

Honors Precalculus

Summer Packet

Richard Montgomery High School

Formulas

General Exponential Equation: $y = ab^x$

Exponential Growth: $A(t) = A_0(1+r)^t$

Exponential Decay: $A(t) = A_0(1-r)^t$

Continuous Growth: $A(t) = A_0e^{rt}$

Continuous Decay: $A(t) = A_0e^{-rt}$

Compound Interest (n compoundings per

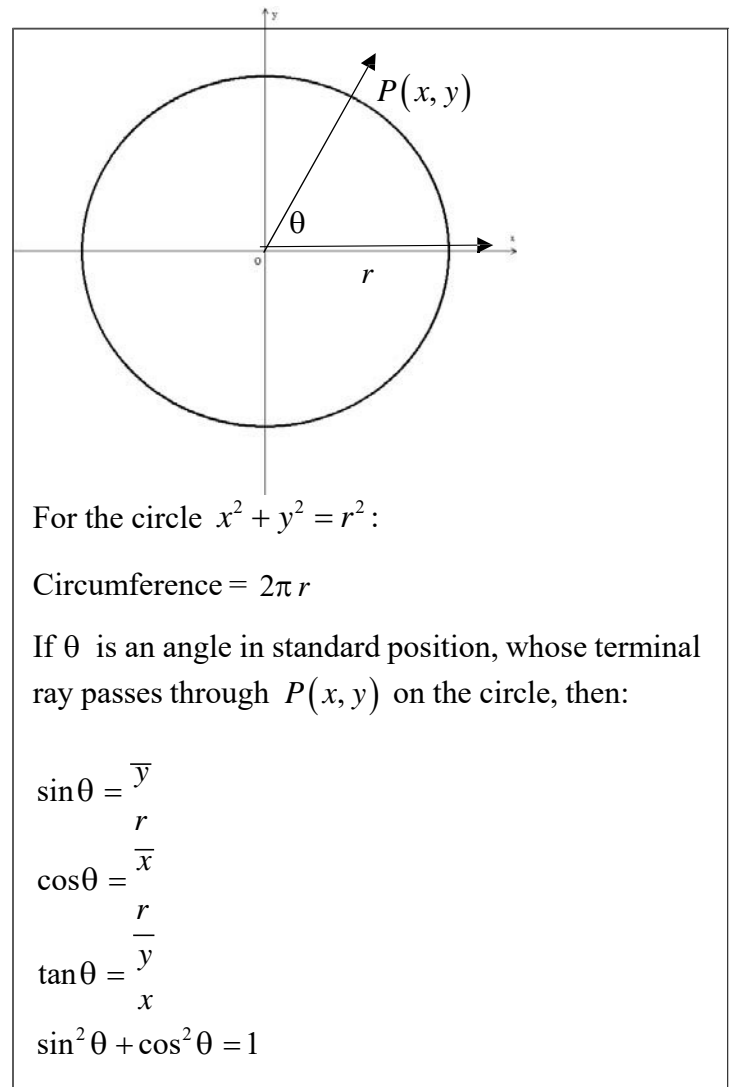
year): $F(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

Compound Interest (continuous
compounding): $F(t) = Pe^{rt}$

$\log_b N = p$ if and only if $b^p = N$

The average rate of change for a function f

on the interval $[a, b]$: $\frac{f(b) - f(a)}{b - a}$



Note: On this Summer packet, items marked with  are items for which a calculator may or may not be allowed.

PART I



Factor completely. If it can't be factored write prime.

1. $2t^2 + 5t - 3$

2. $3x^2 - 8x + 5$

3. $3p^2 - 7p - 6$

4. $4r^2 + 8r + 3$

5. $6x^2 + 7x - 10$

6. $4y^2 - 17y + 15$

7. $4t^2 - 9t + 6$

8. $25u^2 - 20u + 4$

9. $4r^2 + 16rs - 10s^2$

10. $12p^2 - 32pq - 5q^2$

11. $r^4 - 16s^4$

12. $x^4 - 3x^2 - 4$



Solve.

