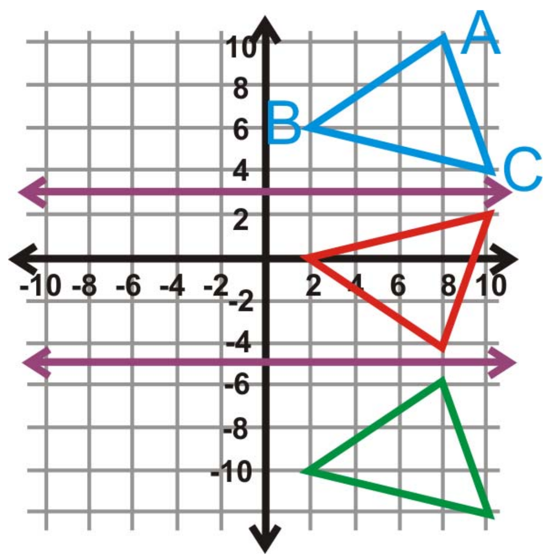
$$A(8,8) \rightarrow A''(-8,0)$$
$$B(2,4) \rightarrow B''(-2,-4)$$
$$C(10,2) \rightarrow C''(-10,-6)$$

Example B

Reflect $\triangle ABC$ over $y = 3$ and then reflect the image over $y = -5$.



Order matters, so you would reflect over $y = 3$ first, (red triangle) then reflect it over $y = -5$ (green triangle).



Example C

A square is reflected over two lines that intersect at a 79° angle. What one transformation will this be the same as?

From the Reflection over Intersecting Lines Theorem, this is the same as a rotation of $2 \cdot 79^\circ = 178^\circ$.

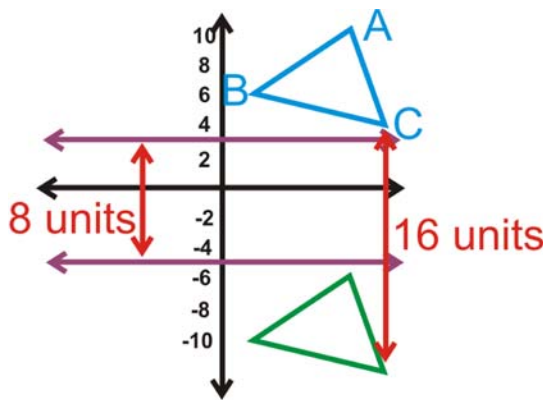
MEDIA

Click image to the left for more content.

Composing Transformations CK-12

Guided Practice

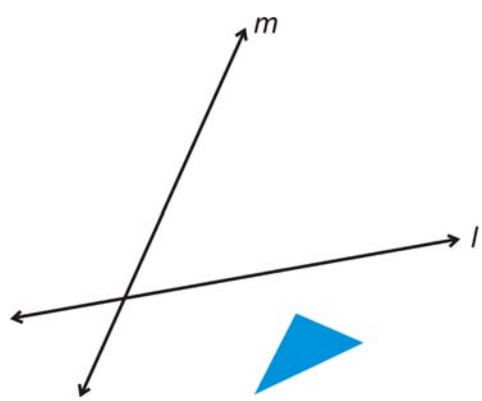
1. Write a single rule for $\triangle ABC$ to $\triangle A''B''C''$ from Example C.



2. $\triangle DEF$ has vertices $D(3,-1), E(8,-3)$, and $F(6,4)$. Reflect $\triangle DEF$ over $x=-5$ and then $x=1$. Determine which one translation this double reflection would be the same as.

3. Reflect $\triangle DEF$ from Question 2 over the $x-$ axis, followed by the $y-$ axis. Find the coordinates of $\triangle D''E''F''$ and the one transformation this double reflection is the same as.

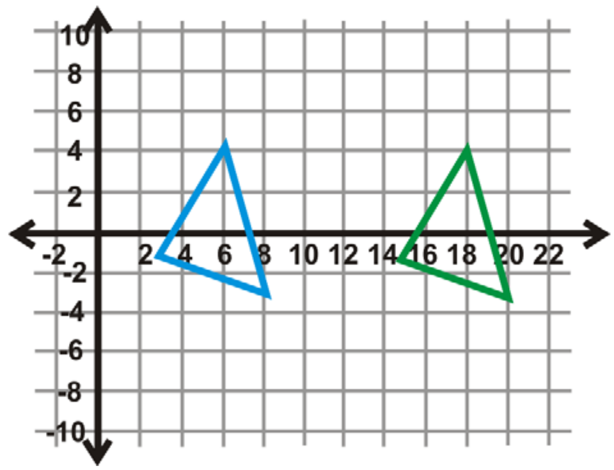
4. Copy the figure below and reflect the triangle over l , followed by m .



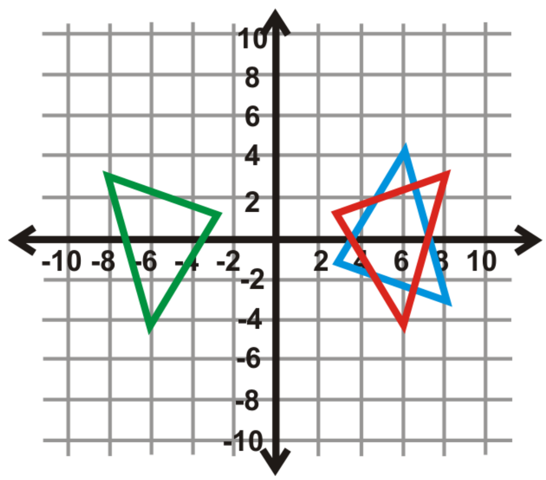
Answers:

1. In the graph, the two lines are 8 units apart $(3-(-5)=8)$. The figures are 16 units apart. The ***double*** reflection is the same as a translation that is ***double*** the distance between the parallel lines. $(x,y) \rightarrow (x,y-16)$.

2. From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of $2(1-(-5))$ or 12 units.

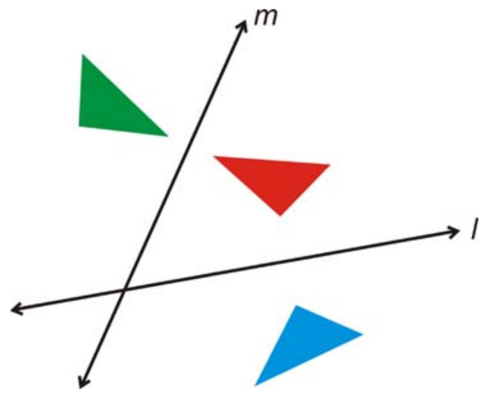


3. $\triangle D''E''F''$ is the green triangle in the graph to the left. If we compare the coordinates of it to $\triangle DEF$, we have:



$$D(3,-1) \to D''(-3,1)$$
$$E(8,-3) \to E''(-8,3)$$
$$F(6,4) \to F''(-6,-4)$$

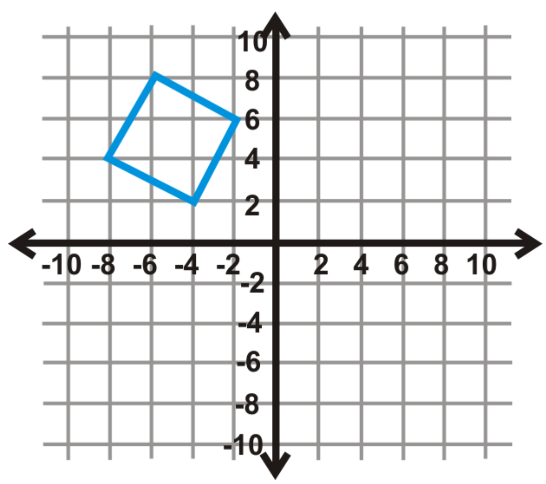
4. The easiest way to reflect the triangle is to fold your paper on each line of reflection and draw the image. The final result should look like this (the green triangle is the final answer):



Practice

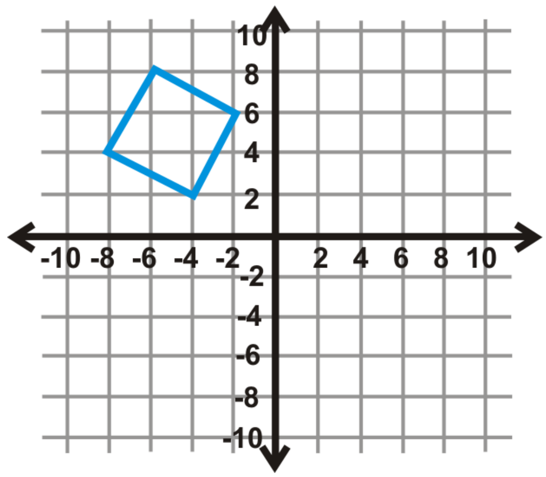
1. *Explain* why the composition of two or more isometries must also be an isometry.
2. What one transformation is the same as a reflection over two parallel lines?
3. What one transformation is the same as a reflection over two intersecting lines?

Use the graph of the square to the left to answer questions 4-6.



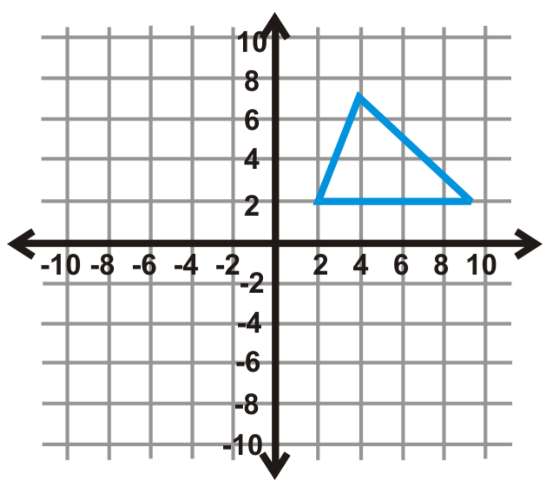
4. Perform a glide reflection over the $x-$ axis and to the right 6 units. Write the new coordinates.
5. What is the rule for this glide reflection?
6. What glide reflection would move the image back to the preimage?

Use the graph of the square to the left to answer questions 7-9.



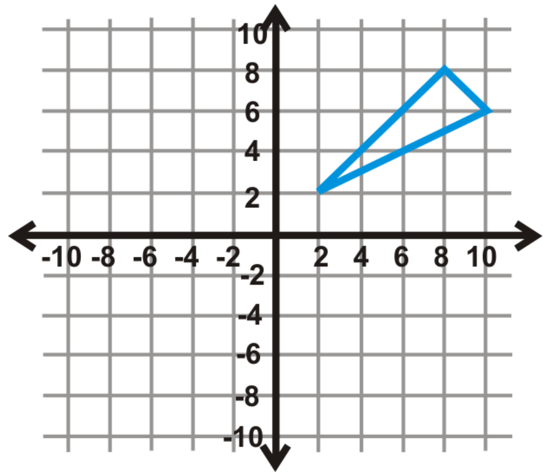
7. Perform a glide reflection to the right 6 units, then over the $x-$ axis. Write the new coordinates.
8. What is the rule for this glide reflection?
9. Is the rule in #8 different than the rule in #5? Why or why not?

Use the graph of the triangle to the left to answer questions 10-12.



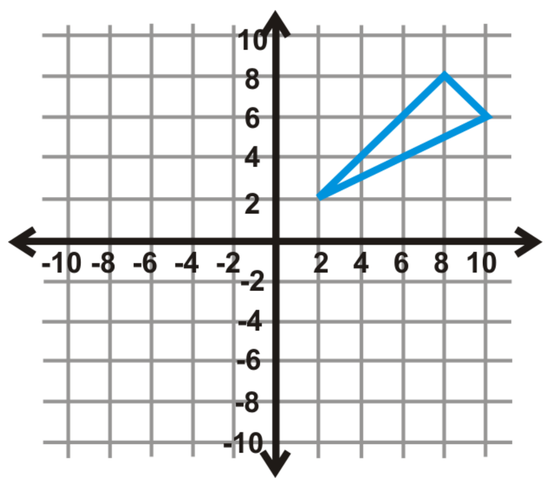
10. Perform a glide reflection over the $y-$ axis and down 5 units. Write the new coordinates.
11. What is the rule for this glide reflection?
12. What glide reflection would move the image back to the preimage?

Use the graph of the triangle to the left to answer questions 13-15.



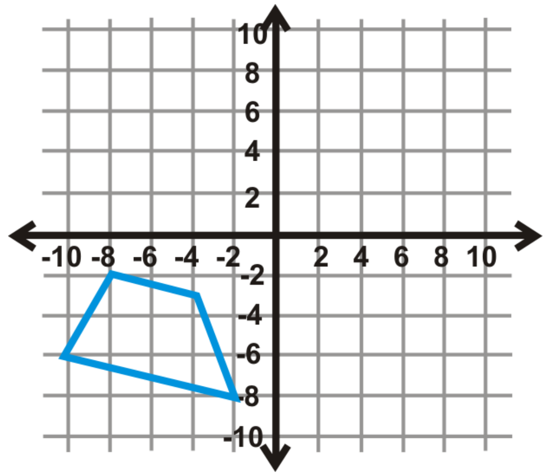
13. Reflect the preimage over $y = -1$ followed by $y = -7$. Draw the new triangle.
14. What one transformation is this double reflection the same as?
15. Write the rule.

Use the graph of the triangle to the left to answer questions 16-18.



16. Reflect the preimage over $y = -7$ *followed* by $y = -1$. Draw the new triangle.
17. What one transformation is this double reflection the same as?
18. Write the rule.
19. How do the final triangles in #13 and #16 differ?

Use the trapezoid in the graph to the left to answer questions 20-22.



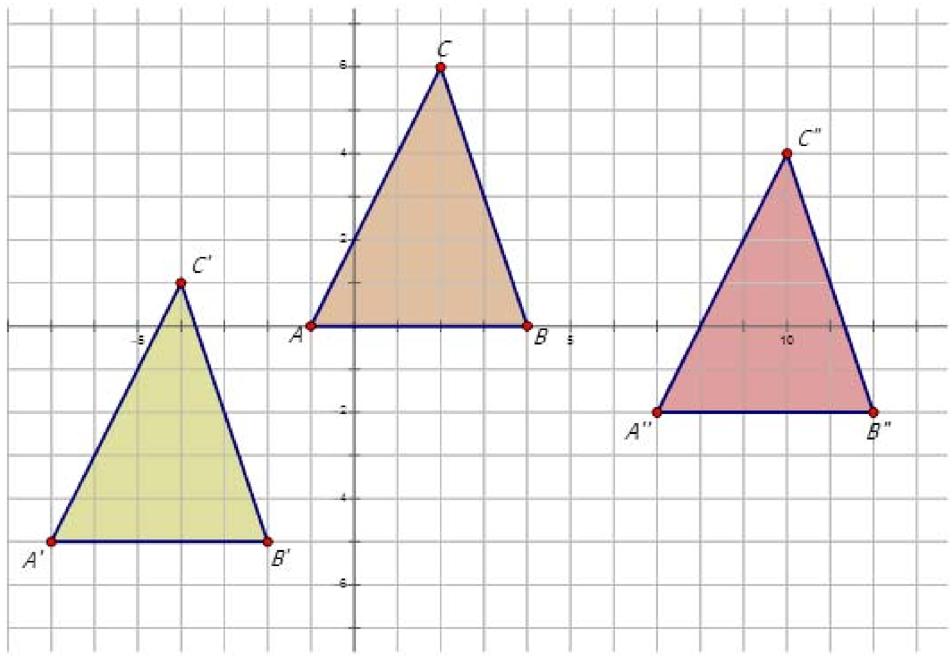
20. Reflect the preimage over the $x-$ axis then the $y-$ axis. Draw the new trapezoid.
21. Now, start over. Reflect the trapezoid over the $y-$ axis then the $x-$ axis. Draw this trapezoid.
22. Are the final trapezoids from #20 and #21 different? Why do you think that is?

Answer the questions below. Be as specific as you can.

23. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
24. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
25. Two lines intersect at a 165° angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
26. What is the center of rotation for #25?
27. Two lines intersect at an 83° angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
28. A preimage and its image are 244° apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
29. A preimage and its image are 98° apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
30. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?

CONCEPT 9 SLT 16 Combine transformations and write the associated function.

Look at the following diagram. It involves two translations. Identify the two translations of triangle *ABC* .



Watch This

First watch this video to learn about composite transformations.

MEDIA
Click image to the left for more content.

CK-12 Foundation Chapter10CompositeTransformationsA

Then watch this video to see some examples.

MEDIA
Click image to the left for more content.

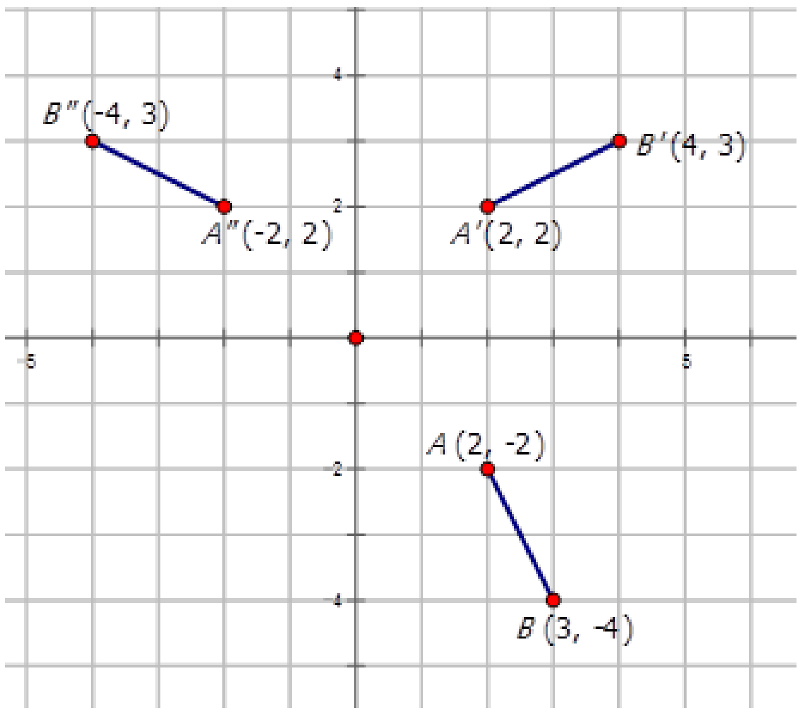
CK-12 Foundation Chapter10CompositeTransformationsB

Guidance

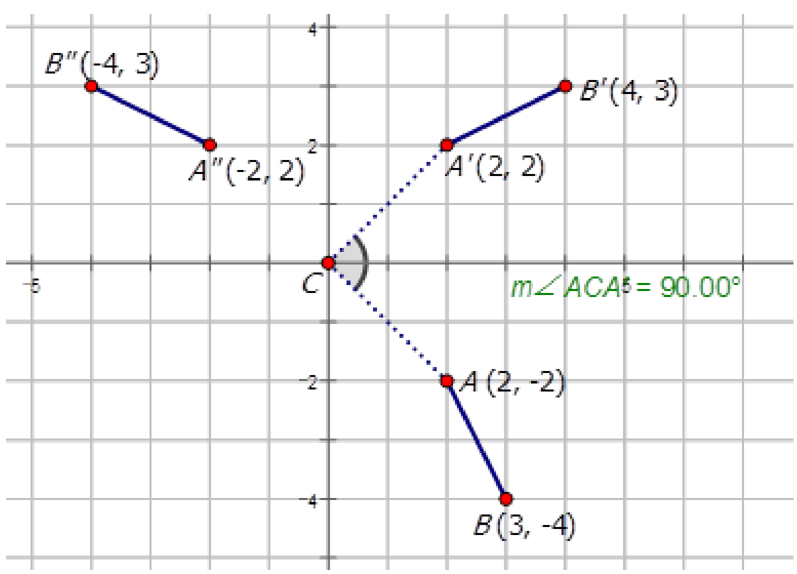
In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image).

Example A

Describe the transformations in the diagram below. The transformations involve a reflection and a rotation.



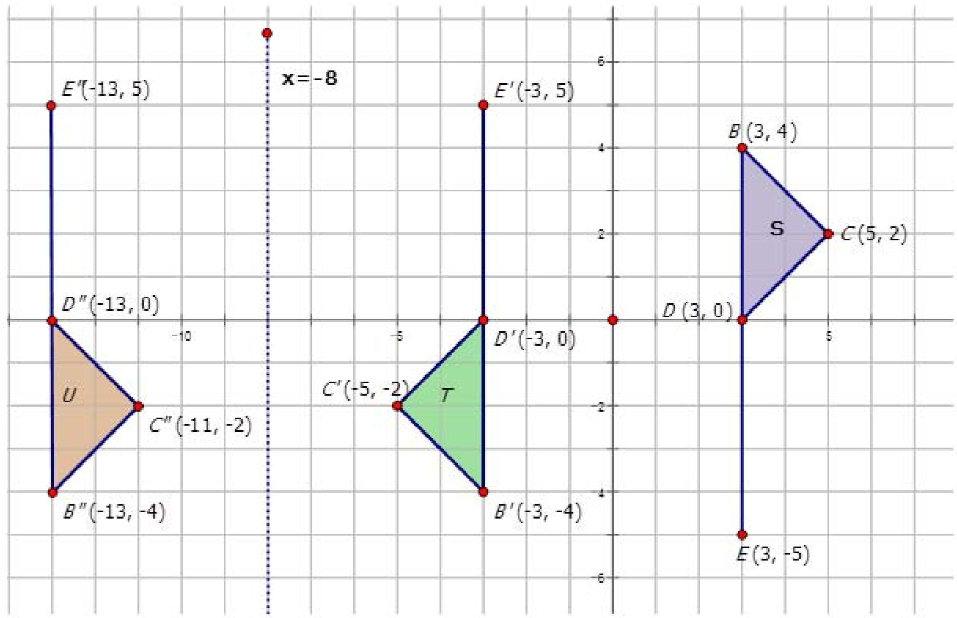
Solution: First line *AB* is rotated about the origin by 90° CCW.



Then the line segment $A'B'$ is reflected about the y -axis to produce line segment $A''B''$. The transformation from pre-image to image is $(x,y) \rightarrow (y,x)$.

Example B

Describe the transformations in the diagram below.



Solution: The flag in diagram S is rotated about the origin 180° to produce flag T. You know this because if you look at one point you notice that both x - and y -coordinate points is multiplied by -1 which is consistent with a 180° rotation about the origin. Flag T is then reflected about the line $x = -8$ to produce Flag U. The transformation from pre-image to image is $(x,y) \rightarrow (x-16,-y)$.

Example C

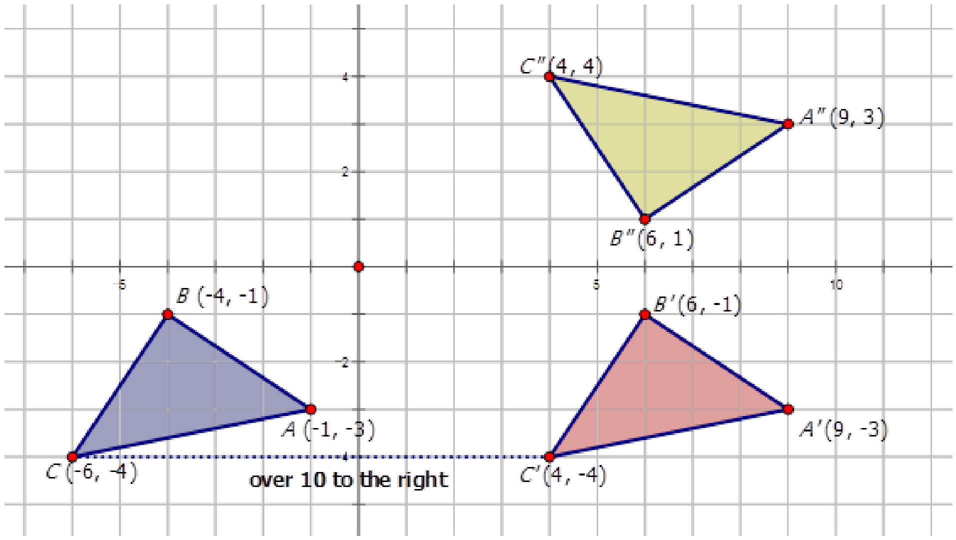
Triangle ABC where the vertices of ΔABC are $A(-1,-3)$, $B(-4,-1)$, and $C(-6,-4)$ undergoes a composition of transformations described as:

a) a translation 10 units to the right, then

b) a reflection in the x -axis.

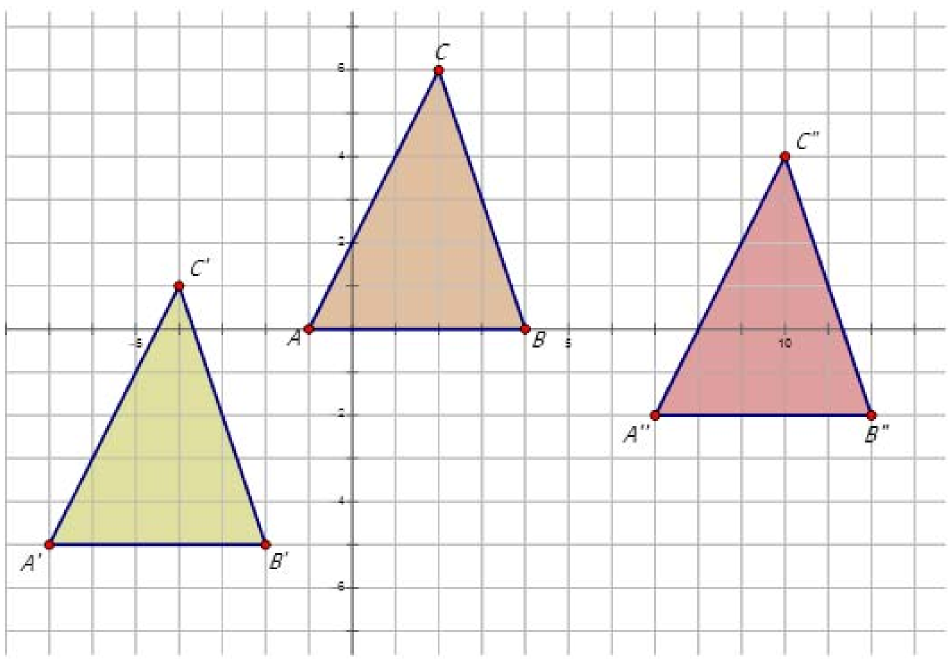
Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied?

Solution:

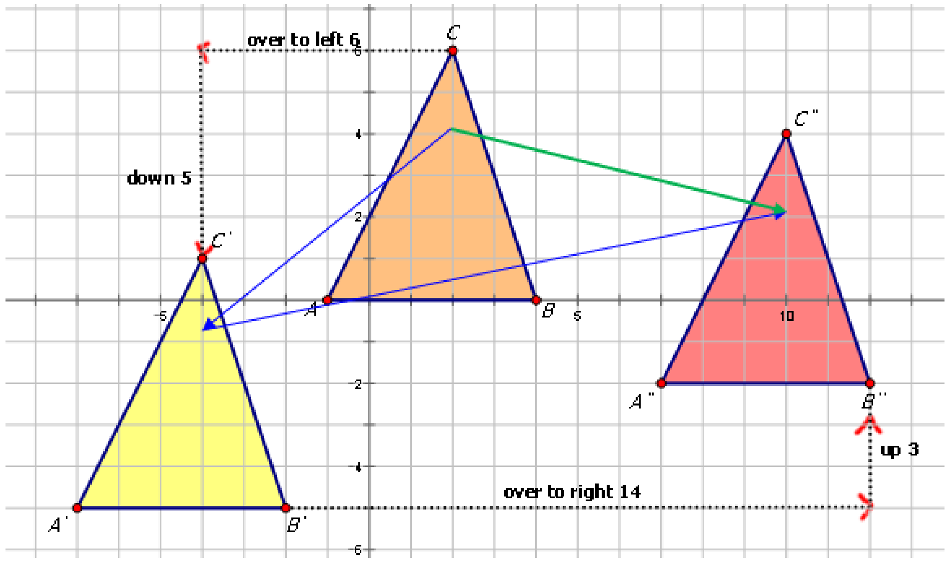


Triangle $A''B''C''$ is the final triangle after all transformations are applied. It has vertices of $A''(9,3)$, $B''(6,1)$, and $C''(4,4)$. The transformation from pre-image to image is $(x,y) \rightarrow (x+10,-y)$.

Concept Problem Revisited



ΔABC moves over 6 to the left and down 5 to produce $\Delta A'B'C'$. Then $\Delta A'B'C'$ moves over 14 to the right and up 3 to produce $\Delta A''B''C''$. These translations are represented by the blue arrows in the diagram.



All together ΔABC moves over 8 to the right and down 2 to produce $\Delta A''B''C''$. The total translations for this movement are seen by the green arrow in the diagram above.

Vocabulary

Image
In a transformation, the final figure is called the ***image*** .

Preimage
In a transformation, the original figure is called the ***preimage.***

Transformation
A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

Translation
A ***translation*** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides** .

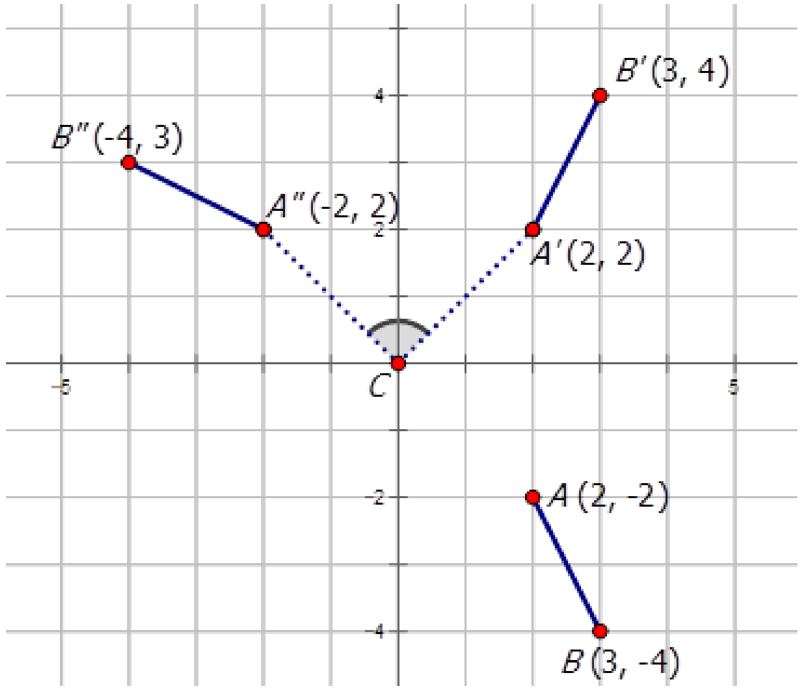
Rotation
A ***rotation*** is a transformation that rotates (turns) an image a certain amount about a certain point.

Reflection
A ***reflection*** is an example of a transformation that flips each point of a shape over the same line.

Composite Transformation
A ***composite transformation*** is when two or more transformations are combined to form a new image from the preimage.

Guided Practice

1. Describe the transformations in the diagram below. The transformations involve a rotation and a reflection.



2. Triangle XYZ has coordinates $X(1,2)$, $Y(-3,6)$ and $Z(4,5)$ .The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle $X'Y'Z'$. Triangle $X'Y'Z'$ is then reflected about the y -axis to form triangle $X''Y''Z''$. Draw the diagram of this composite transformation and determine the vertices for triangle $X''Y''Z''$.

3. The coordinates of the vertices of ΔJAK are $J(1,6)$, $B(2,9)$, and $C(7,10)$.

a) Draw and label ΔJAK .