13. $x^2 + 5x + 6 = 0$

14. $t^2 + 2t - 19 = 5$

15. $2x^2 + 6x + 4 = 0$

16. $4n^2 + 12n + 9 = 0$

PART II



1. Let f and g be functions that are inverses of each other.

Complete the following statements.

- If the point (a, b) is on the graph of f , then the point _____ is on the graph of g .
- If $f(3) = 7$, then $g(7) = \underline{\hspace{2cm}}$.
- The graphs of f and g are symmetric with respect to the line _____.
- The range of f is the same as the _____ of g .
- The domain of f is the same as the _____ of g .



2. Let f and g be functions that are inverses of each other.

- If $f(x) = 3x - 2$, then $g(x) = \underline{\hspace{2cm}}$.
- If $f(x) = 2x^3 + 9$, then $g(x) = \underline{\hspace{2cm}}$.
- If $f(x) = \sqrt[3]{5x}$, then $g(x) = \underline{\hspace{2cm}}$.
- If $f(x) = 10^x$, then $g(x) = \underline{\hspace{2cm}}$.
- If $f(x) = \ln x$, then $g(x) = \underline{\hspace{2cm}}$.



3. Jill sells lemonade. The profit, p , in dollars is a function of the number of glasses of lemonade, g , that she sells. The function that represents this relationship is $p(g) = 2g - 18$.

- Write the function that gives the number of glasses that she will need to sell to earn a profit of p dollars.

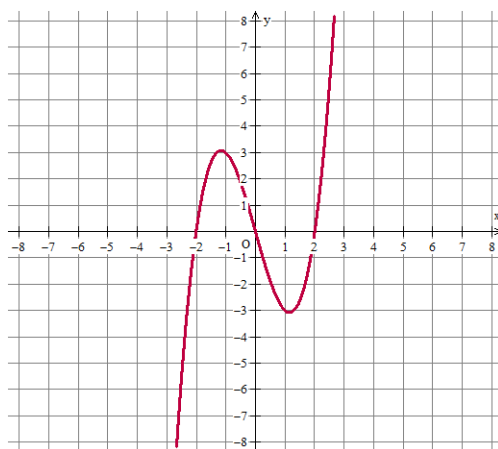
- If Jill made a profit of \$32, how many glasses did she sell?

4. On a national test, a student receives a score based on the number of correct items. The score, s , in points is a function of the number of correct items c . The function that represents this relationship is $s(c) = 200 + 2.5c$

- a. Write a function that gives the number of correct items that it will take to receive a score of s .
- b. A student received a score of 325. How many items did the student get correct?



5. Look at the graph of a function below.



- a. Explain why the inverse of this function is NOT a function.
- b. If the domain of the function is restricted to $x \geq 1$, would the inverse be a function?



6. Write the following expression as a radical.

- a. $x^{\frac{1}{5}}$ b. $y^{\frac{1}{3}}$ c. $z^{\frac{2}{3}}$



7. Determine the exponent that goes into the box.

a. $\frac{1}{\sqrt{x}} = x^{\square}$

b. $\sqrt[5]{x^4} = x^{\square}$

c. $\left(\begin{matrix} 2 \\ x^3 \\ x \\ x^6 \end{matrix} \right)^{18} = x^{\square}$

d. $(\sqrt{x})^6 = x^{\square}$

e. $\frac{x^{\square}}{\frac{1}{x^6}} = x^{\frac{2}{3}}$



8. Compute.

a. $81^{\frac{1}{2}}$

b. $27^{\frac{2}{3}}$

c. $\left(\frac{1}{10000} \right)^{\frac{1}{4}}$



9. Determine the value of n .

a. $(\sqrt[4]{x})^n = x^8$

b. $(\sqrt[n]{3})^4 = 9$



10. Sally solves the radical equation $3\sqrt{x} = -15$ and obtains the solution $x = 25$. Is this solution extraneous? Justify your answer.

11. The population of a town can be modeled by the function $P(x) = 60,000\sqrt[3]{x-1970}$, where x is the year. In what year was the population 120,000?

12. For each equation in column 1, choose the interval in which the solution lies from column 2.

Column 1	Column 2
a. $\log_2 18 = x$	x is between 0 and 1
b. $3^x = 10$	x is between 1 and 2
c. $\log 20$	x is between 2 and 3
d. $5^{-x} = \frac{1}{6}$	x is between 4 and 5



13. Write the logarithmic equation that is equivalent to each exponential equation

a. $4^2 = 16$

b. $10^3 = 1000$

c. $e^0 = 1$



14. Write the exponential equation that is equivalent to each logarithmic equation.

a. $\log \frac{1}{100} = -2$

b. $\ln \frac{1}{e} = -1$

c. $\log_9 81 = 2$



15. Evaluate the following logarithms

a. $\log_3 9$ _____

b. $\log_4 \frac{1}{16} =$ _____

c. $\log 10,000$ _____

d. $\ln e =$ _____

e. $\log_6 1$ _____

f. $\ln \frac{1}{e} =$ _____

g. $\log_4 8 =$ _____

h. $\log_{100} 10 =$ _____



16. Does each function below represent exponential growth or decay?

a. $f(x) = \left(\frac{1}{2}\right)^x$

b. $g(x) = 3^{-x}$

c. $h(x) = 5^x$



17. Write an exponential function in terms of time t (t in years) for each situation.

a. There are 300 bacteria at time 0. The bacteria has a continuous growth rate of 70% per year.

b. The population of a town is currently 2000. The population is growing at an annual rate of 11% per year.

c. Jack puts \$500 into a savings account. It earns interest at a nominal annual rate of 6% per year, compounded monthly.

d. The number of deer in a forest is decreasing at an annual rate of 8%. There are currently 700 deer in the forest.

e. The number of gnats in a swamp decreases at a continuous decay rate of 12% per year. There are currently 4 billion gnats in the swamp.

18. Solve each equation. Your answer must be exact or expressed as a decimal to at least three places beyond the decimal point.

a. $10^{3t-2} = 9800$

b. $5e^{t+4} = 275$

19. The number of apples that have been harvested from an orchard has been increasing at an exponential rate. The table below shows the number of apples harvested in thousands, y , where x is the number of years after 2000.

Years since 2000 (x)	Number of apples in thousands (y)
0	5
3	40

Write an equation in the form $y = ab^x$ that represents the number of apples harvested x years after 2000.

20. The number of tons of flour that a company manufactured can be described by the function $F(t) = 50e^{0.25t}$, where t is the number of years since the beginning of 2010.

- a. What is the continuous percentage rate of increase of this function?
- b. How many tons of flour were produced at the beginning of 2012?
- c. To the nearest hundredth of a year, when will the company have manufactured twice as much flour as they had at the beginning of 2010?

21. Sammy deposits \$2,500 in an account that pays a nominal interest rate of 12%, compounded monthly. How much money will be in the account after 4 years?



22. Look at the functions below.

$$f(x) = 2^x$$

$$g(x) = \log_3 x$$

$$h(x) = 10^{-x}$$

$$p(x) = -\ln x$$

Several properties are listed below. For each property, write the function(s) that have this property. You may use f , g , h , or p as your answers.

- a. _____ The graph of the function has a horizontal asymptote of $y = 0$.

- b. _____ The function has a range of all real numbers.
- c. _____ The function is increasing on its entire domain.
- d. _____ The graph of the function has a y-intercept at the point $(0,1)$.
- e. _____ The domain of the function is the positive real numbers.
- f. _____ The graph of the function has a vertical asymptote of $x = 0$.
- g. _____ The function has a domain of all real numbers.
- h. _____ The function is decreasing on its entire domain.
- i. _____ The graph of the function has an x-intercept at the point $(1,0)$.
- j. _____ The range of the function is the positive real numbers.