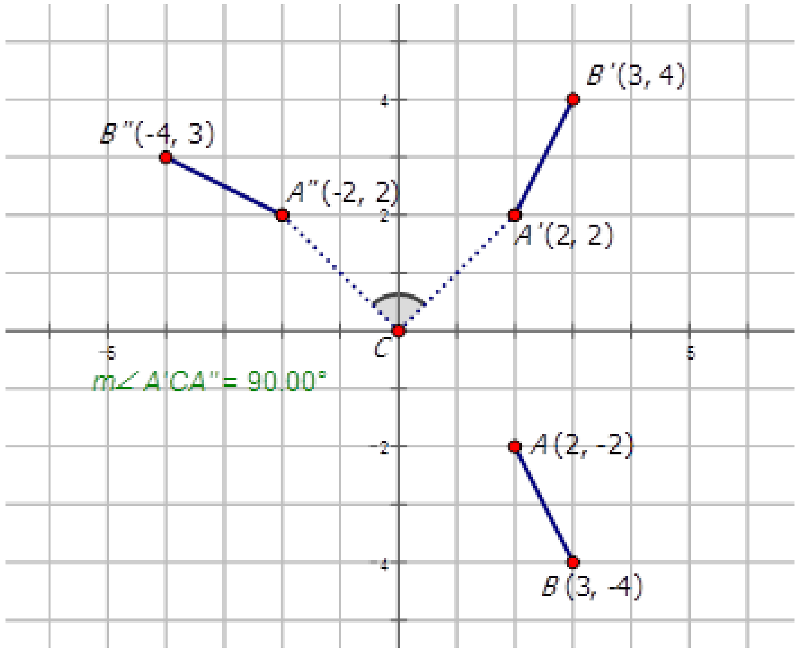
b) ΔJAK is reflected over the line $y = x$. Graph and state the coordinates of $\Delta J'A'K'$.

c) $\Delta J'A'K'$ is then reflected about the x -axis. Graph and state the coordinates of $\Delta J''A''K''$.

d) $\Delta J''A''K''$ undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of $\Delta J'''A'''K'''$.

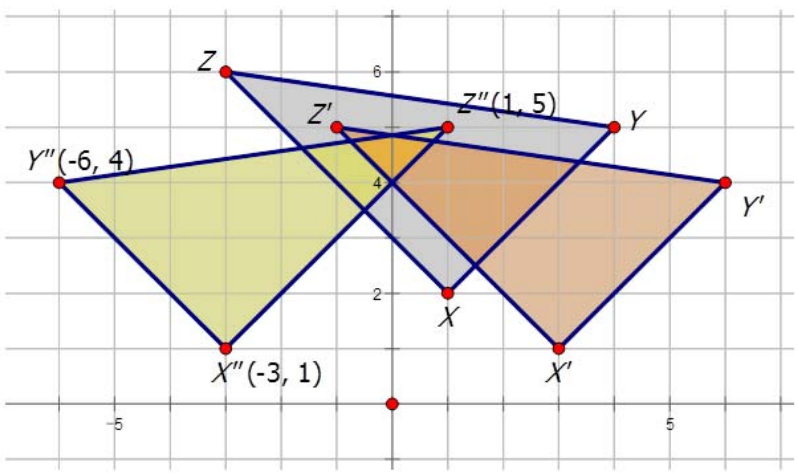
Answers:

1. The transformations involve a reflection and a rotation. First line AB is reflected about the y -axis to produce line $A'B'$.

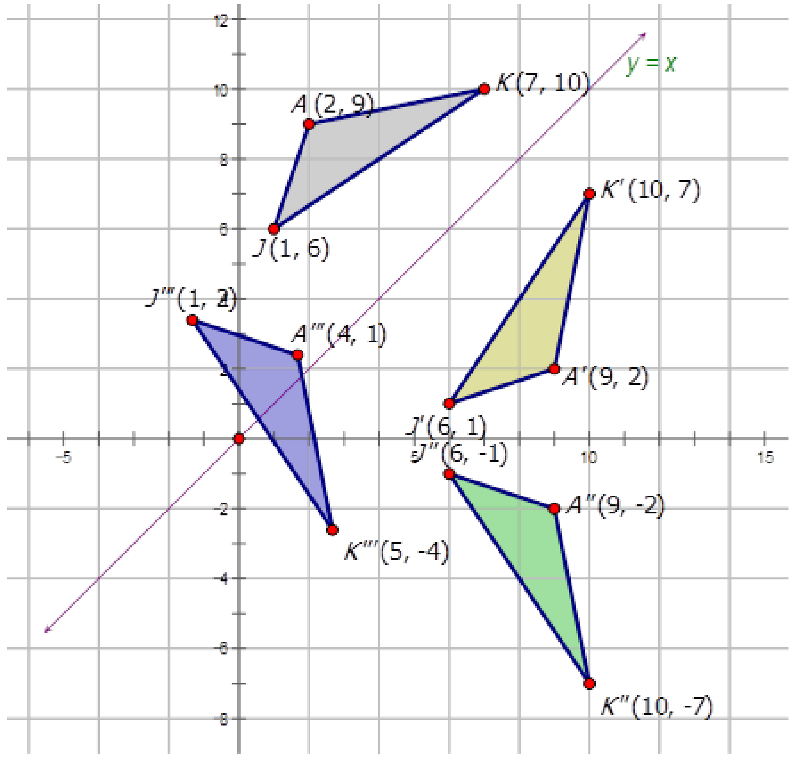


Then the line $A'B'$ is rotated about the origin by 90° CCW to produce line $A''B''$.

2.



3.

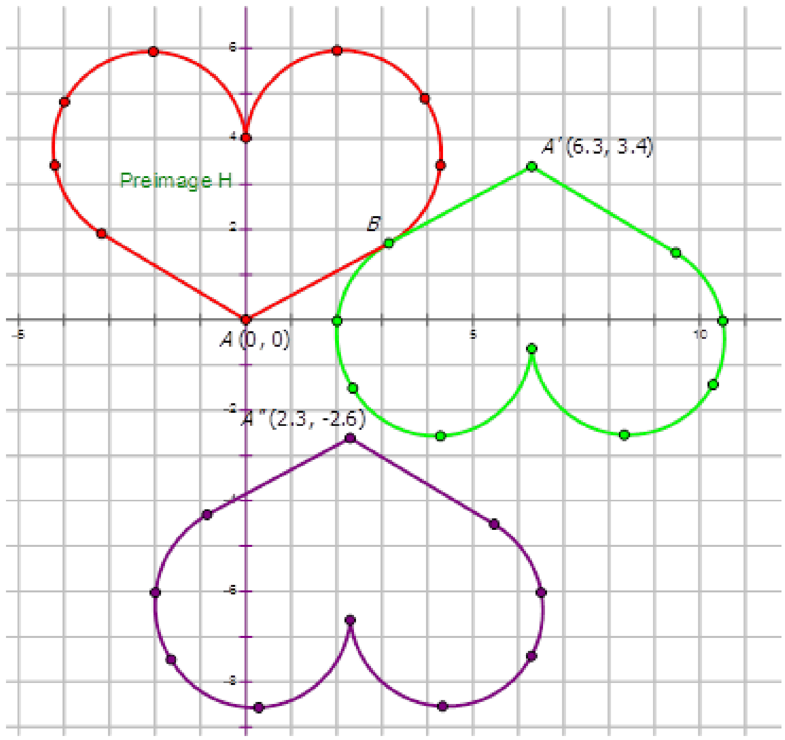


Practice

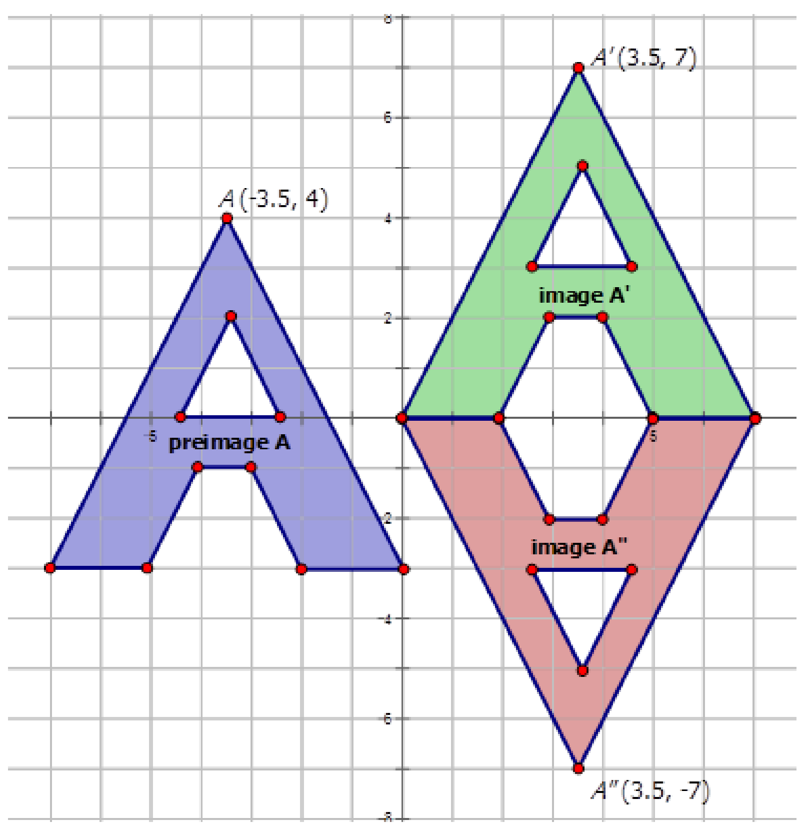
1. A point X has coordinates (-1, -8). The point is reflected across the y -axis to form X' . X' is translated over 4 to the right and up 6 to form X'' . What are the coordinates of X' and X'' ?
2. A point A has coordinates (2, -3). The point is translated over 3 to the left and up 5 to form A' . A' is reflected across the x -axis to form A'' . What are the coordinates of A' and A'' ?
3. A point P has coordinates (5, -6). The point is reflected across the line $y=-x$ to form P' . P' is rotated about the origin 90° CW to form P'' . What are the coordinates of P' and P'' ?
4. Line JT has coordinates $J(-2,-5)$ and $T(2,3)$. The segment is rotated about the origin 180° to form $J'T'$. $J'T'$ is translated over 6 to the right and down 3 to form $J''T''$. What are the coordinates of $J'T'$ and $J''T''$?
5. Line SK has coordinates $S(-1,-8)$ and $K(1,2)$. The segment is translated over 3 to the right and up 3 to form $S'K'$. $S'K'$ is rotated about the origin 90° CCW to form $S''K''$. What are the coordinates of $S'K'$ and $S''K''$?
6. A point K has coordinates (-1, 4). The point is reflected across the line $y=x$ to form K' . K' is rotated about the origin 270° CW to form K'' . What are the coordinates of K' and K'' ?

Describe the following composite transformations:

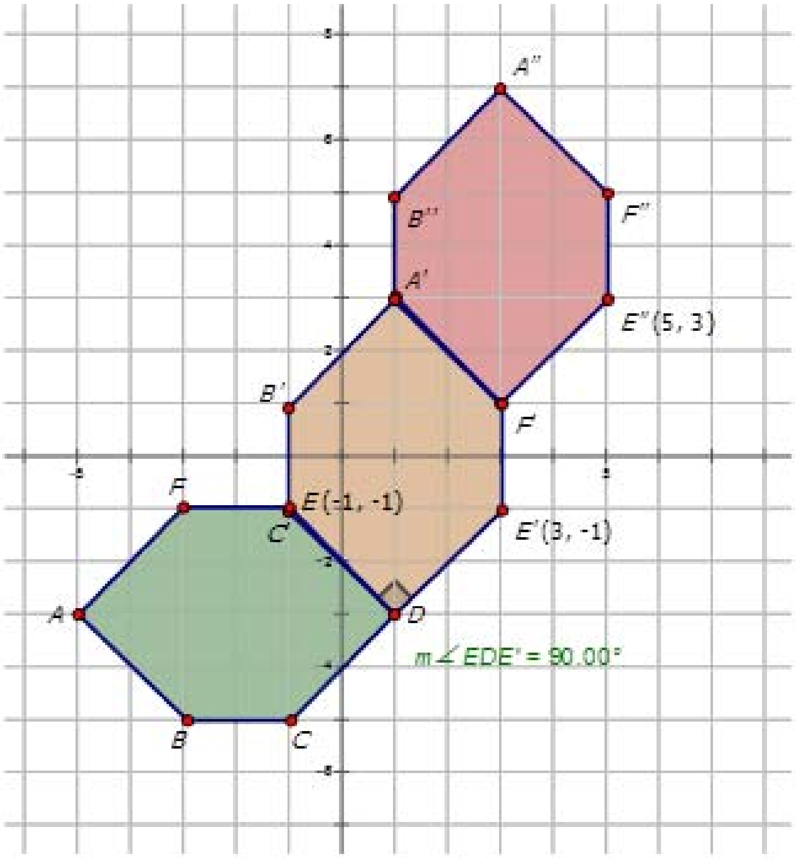
7.



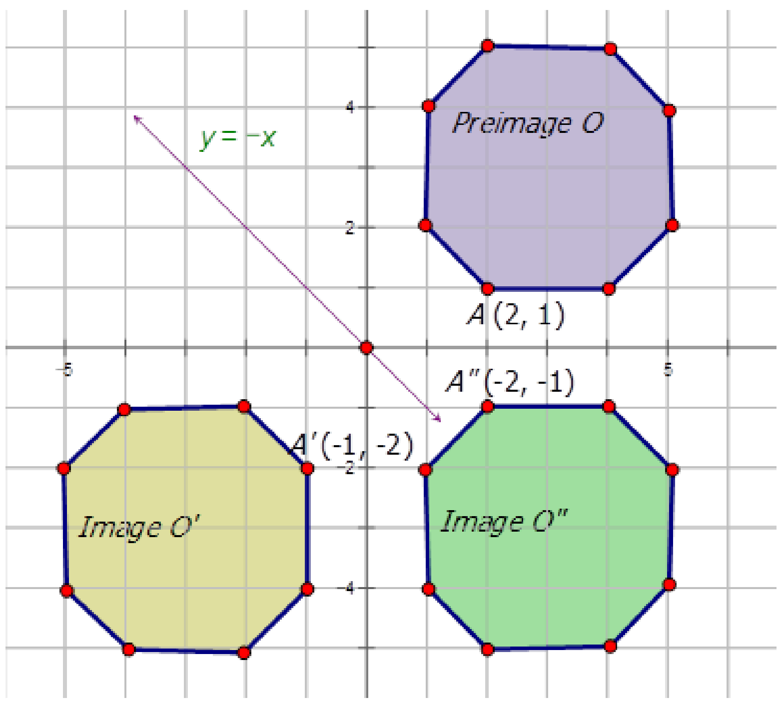
8.

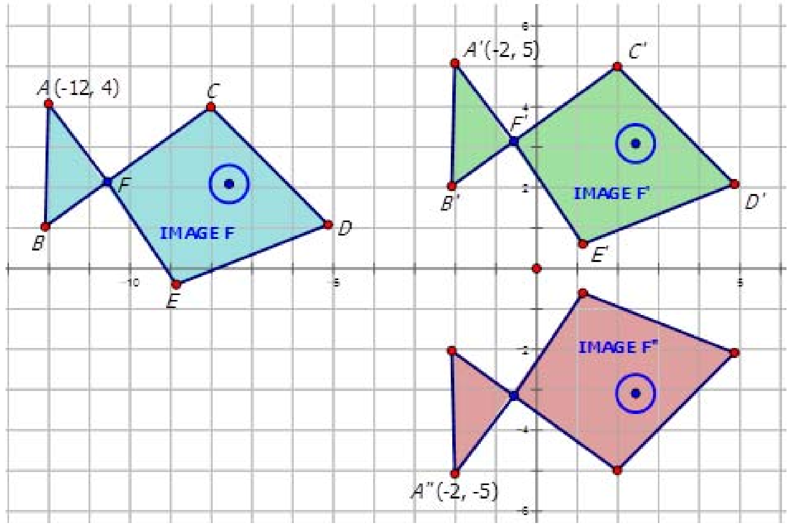


9.