23. Each function below is a transformation of the function $f(x) = e^x$. After each given transformation, write the function rule.

a. The graph of function g is the graph of $f(x) = e^x$ translated one unit to the right.

$g(x) =$ _____

b. The asymptote of the graph of function h has the equation $y = -4$.

$h(x) =$ _____

c. The graph of function p is the graph of $f(x) = e^x$ reflected across the x -axis.

$p(x) =$ _____

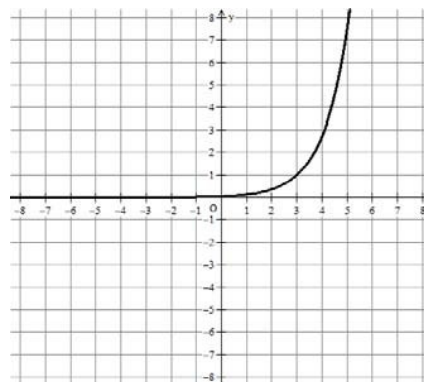
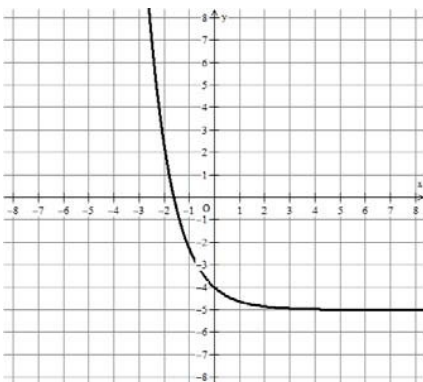
d. The graph of function s is the graph of $f(x) = e^x$ reflected across the y -axis.

$s(x) =$ _____

e. The graph of function w is the graph of $f(x) = e^x$ dilated by a factor of 2 with respect to the x -axis. $w(x) =$ _____

f. $t(x) =$ _____

g. $z(x) =$ _____





24. Each function below is a transformation of the function $f(x) = \log_2 x$. After each given transformation, write the function rule.

a. The graph of function g is the graph of $f(x) = \log_2 x$ translated two units to the left.
 $g(x) = \underline{\hspace{2cm}}$

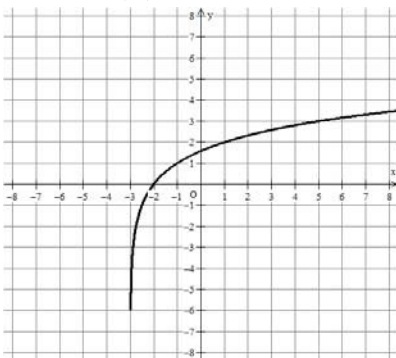
b. The asymptote of the graph of function h has the equation $x = 2$.
 $h(x) = \underline{\hspace{2cm}}$

c. The graph of function p is the graph of $f(x) = \log_2 x$ reflected across the x -axis.
 $p(x) = \underline{\hspace{2cm}}$

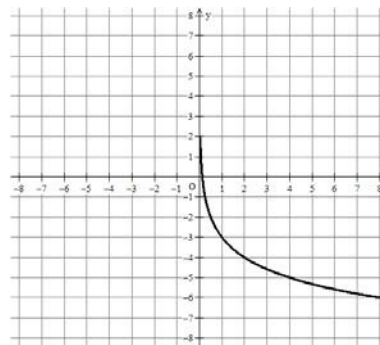
d. The graph of function s is the graph of $f(x) = \log_2 x$ reflected across the y -axis.
 $s(x) = \underline{\hspace{2cm}}$

e. The graph of function w is the graph of $f(x) = \log_2 x$ dilated by a factor of $\frac{1}{3}$ with respect to the x -axis. $w(x) = \underline{\hspace{2cm}}$

f. $t(x) = \underline{\hspace{2cm}}$



g. $z(x) = \underline{\hspace{2cm}}$



25. Let $f(x) = b^x$.

- The value of b must be _____ than zero, but not equal to _____.
- The function f has a _____ asymptote whose equation is _____.
- The domain of f is _____.
- The range of f is _____.
- The function is increasing when the value of b is _____.
- The function is decreasing when the value of b is _____.
- The y -intercept is at the point _____.



26. Let $g(x) = \log_b x$.

- The value of b must be _____ than zero, but not equal to _____.
- The function g has a _____ asymptote whose equation is _____.
- The domain of g is _____.
- The range of g is _____.
- The function is increasing when the value of b is _____.
- The function is decreasing when the value of b is _____.
- The x -intercept is at the point _____.

In the following problems, $i = \sqrt{-1}$ and $a + bi$ represents a complex number where a and b are real numbers.



27. What is the value of $i + i^2 + i^3 + i^4$?



28. Perform the following operations. Write your answer in the form $a + bi$.

- $(3 + i) + (6 - 5i)$
- $(4 + 7i) - (2 - 6i)$
- $(3 - 2i)(4 + 9i)$
- $(4 - 3i)(4 + 3i)$
- $(7 + 5i)^2$

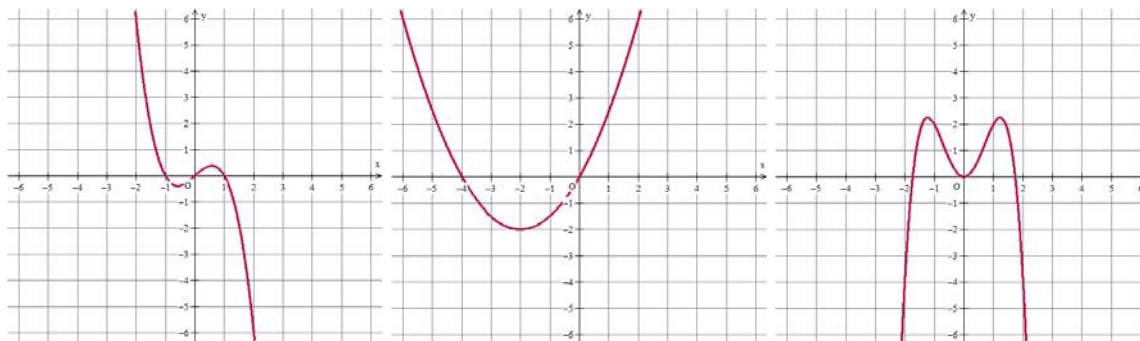
29. Solve the following quadratic equations over the set of complex numbers. Show how you determined your solutions.

- $(x - 3)^2 = -49$
- $(x + 4)^2 = -11$
- $x^2 + 8x = -30$
- $3x^2 - 6x - 7 = -17$
- $(x - 2)(x + 1) = (2x + 1)^2$



30. Classify each function represented by the graph as even, odd, or neither even nor odd.

- a. _____ b. _____ c. _____



31. Let $f(x) = x^2 + 2x - 7$, $g(x) = 2x^2 + 9$, and $h(x) = 5x + 3$.

- Write $(f + g)(x)$ as a polynomial in standard form.
- Write $(f - g)(x)$ as a polynomial in standard form.
- What is the degree of the product $(f \cdot g)(x)$?
- What is the degree of the product $(g \cdot h)(x)$?
- Write $(f \cdot h)(x)$ as a polynomial in standard form.
- Write $(f \cdot g)(x)$ as polynomial in standard form.

32. Jillian divides $x^3 - 8x^2 + 7x + 1$ by $x - 9$ and gets a quotient of $x^2 + x + 16$ with a remainder of 145.

- Is $x - 9$ a factor of $x^3 - 8x^2 + 7x + 1$? Justify your answer.
- If $f(x) = x^3 - 8x^2 + 7x + 1$, what is the value of $f(9)$?

33. Jake divides $x^3 - 3x^2 + 6x - 18$ by $x - 3$ and gets a quotient of $x^2 + 6$ with no remainder.

a. If $f(x) = x^3 - 3x^2 + 6x - 18$, what is the value of $f(3)$?