A''
B''
F''
A'
E''(5, 3)
B'
F'
F
E(-1, -1)
C'
E'(3, -1)
A
D
m∠EDE' = 90.00°
B
C

10.

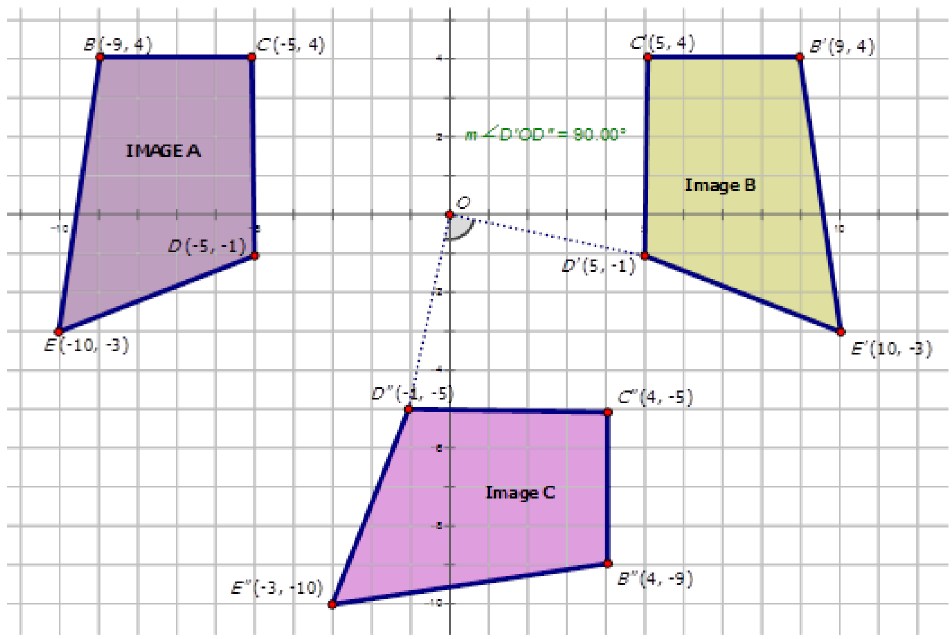

Preimage O
y = −x
A (2, 1)
A''(-2, -1)
A'(-1, -2)
Image O'
Image O''

11. 

12. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?
13. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?
14. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?
15. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it's called a **glide** reflection?

CONCEPT 10 SLT 17 Combine transformations and write the associated function.

The figure below shows a composite transformation of a trapezoid. Write the mapping rule for the composite transformation.



Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image). The order of transformations performed in a composite transformation matters.

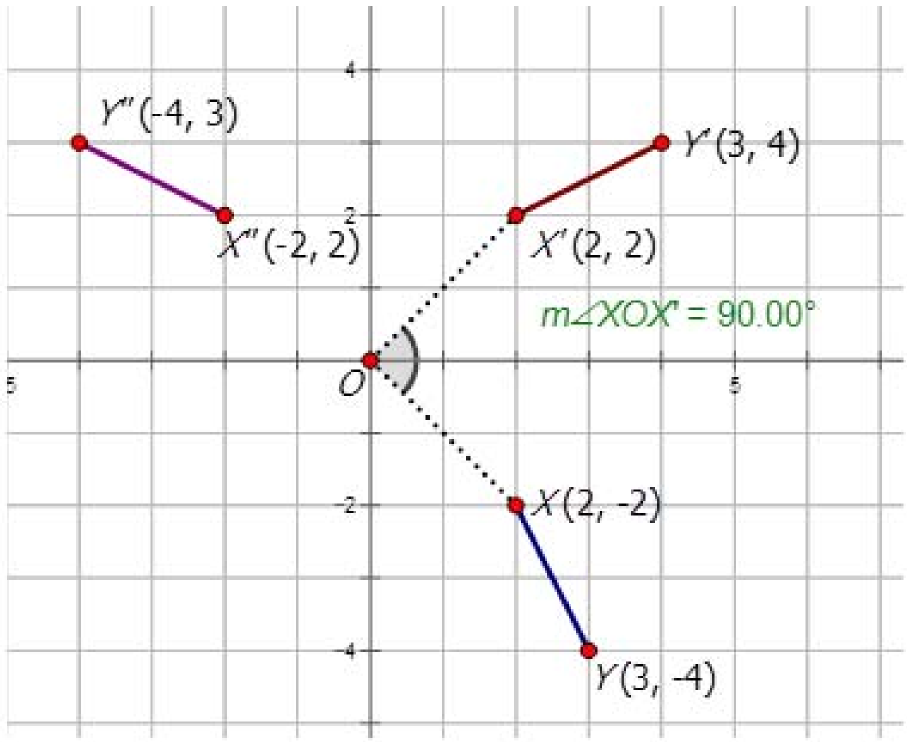
To describe a composite transformation using notation, state each of the transformations that make up the composite transformation and link them with the symbol $\circ$. The transformations are performed in order from right to left. Recall the following notation for translations, reflections, and rotations:

- Translation: $(x,y) \rightarrow (x+a, y+b)$ is a translation of a units to the right and b units up.
- Reflection about the y -axis: $(x,y) \rightarrow (-x,y)$.
- 90° Rotation: $(x,y) = (-y,x)$

Example A

Graph the line segment XY given that $X(2,-2)$ and $Y(3,-4)$. Reflect the line segment about the y-axis and then rotate it 90 °.

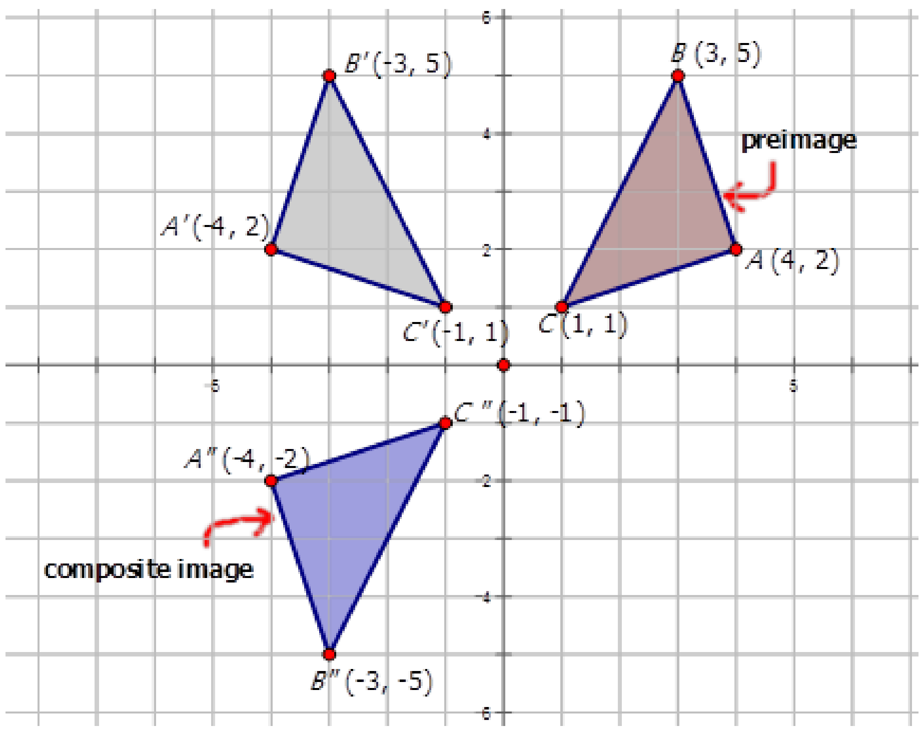
Solution: The first translation is a 90° CCW turn about the origin to produce $X'Y'$. The second translation is a reflection about the y -axis to produce $X''Y''$. The transformation from preimage to image is $(x,y) \rightarrow (y,x)$.



Example B

Image A with vertices $A(3,5), B(4,2)$ and $C(1,1)$ undergoes a composite transformation that is a reflection about the x-axis and then a reflection about the y -axis. Draw the preimage and the composite image and show the vertices of the composite image.

Solution:

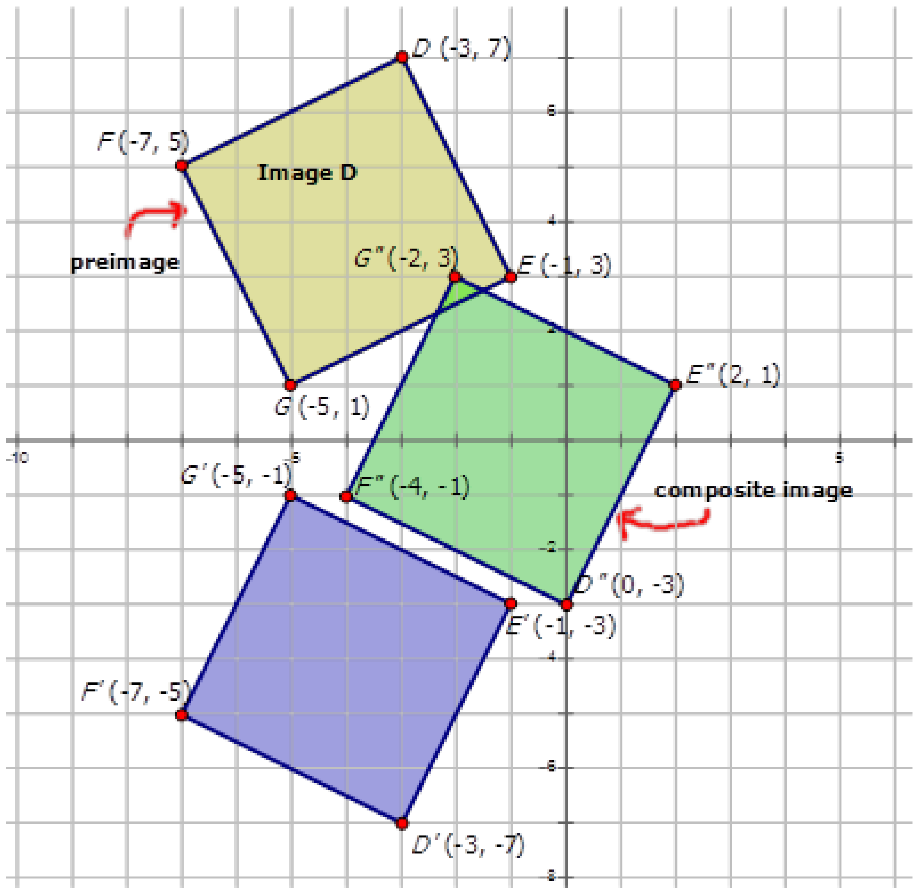


The transformation from preimage to image is $(x,y) \rightarrow (-x,-y)$.

Example C

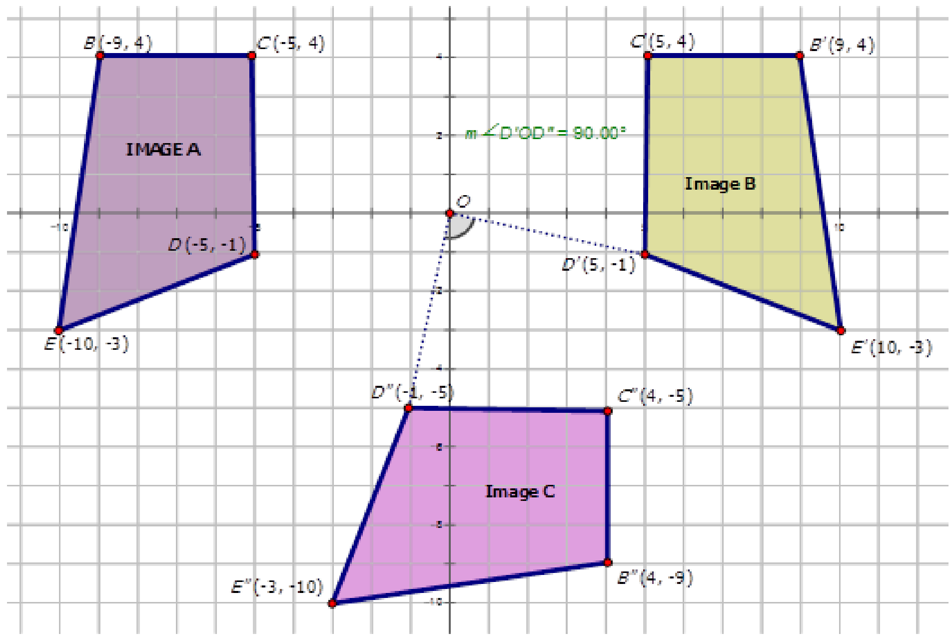
Image D with vertices $D(-3,7), E(-1,3), F(-7,5)$ and $G(-5,1)$ undergoes a composite transformation that is a horizontal translation to the right 3, a vertical translation up 4, and a reflection about the x -axis. Draw the preimage and the composite image and show the vertices of the composite image.

Solution:



The transformation from preimage to image is $(x,y) \rightarrow (x+3, -y+4)$.

Concept Problem Revisited



The transformation from Image A to Image B is a reflection across the y -axis or $(x,y) \rightarrow (-x,y)$. The transformation for image B to form image C is a rotation about the origin of 90° CW or $(x,y) \rightarrow (y,-x)$. Therefore, the notation to describe the transformation of Image A to Image C is $(x,y) \rightarrow (y,x)$.

Vocabulary

Image
In a transformation, the final figure is called the ***image*** .

Preimage
In a transformation, the original figure is called the ***preimage.***

Transformation
A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

Translation
A ***translation*** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides** .

Rotation
A ***rotation*** is a transformation that rotates (turns) an image a certain amount about a certain point.

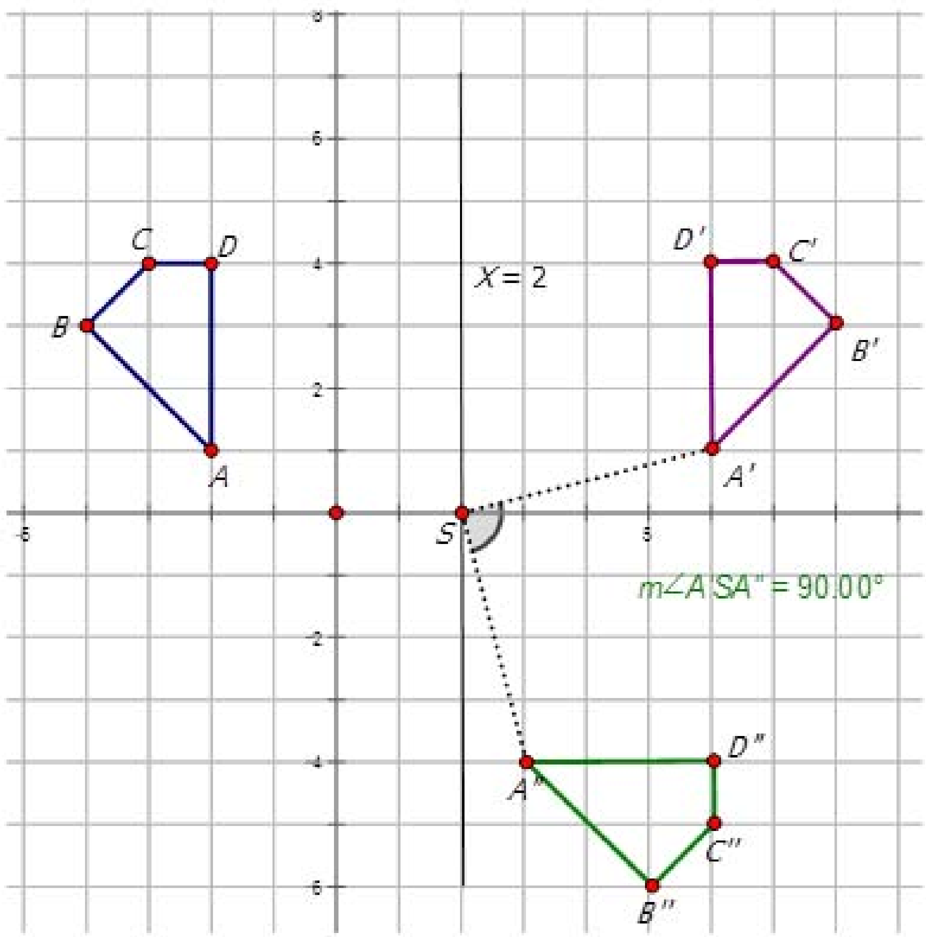
Reflection
A ***reflection*** is an example of a transformation that flips each point of a shape over the same line.

Composite Transformation
A ***composite transformation*** is when two or more transformations are combined to form a new image from the preimage.

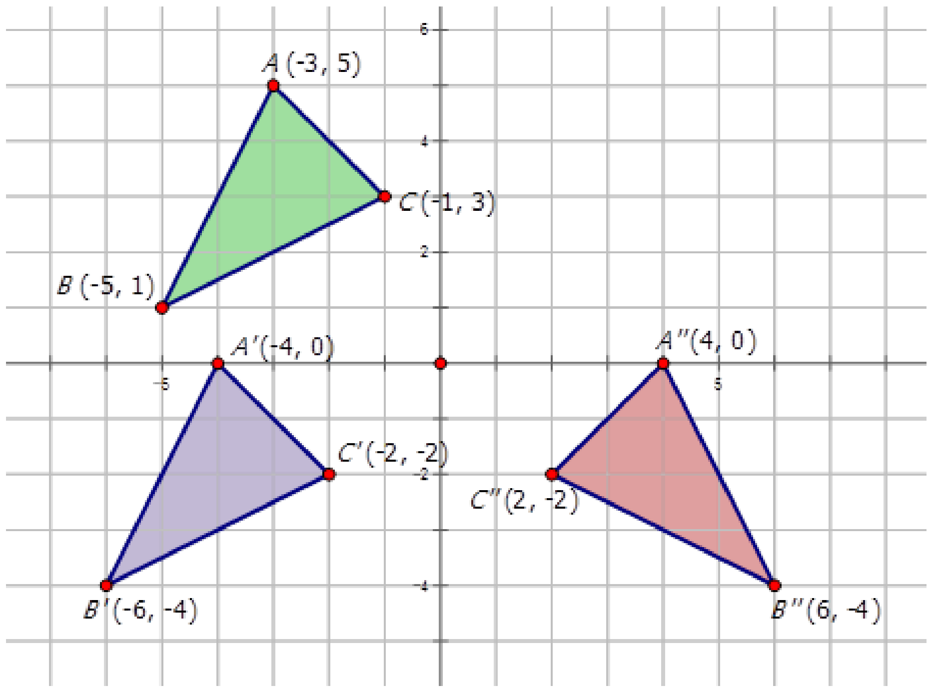
Guided Practice

1. Graph the line XY given that $X(2,-2)$ and $Y(3,-4)$. Also graph the composite image that shows a reflection about the y -axis and a counterclockwise rotation of 90 .

2. Describe the composite transformations in the diagram below and write the notation to represent the transformation of figure $ABCD$ to $A''B''C''D''$.

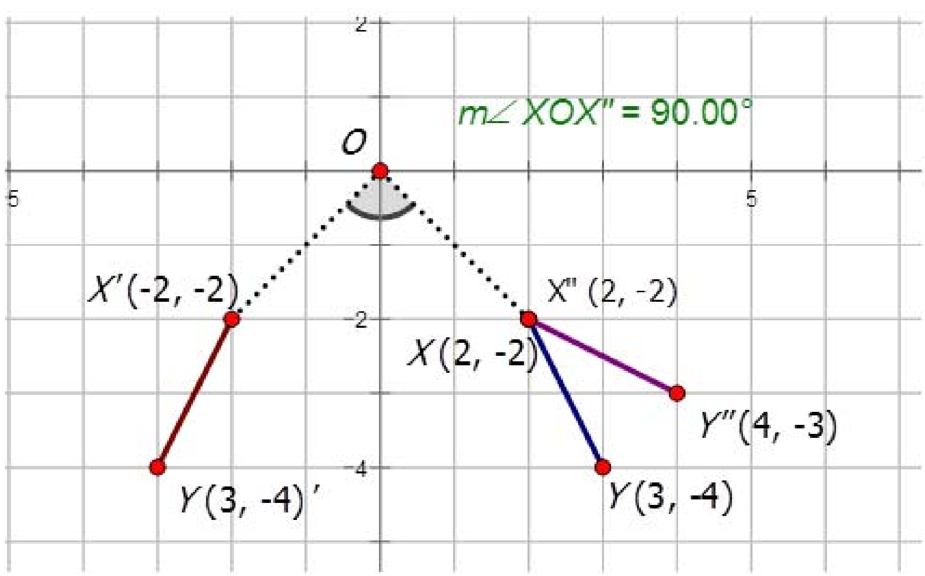


3. Describe the composite transformations in the diagram below and write the notation to represent the transformation of figure ABC to $A''B''C''$.



Answers:

1. The first transformation is a reflection about the y -axis to produce $X'Y'$. The second transformation is a 90° CCW turn about the origin to produce $X''Y''$.

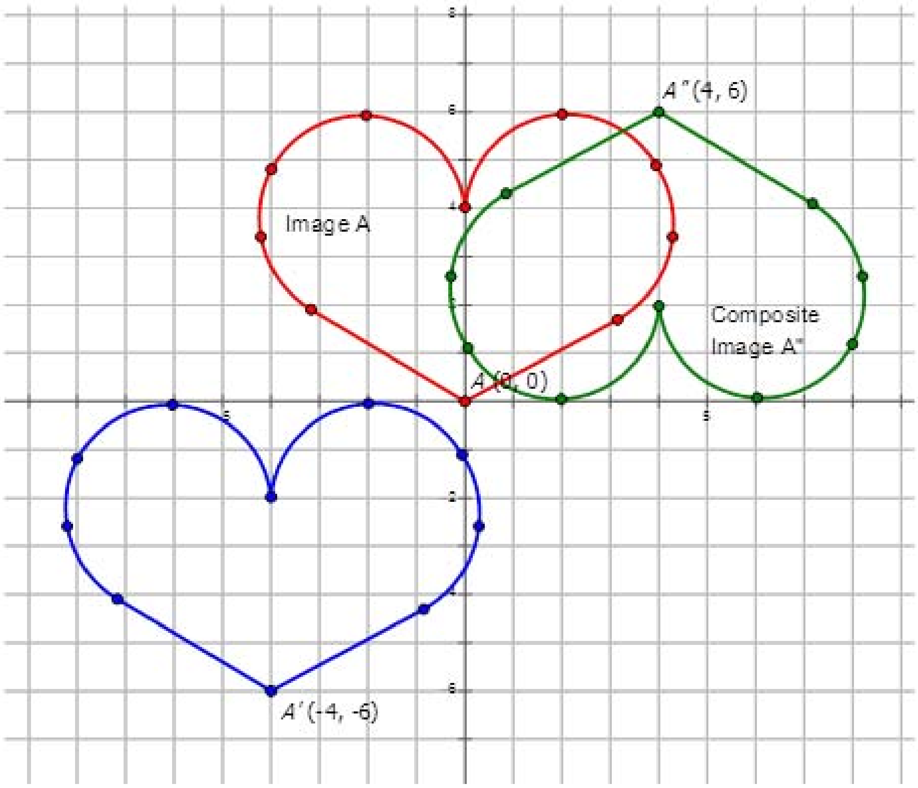


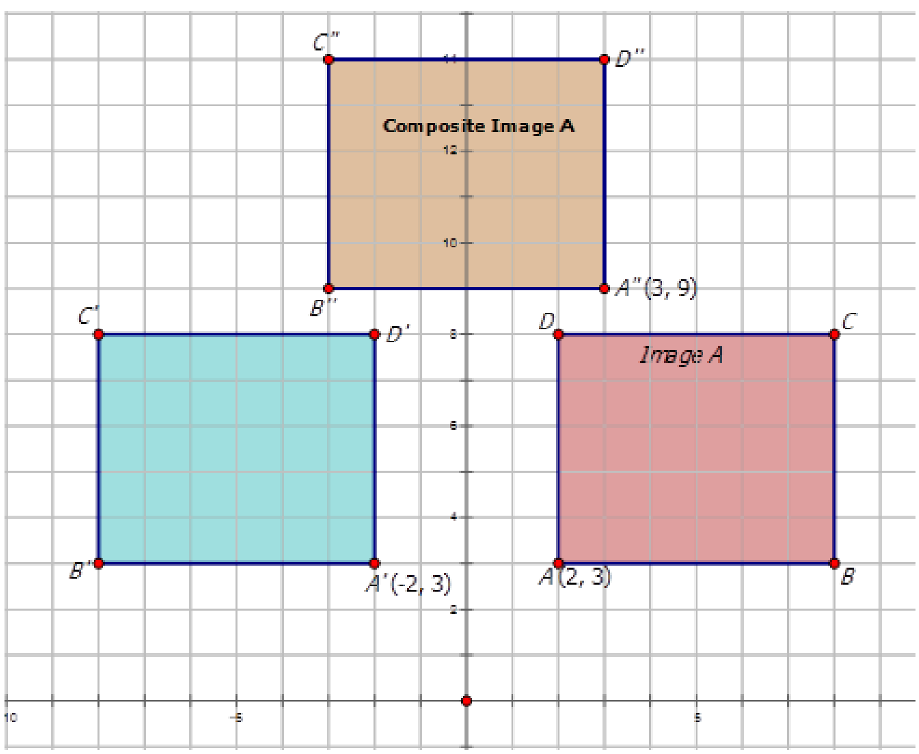
2. There are two transformations shown in the diagram. The first transformation is a reflection about the line $X = 2$ to produce $A'B'C'D'$. The second transformation is a 90° CW (or 270° CCW) rotation about the point (2, 0) to produce the figure $A''B''C''D''$.

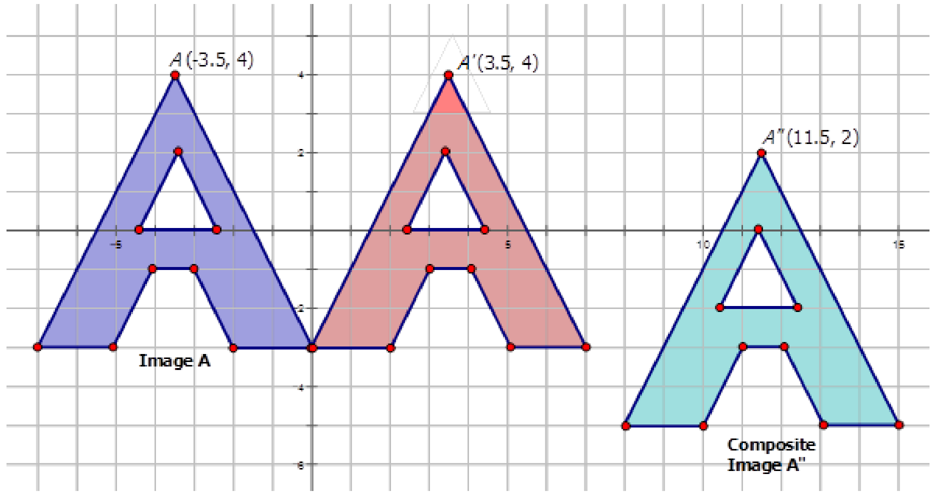
3. There are two transformations shown in the diagram. The first transformation is a translation of 1 unit to the left and 5 units down to produce $A'B'C'$. The second reflection in the y -axis to produce the figure $A''B''C''$.

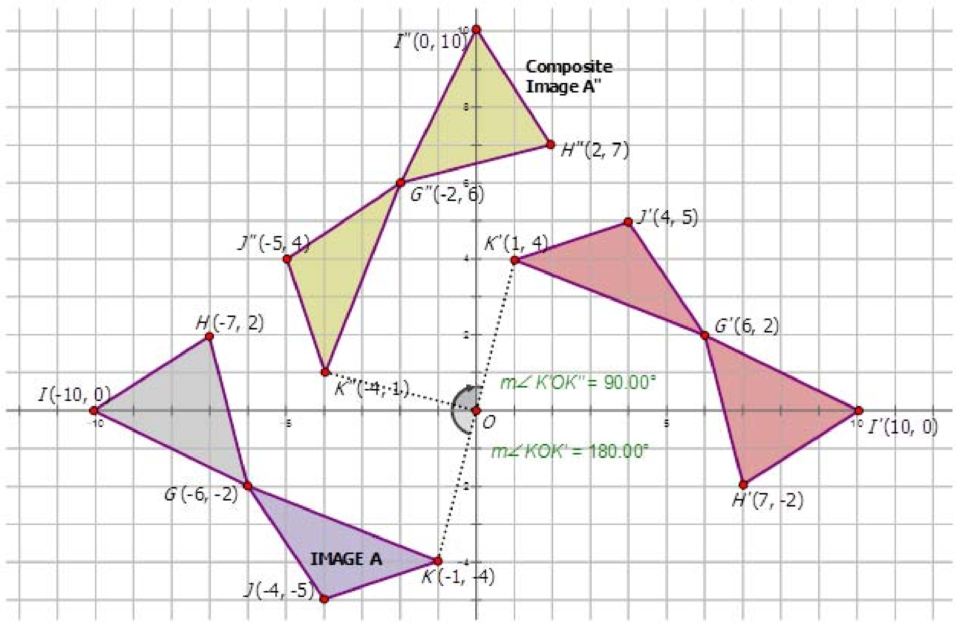
Practice

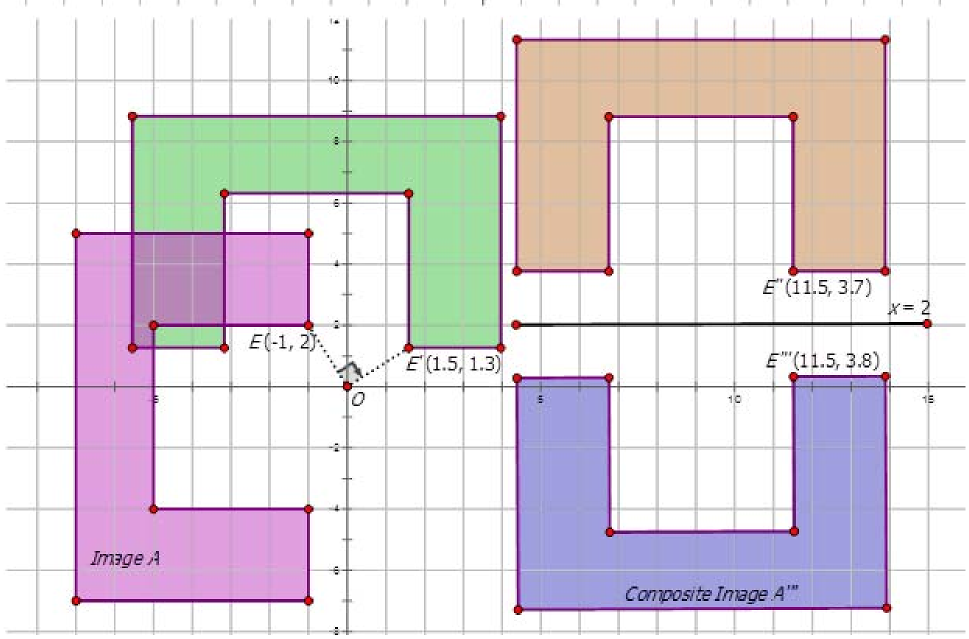
Write the notation that represents the composite transformation of the preimage A to the composite images in the diagrams below.

11. 

12. 