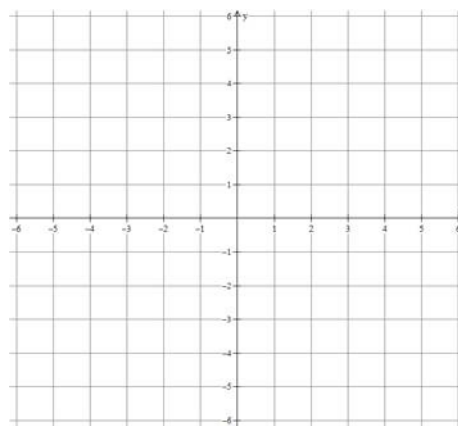
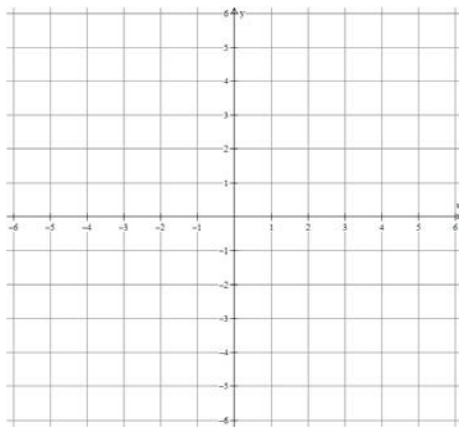
b. Why is $x - 3$ a factor of $x^3 - 3x^2 + 6x - 18$?

c. Write a factorization of $f(x) = x^3 - 3x^2 + 6x - 18$.

PART III

1. Graph the following. a. $y = 3 + \frac{1}{x-1}$

b. $y = -2 + \frac{1}{x-3}$

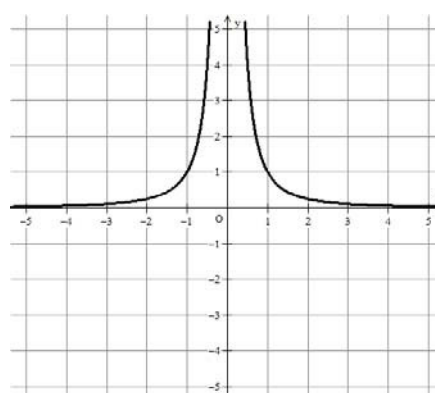
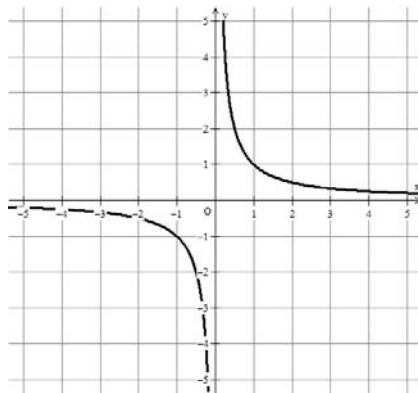


2. Write the least common multiple of the denominators in each equation, then solve the equations. Check for extraneous solutions.

a. $\frac{7}{x-3} = \frac{5}{x+2}$

b. $\frac{3x+6}{(x+2)(x-2)} = \frac{x+1}{x-2}$

3. Look at the graphs of $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$.



Complete the table below for each function.

	Function $f(x) = \frac{1}{x}$	Function $g(x) = \frac{1}{x^2}$
Even, odd, or neither		
Domain		
Range		
End behavior as $x \rightarrow -\infty$		
End behavior as $x \rightarrow \infty$		



4. a. Show that the functions f and g below are equivalent.

$$f(x) = 2 + \frac{1}{x+3} \qquad g(x) = \frac{2x+7}{x+3}$$

- b. What are the equations of the asymptotes of the graphs?



5. a. Show that the functions h and z are equivalent.

$$h(x) = 5 - \frac{1}{x-4} \qquad z(x) = \frac{5x-21}{x-4}$$

- b. What are the equations of the asymptotes of the graphs?