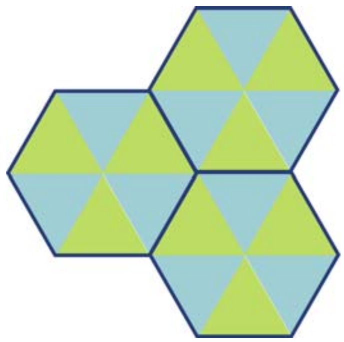
13. 
A(-3.5, 4)
A'(3.5, 4)
A"(11.5, 2)
Image A
Composite
Image A"

14. 
I"(0, 10)
Composite
Image A"
H"(2, 7)
G"(-2, 6)
J'(4, 5)
J"(-5, 4)
K'(1, 4)
H(-7, 2)
G'(6, 2)
K"(-4, 1)
m∠K'OK" = 90.00°
I(-10, 0)
O
I'(10, 0)
m∠KOK' = 180.00°
G(-6, -2)
H'(7, -2)
IMAGE A
K(-1, -4)
J(-4, -5)

15. 
E"(11.5, 3.7)
x = 2
E(-1, 2)
E'(1.5, 1.3)
E'"(11.5, 3.8)
O
Image A
Composite Image A'"

CONCEPT 11 SLT 18 Construct an equilateral triangle and describe its rotational and reflectional symmetry.

What if your parents want to redo the bathroom? Below is the tile they would like to place in the shower. The blue and green triangles are all equilateral. What type of polygon is dark blue outlined figure? Can you determine how many degrees are in each of these figures? Can you determine how many degrees are around a point? After completing this Concept, you'll be able to apply important properties about equilateral triangles to help you solve problems like this one.

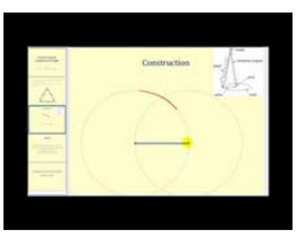


Watch This

MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter4EquilateralTrianglesA



MEDIA

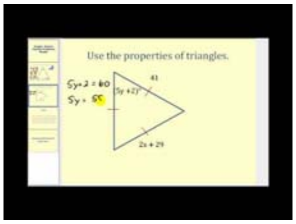
Click image to the left for more content.

James Sousa: Constructing an Equilateral Triangle

MEDIA

Click image to the left for more content.

James Sousa: Equilateral Triangles Theorem



MEDIA

Click image to the left for more content.

James Sousa: Using the Properties of Equilateral Triangles

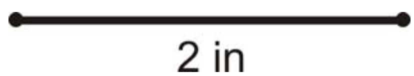
Guidance

By definition, all sides in an equilateral triangle have exactly the same length.

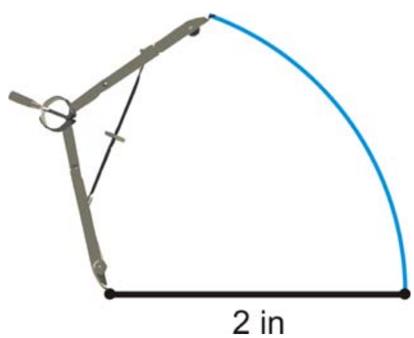
Investigation: Constructing an Equilateral Triangle

Tools Needed: pencil, paper, compass, ruler, protractor

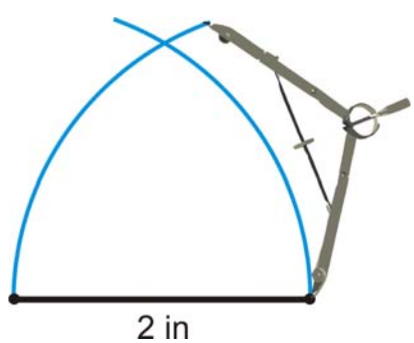
1. Because all the sides of an equilateral triangle are equal, pick a length to be all the sides of the triangle. Measure this length and draw it horizontally on your paper.



2. Put the pointer of your compass on the left endpoint of the line you drew in Step 1. Open the compass to be the same width as this line. Make an arc above the line.

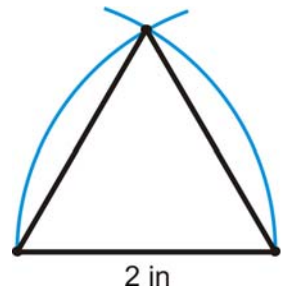


3. Repeat Step 2 on the right endpoint.



4. Connect each endpoint with the arc intersections to make the equilateral triangle.

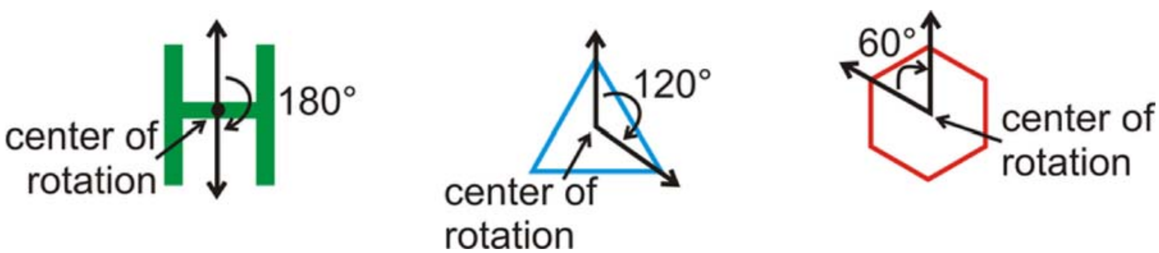
Use the protractor to measure each angle of your constructed equilateral triangle. What do you notice?



From the Base Angles Theorem, the angles opposite congruent sides in an isosceles triangle are congruent. So, if all three sides of the triangle are congruent, then all of the angles are congruent or 60° each.

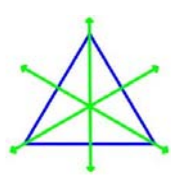
Equilateral Triangles Theorem: All equilateral triangles are also equiangular. Also, all equiangular triangles are also equilateral.

Rotational symmetry is present when a figure can be rotated (less than 360°) such that it looks like it did before the rotation. The **center of rotation** is the point a figure is rotated around such that the rotational symmetry holds.



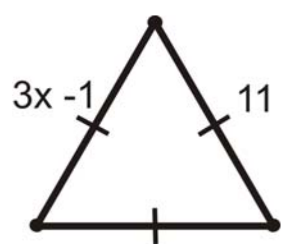
For the H , we can rotate it twice, the triangle can be rotated 3 times and still look the same and the hexagon can be rotated 6 times.

Reflectional Symmetry is present when a figure has one or more lines of symmetry. An equilateral triangle has three lines of symmetry.



Example A

Find the value of x .

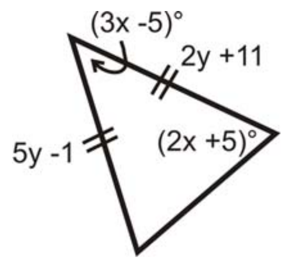


Because this is an equilateral triangle $3x-1=11$. Now, we have an equation, solve for x .

$$\begin{aligned}3x-1&=11\\3x&=12\\x&=4\end{aligned}$$

Example B

Find the values of x and y .



Let's start with y . Both sides are equal, so set the two expressions equal to each other and solve for y .

$$\begin{aligned}5y-1&=2y+11\\3y&=12\\y&=4\end{aligned}$$

For x , we need to use two $(2x+5)^\circ$ expressions because this is an isosceles triangle and that is the base angle measurement. Set all the angles equal to 180° and solve.

$$\begin{aligned}(2x+5)^\circ+(2x+5)^\circ+(3x-5)^\circ&=180^\circ\\(7x+5)^\circ&=180^\circ\\7x&=175^\circ\\x&=25^\circ\end{aligned}$$

Example C

Two sides of an equilateral triangle are $2x+5$ units and $x+13$ units. How long is each side of this triangle?

The two given sides must be equal because this is an equilateral triangle. Write and solve the equation for x .

$$\begin{aligned}2x+5&=x+13\\x&=8\end{aligned}$$

To figure out how long each side is, plug in 8 for x in either of the original expressions. $2(8)+5=21$. Each side is 21 units.

Watch this video for help with the Examples above.

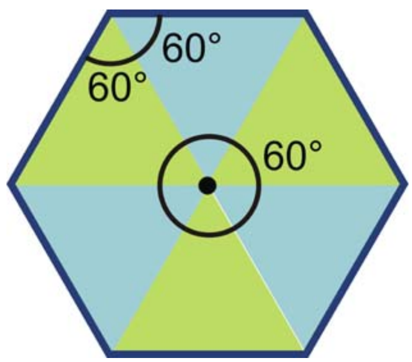
MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter4EquilateralTrianglesB

Concept Problem Revisited

Let's focus on one tile. First, these triangles are all equilateral, so this is an equilateral hexagon (6 sided polygon). Second, we now know that every equilateral triangle is also equiangular, so every triangle within this tile has 360° angles. This makes our equilateral hexagon also equiangular, with each angle measuring 120° . Because there are 6 angles, the sum of the angles in a hexagon are 6.120° or 720° . Finally, the point in the center of this tile, has 660° angles around it. That means there are 360° around a point.

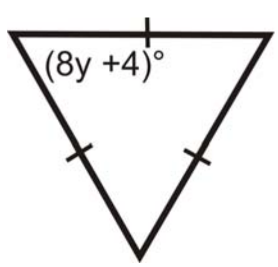


Vocabulary

An ***isosceles triangle*** is a triangle that has **at least** two congruent sides. The congruent sides of the isosceles triangle are called the ***legs*** . The other side is called the ***base*** . The angles between the base and the legs are called ***base angles*** and are always congruent by the ***Base Angles Theorem*** . The angle made by the two legs is called the ***vertex angle*** . An ***equilateral triangle*** is a triangle with three congruent sides. ***Equiangular*** means all angles are congruent. All equilateral triangles are equiangular.

Guided Practice

1. Find the measure of y .



2. Fill in the proof:

Given : Equilateral $\triangle RST$ with

$\overline{RT} \cong \overline{ST} \cong \overline{RS}$

Prove : $\triangle RST$ is equiangular

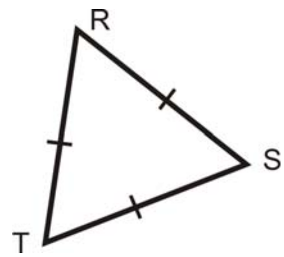


TABLE 11.1:

Statement	*Reason*
1.	1. Given
2.	2. Base Angles Theorem
3.	3. Base Angles Theorem
4.	4. Transitive PoC
5. $\triangle RST$ is equiangular	5.

3. True or false: All equilateral triangles are isosceles triangles.

Answers:

1. The markings show that all angles are congruent. Since all three angles must add up to 180° this means that each angle must equal 60° . Write and solve an equation:

$$\begin{aligned} 8y+4 &= 60 \\ 8y &= 56 \\ y &= 7 \end{aligned}$$

2.

TABLE 11.2:

Statement	*Reason*
1. $\overline{RT} \cong \overline{ST} \cong \overline{RS}$	1. Given
2. $\angle R \cong \angle S$	2. Base Angles Theorem
3. $\angle T \cong \angle R$	3. Base Angles Theorem
4. $\angle T \cong \angle S$	4. Transitive PoC
5. $\triangle RST$ is equiangular	5. Definition of equiangular.

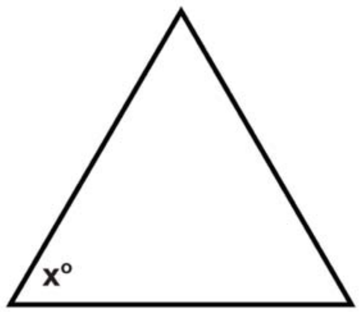
3. This statement is true. The definition of an isosceles triangle is a triangle with at least two congruent sides. Since all equilateral triangles have three congruent sides, they fit the definition of an isosceles triangle.

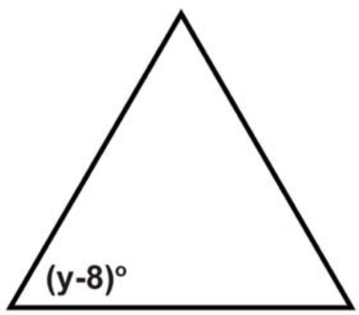
Interactive Practice

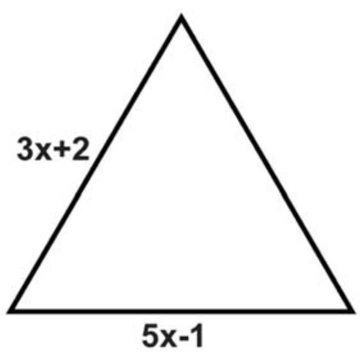
MEDIA
Click image to the left for more content.

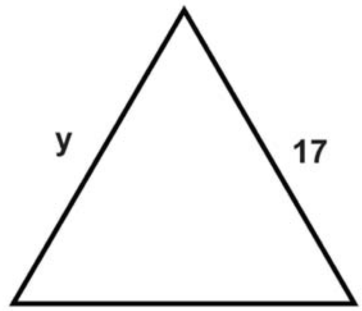
Practice

The following triangles are equilateral triangles. Solve for the unknown variables.

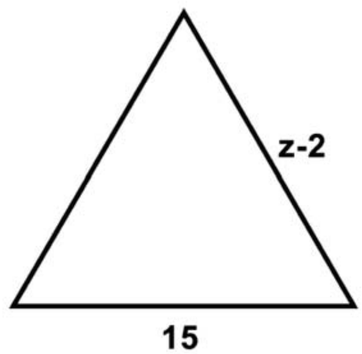
1. 

2. 

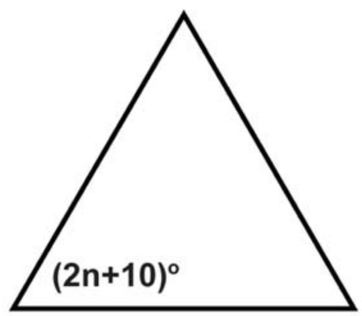
3. 



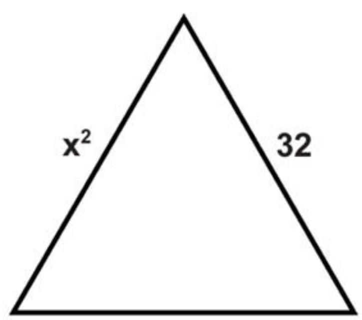
4.



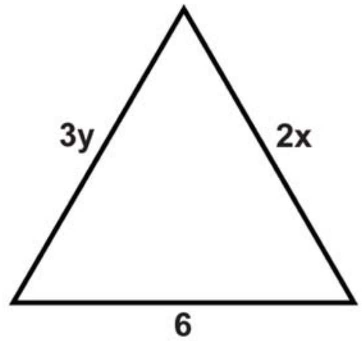
5.



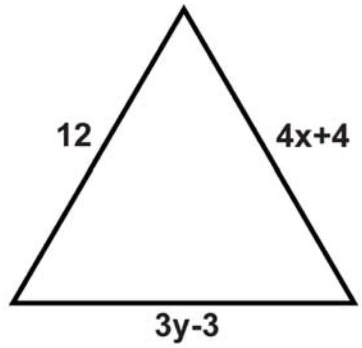
6.



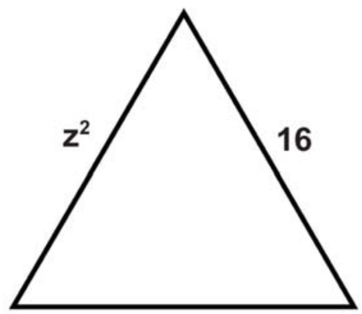
7.



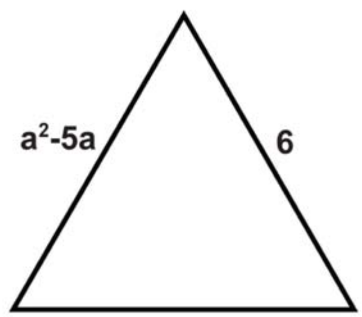
8.



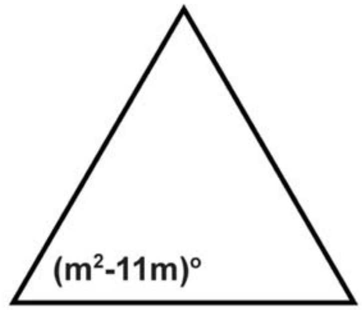
9.



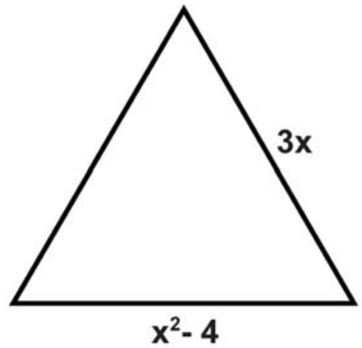
10.



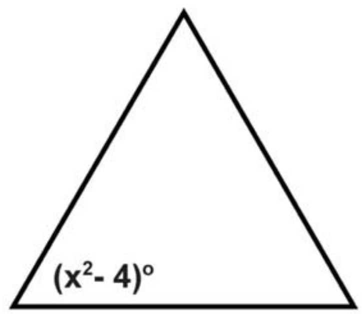
11.



12.

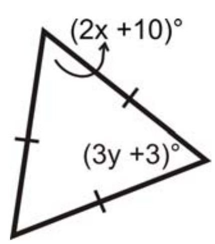


13.



14.

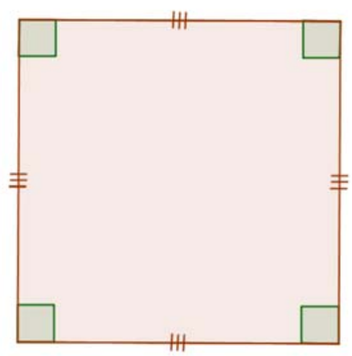
15. Find the measures of x and y .



CONCEPT 12 SLT 19 Construct a square and describe its rotational and reflectional symmetry.

Guidance

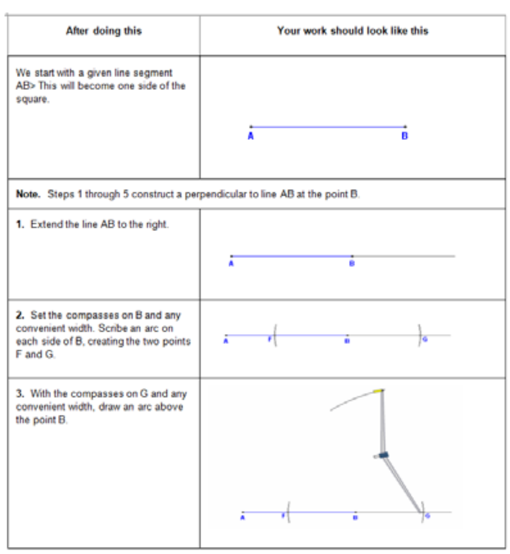
A square is a quadrilateral with four right angles and four congruent sides. All squares are rectangles and rhombuses.



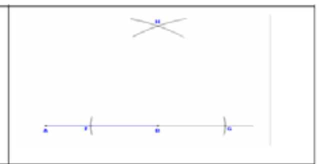
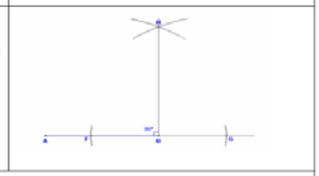
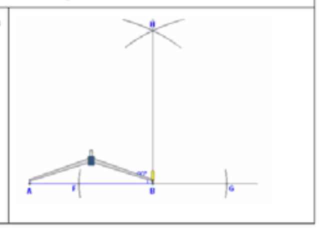
Watch This

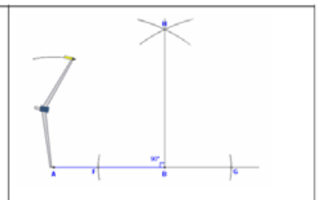
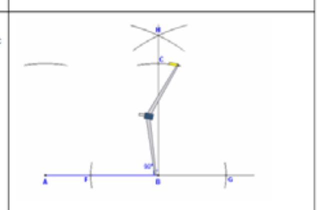
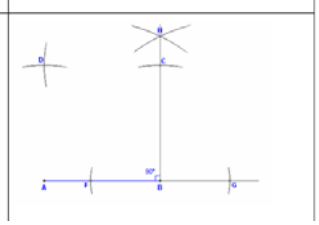
http://www.mathopenref.com/constsquare.html Constructing a square

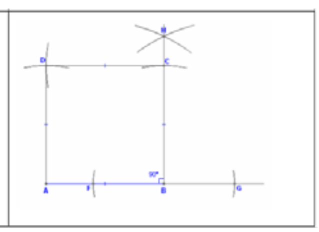
How to Construct a Square



After doing this	Your work should look like this
We start with a given line segment AB> This will become one side of the square.	
Note. Steps 1 through 5 construct a perpendicular to line AB at the point B.	
1. Extend the line AB to the right.	
2. Set the compasses on B and any convenient width. Scribe an arc on each side of B, creating the two points F and G.	
3. With the compasses on G and any convenient width, draw an arc above the point B.	

4. Without changing the compasses' width, place the compasses on F and draw an arc above B, crossing the previous arc, and creating point H	
5. Draw a line from B through H. This line is perpendicular to AB, so the angle ABH is a right angle (90°); This will become the second side of the square	
We now create four sides of the square the same length as AB	
6. Set the compasses on A and set its width to AB. This width will be held unchanged as we create the square's other three sides.	

7. Draw an arc above point A.	
8. Without changing the width, move the compasses to point B. Draw an arc across BH creating point C - a vertex of the square.	
9. Without changing the width, move the compasses to C. Draw an arc to the left of C across the exiting arc, creating point D - a vertex of the square.	

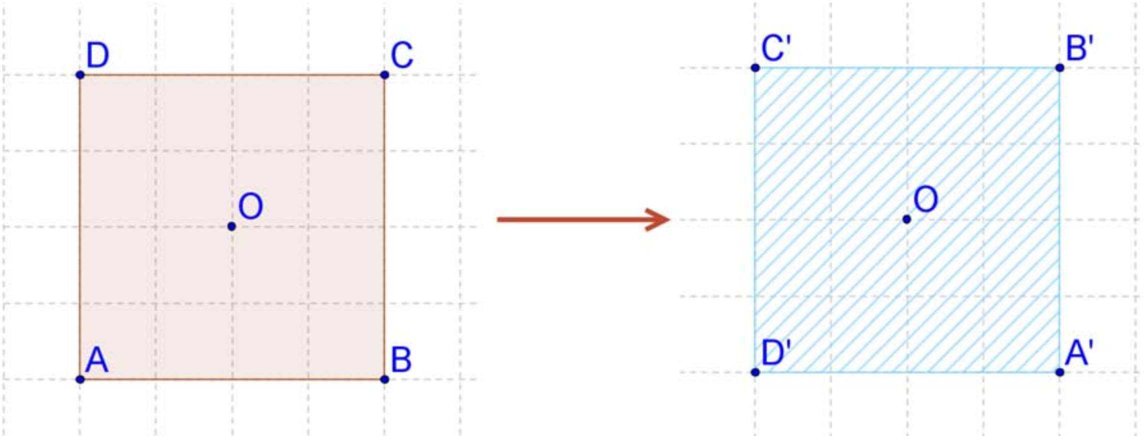
10. Draw the lines CD and AD	

Rotational symmetry is present when a figure can be rotated (less than 360°) such that it looks like it did before the rotation. The **center of rotation** is the point a figure is rotated around such that the rotational symmetry holds.

Example A

Does a square have rotation symmetry?

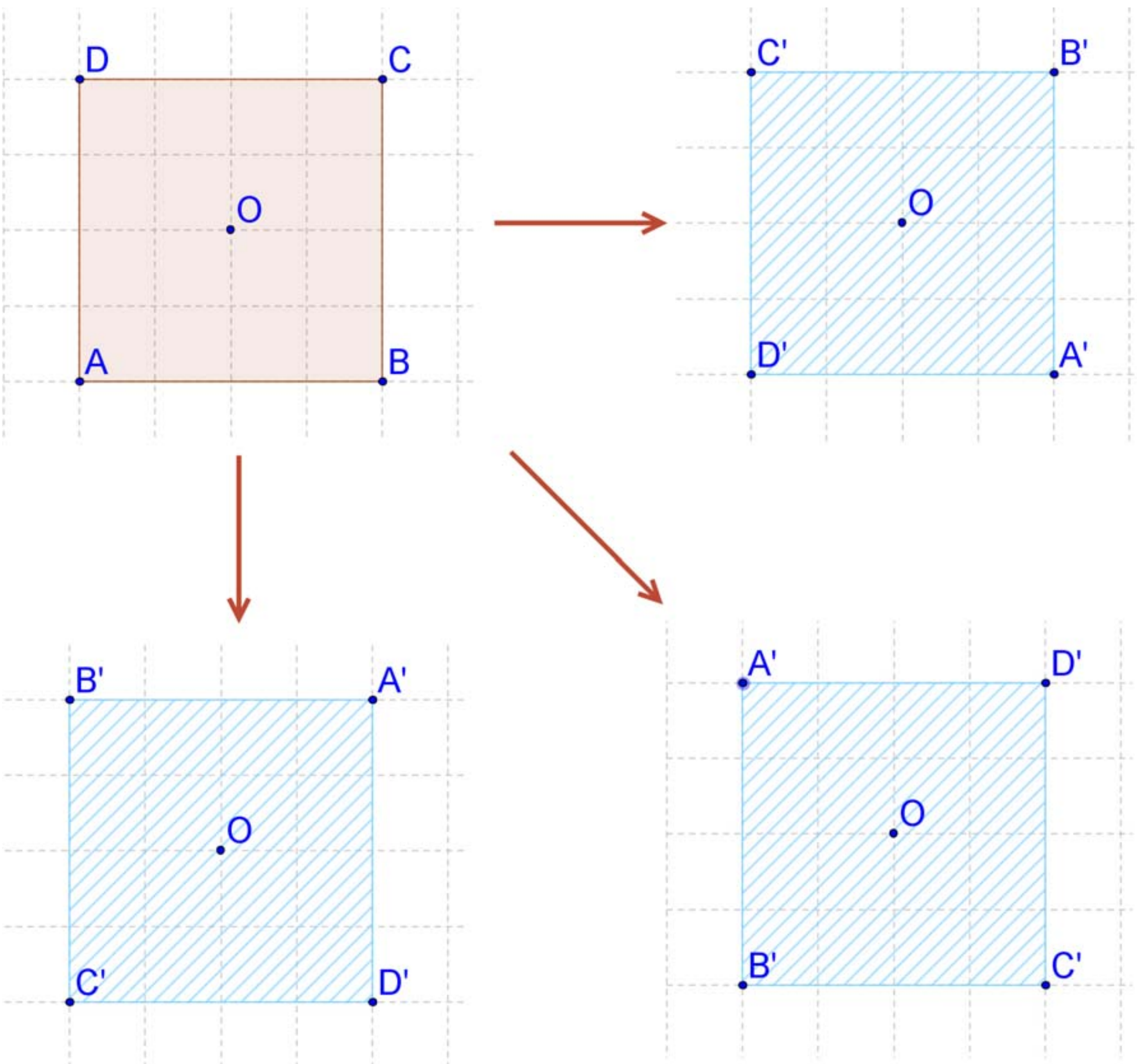
Solution: Yes, a square can be rotated 90° counterclockwise (or clockwise) about its center and the image will be indistinguishable from the original square.



Example B

How many angles of rotation cause a square to be carried onto itself?

Solution: Rotations of 90° , 180° and 270° counterclockwise will all cause the square to be carried onto itself.



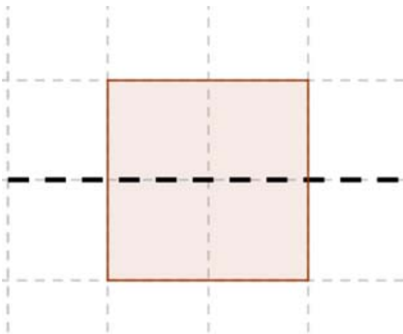
The square can be rotated 4 times and still look the same.

Reflectional Symmetry is present when a figure has one or more lines of symmetry. A square has four lines of symmetry.

Example C

Does a square have reflection symmetry?

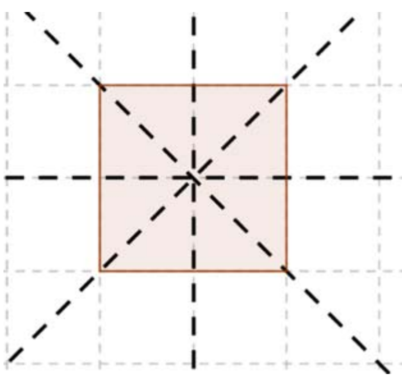
Solution: Yes, because there exists at least one line of reflection that carries the square onto itself. Like a rectangle, a line through the midpoints of opposite sides will be a line of symmetry.



Example D

How many lines of symmetry does a square have?

Solution: A square has 4 lines of symmetry. Lines through the midpoints of opposites sides and lines through opposite vertices are all lines of symmetry.



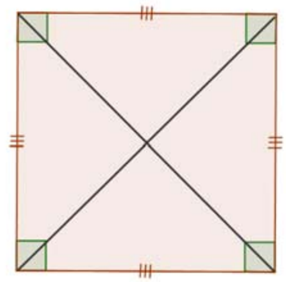
Guided Practice

Draw a square. Draw in the diagonals of the square. Make at least one conjecture about the diagonals of the square.

Answer :

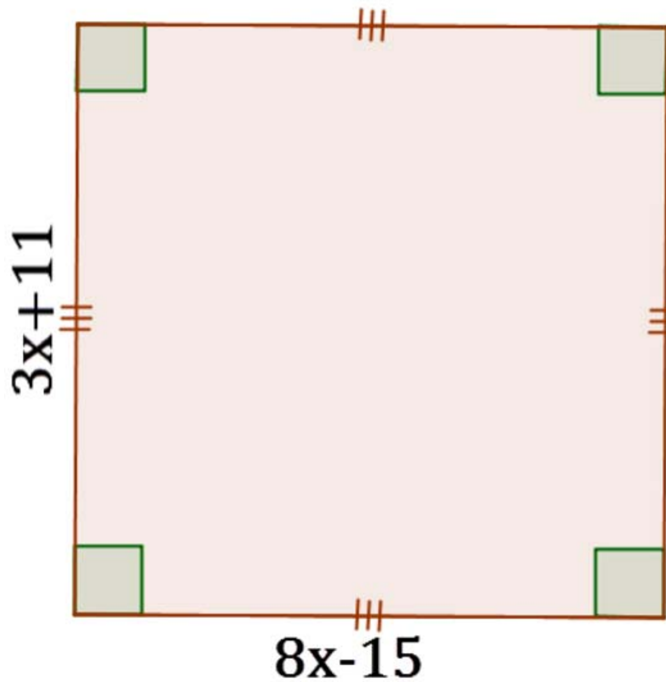
To make a conjecture means to make an educated guess. There are a few conjectures you might make about the diagonals of a square. These conjectures will be proved in a later concept. Here are some possible conjectures:

- diagonals of a square are congruent
- diagonals of a square are perpendicular
- diagonals of a square bisect each other (cut each other in half)
- diagonals of a square bisect the angles (cut the 90° angles in half)



Practice

Use the markings on the shape below to identify the shape. Then, solve for x . *Note: pictures are not drawn to scale.*

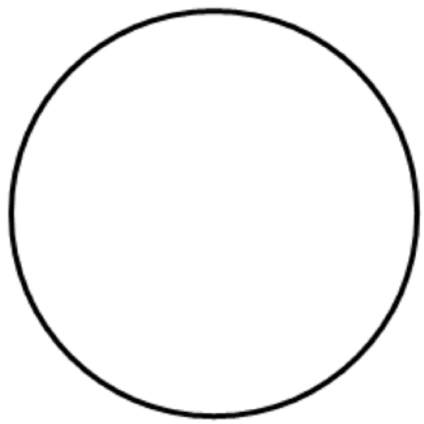


References

1. . . CC BY-NC-SA
2. . . CC BY-NC-SA
3. . . CC BY-NC-SA

CONCEPT 13 SLT 20 Construct a regular hexagon and describe its rotational and reflectional symmetry.

Use your compass to construct a circle like the one shown below on a piece of paper. Describe how to fold the paper two times in order to help you construct a square.



Guidance

A **regular polygon** is a polygon that is **equiangular** and **equilateral** . This means that all its angles are the same measure and all its sides are the same length.

The most basic example of a regular polygon is an **equilateral triangle** , a triangle with three congruent sides and three congruent angles. **Squares** are also regular polygons, because all their angles are the same (90°) and all their sides are the same length. Regular polygons with five or more sides do not have special names. Instead, the word *regular* is used to describe them. For example, a **regular hexagon** is a hexagon (6 sided polygon) whose angles are all the same measure and sides are all the same length.

All regular polygons have **rotation symmetry** . This means that a rotation of less than 360° will carry the regular polygon onto itself. In fact, a regular n -sided polygon has rotation symmetry for any multiple of $\frac{360^\circ}{n}$.

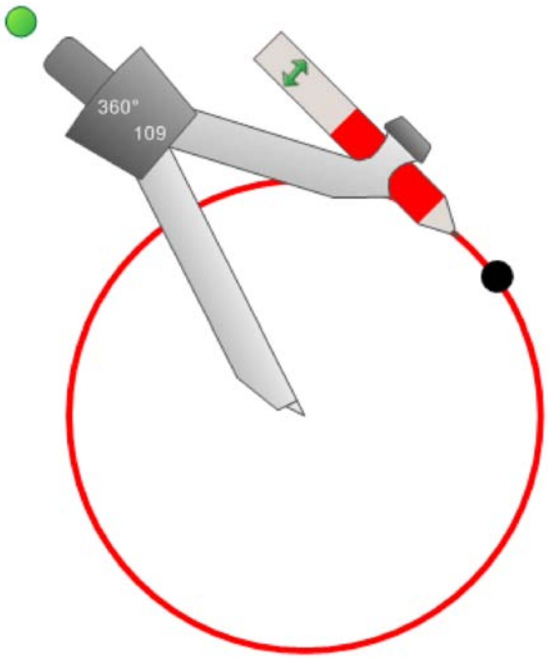
Constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

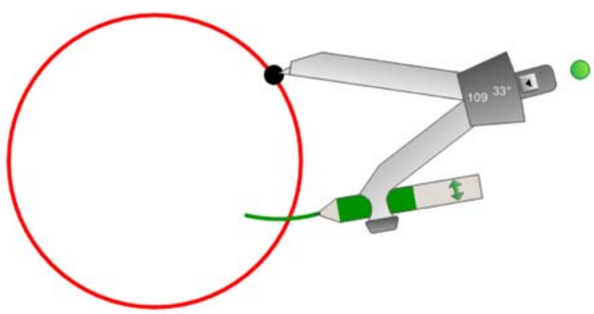
You can construct ***some*** regular polygons by hand if you remember the definitions and properties of these regular polygons. With the additional help of geometry software or a protractor, you can construct ***any*** regular polygon.

Constructing a Hexagon

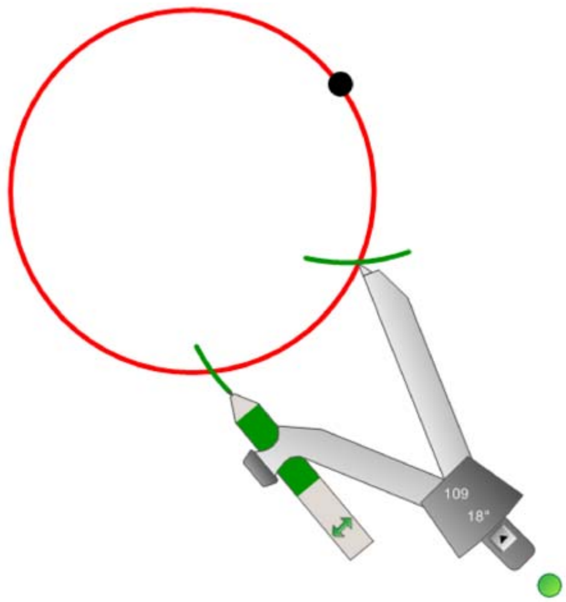
Start by constructing a circle and a point on the circle.

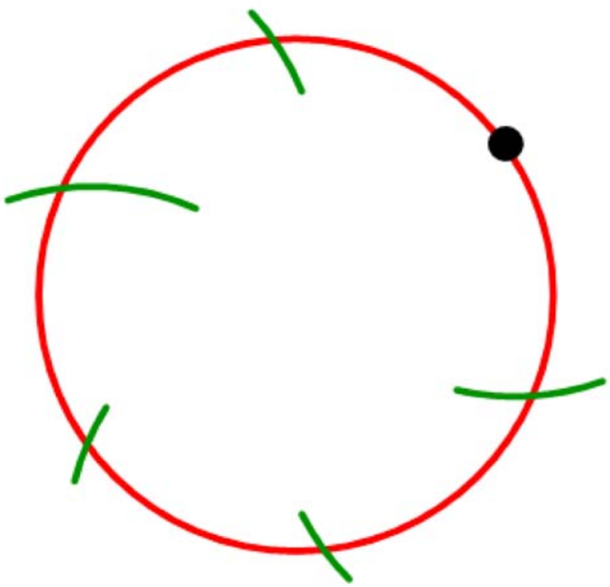


You know that the radius of the circle is the same as the length of each side of the circle (see guided practice #1). Therefore, your goal is to place six points around the circle that are the same distance apart from one another as the radius of the circle. Keep your compass open to the same width as the radius of the circle and make one new mark on the circle.

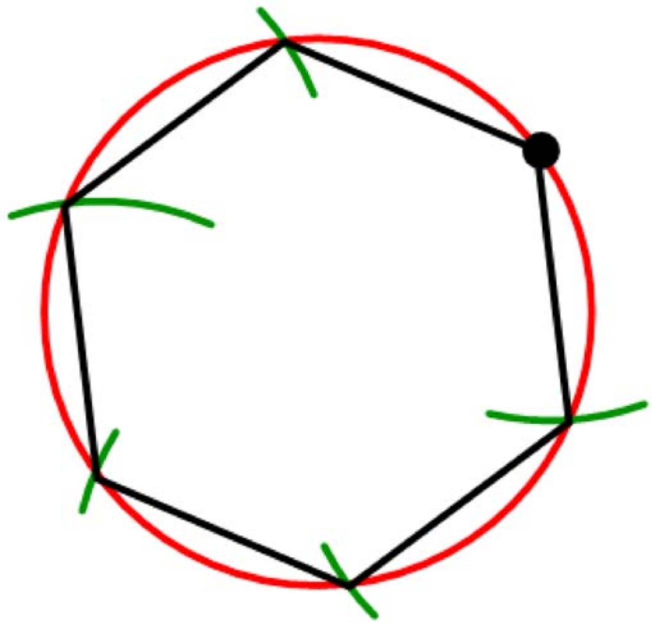


Continue to make new marks around the circle that are the same distance apart from one another.

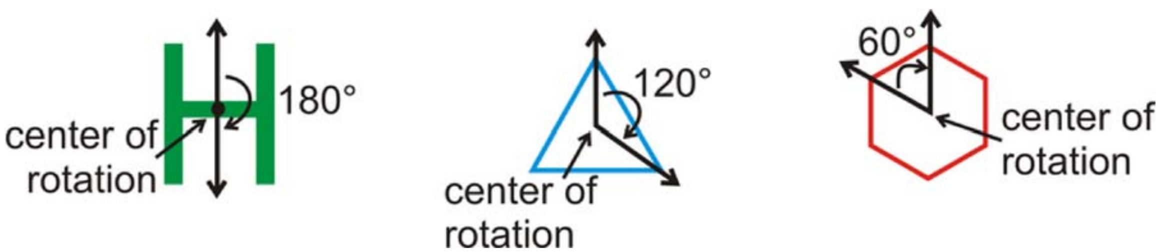




Connect the intersection points to form the regular hexagon.

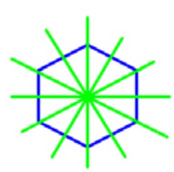


Rotational symmetry is present when a figure can be rotated (less than 360°) such that it looks like it did before the rotation. The **center of rotation** is the point a figure is rotated around such that the rotational symmetry holds.



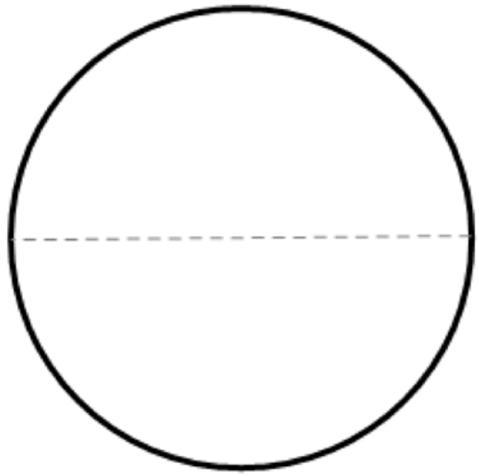
For the H , we can rotate it twice, the triangle can be rotated 3 times and still look the same and the hexagon can be rotated 6 times.

Reflectional Symmetry is present when a figure has one or more lines of symmetry. A regular hexagon has six lines of symmetry.

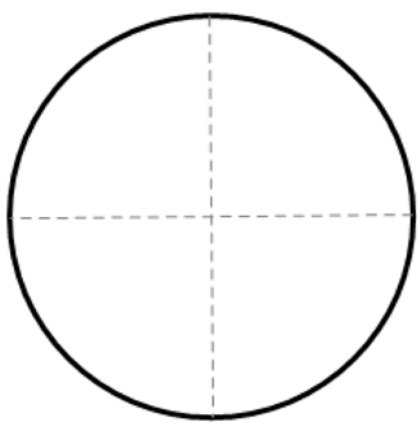


Concept Problem Revisited

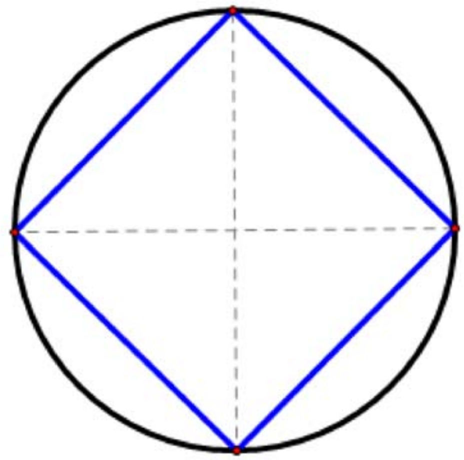
Fold the circle so that the two halves overlap to create a crease that is the diameter.



Fold the circle in half again to create the perpendicular bisector of the diameter. To do this, fold so that the two endpoints of the diameter meet. The second crease will also be a diameter.



Note that the two diameters are perpendicular to one another. Connect the four points of intersection on the circle to construct the square.



You can be certain that this is a square due to the proof in Example C.

Vocabulary

A ***regular polygon*** is a polygon that is ***equiangular*** (all angles the same measure) and ***equilateral*** (all sides the same length).

A ***drawing*** is a rough sketch used to convey an idea.

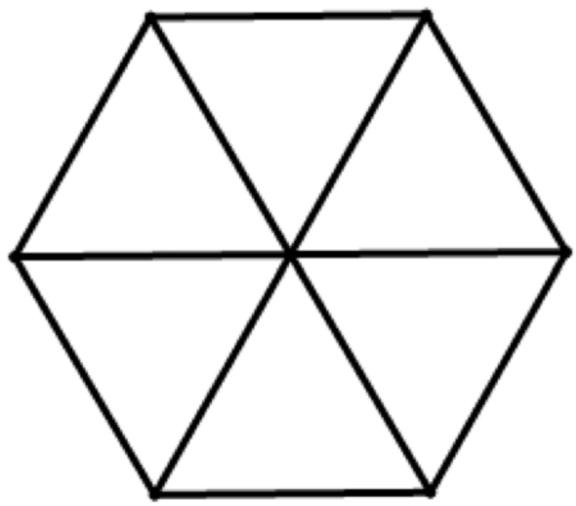
A ***construction*** is a step-by-step process used to create an accurate geometric figure.

A ***compass*** is a device that allows you to create a circle with a given radius. Compasses can also help you to copy distances.

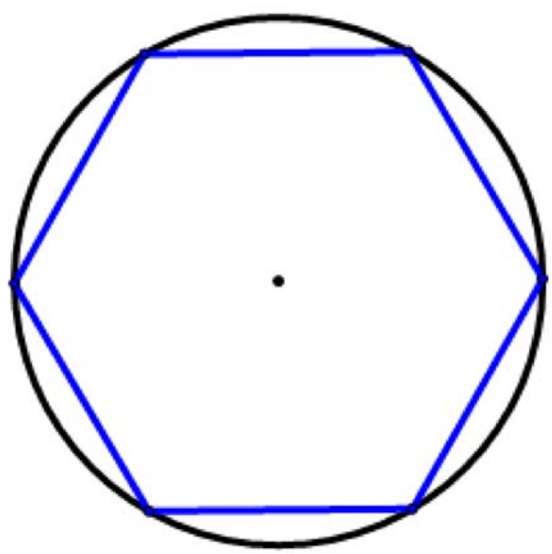
A ***straightedge*** is anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.

Guided Practice

1. The regular hexagon below has been divided into six congruent triangles. What type of triangles are they? Explain.



2. Six points have been evenly spaced around the circle below. Explain why a regular hexagon is created when these points are connected.



Answers:

1. They must be equilateral triangles.

- A full circle is 360° , so each angle at the center of the hexagon must be $\frac{360^\circ}{6} = 60^\circ$. * *This is also why regular hexagons demonstrate rotation symmetry at multiples of* 60° .*
- The six triangles are congruent, so the six segments connecting the center of the hexagon to the vertices must be congruent. This means the six triangles are all isosceles.
- The base angles of each of the isosceles triangles must be $\frac{180-60}{2} = 60^\circ$.
- The measure of each angle of all of the triangles is 60° , so all triangles are equilateral.

2. Because the six points are evenly spaced, each of the segments connecting the six points must be the same length. Therefore, the polygon must be regular. Because there are six sides, it must be a regular hexagon.

Practice

1. Construct an equilateral triangle.

2. Construct another equilateral triangle.

3. Explain why your process for constructing equilateral triangles works.

4. Construct a square inscribed in a circle by making two folds.

5. Justify why the polygon you've created is actually a square.

Use your straightedge to construct $\overline{AB}$.

6. Construct the perpendicular bisector of $\overline{AB}$.

7. Construct a circle with diameter $\overline{AB}$.

8. Construct a square inscribed in the circle by connecting the four endpoints of the diameters.

9. Extend your construction to a regular octagon by bisecting each of the right angles at the center of the circle.

10. Construct a regular hexagon inscribed in a circle.

11. Explain why the method for constructing a regular hexagon relies on a circle.

12. Explain how you could extend your construction of the regular hexagon to a construction of a regular 12-gon.

13. Construct an equilateral triangle. Explain how you could construct the circle that passes through the three points of the equilateral triangle.

14. Given an equilateral triangle inscribed in a circle, how could you extend the construction to construct a regular hexagon?

15. Given a circle and a protractor, explain how you could create a regular pentagon.

References

1. . . CC BY-NC-SA
2. . . CC BY-NC-SA
3. . . CC BY-NC-SA
4. . . CC BY-NC-SA
5. . . CC BY-NC-SA
6. . . CC BY-NC-SA