

Name:

Kennedy High School
Summer 2021 Math Packet
Precalculus / IB HL Year 1 Students

The problems in this packet are designed to help rising Precalculus students (**Honors or Regular**) with review topics from previous mathematics courses. When you do the problems, show all work that leads you to the solution.

Algebra

1. Simplify: $[2^3 + 4(7 - 3)] \div 8$

2. Evaluate: $3y^3 + 2y^2 - y + 7$ if $y = -4$

Solve each equation.

3. $5 - (4 - 2x) = 2x + 8$

4. $3(1 + x) = 2[3(x + 2) - (x + 1)]$

5. $\frac{3}{4}(8x - 12) - \frac{2}{3}(15x + 4) = -6x + 14$

6. $5|2x - 5| + 4 = 29$

7. $x^2 = 16$

8. $25x^2 = 225$

9. Solve for h: $T = 2\pi rh + 2\pi r^2$

Multiply: 10. $(2r - 3s)(5r + 7s)$

11. $(h^2 + 6h - 7)(2h^2 + 7h + 10)$

Exponents:

Rules of 1: $a^1 = a$ and $1^a = 1$

Zero Rule $a^0 = 1$

Product Rule $a^m \cdot a^n = a^{m+n}$

Quotient Rule $\frac{a^m}{a^n} = a^{m-n}$

Power Rule $(a^m)^n = a^{m \cdot n}$

Negative Exponents $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Root Rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Simplify. All answers must have positive exponents.

1. $x^5 \cdot x^7$

2. $(x^7)^5$

3. $\frac{x^7}{x^5}$

4. $\frac{x^5}{x^7}$

5. $\frac{(3x^2)^3}{(9xy)^5}$

6. $2(3x^3)(5x^2)^2$

7. $\left(\frac{3}{4}\right)^{-2}$

8. $(-3x^2y^3)^4$

9. $(5x^4y^3)^2$

10. $(-4xy^5) \cdot (2x^3y^4)$

11. $\frac{36x^6y^7}{4x^3y^9}$

12. $\frac{15x^3y^{-2}}{3x^2y^3}$

Evaluate.

13. $9^{\frac{1}{2}} \cdot 81^{\frac{3}{4}}$

14. $27^{\frac{-2}{3}}$

15. $\left(\frac{3^4}{7^4}\right)^{\frac{-1}{4}}$

16. $\left(2^{\frac{1}{5}} \cdot 2^{\frac{1}{2}}\right)^{10}$

Functions - To evaluate a function for a given value, substitute the value into the function for x and simplify.

Remember: $(f \circ g)(x) = f(g(x))$ read “ f of g of x ” Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case $f(x)$).

Ex: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $(f \circ g)(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f(x - 4) \\ &= 2(x - 4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ (f \circ g)(x) &= f(g(x)) = 2x^2 - 16x + 33\end{aligned}$$

Given $f(x) = 7 - x^2$ and $g(x) = x - 4$, find each value.

1. $f(-4)$ 2. $g(-4)$ 3. $f(g(-2))$ 1. _____

2. _____

3. _____

4. $f(4a)$ 5. $f(t+1)$ 4. _____

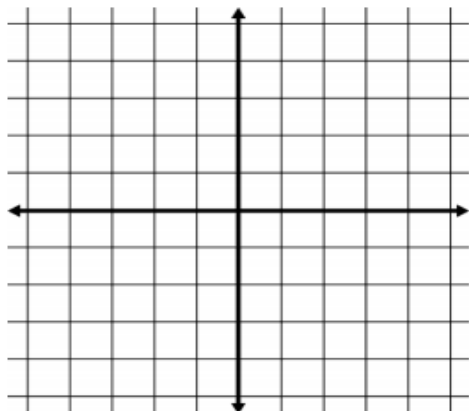
5. _____

6. _____

6. $(f \circ g)(x)$ 7. $(g \circ f)(x)$ 7. _____

Graph each function and describe the transformations.

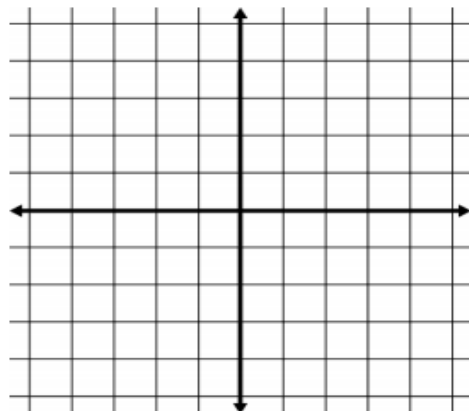
$$g(x) = (x + 2)^2$$



Describe the transformation of $g(x)$ from function $f(x) = x^2$

The graph is _____

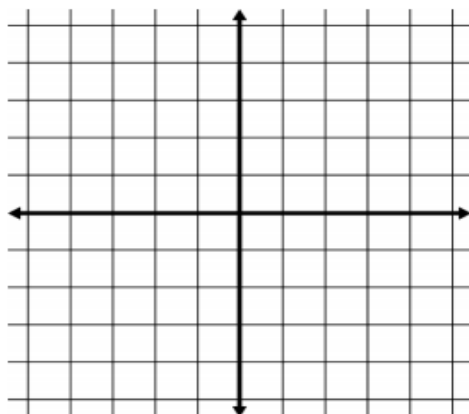
$$g(x) = |x| - 3$$



Describe the transformation of $g(x)$ from the parent the parent function $f(x) = |x|$

The graph is _____

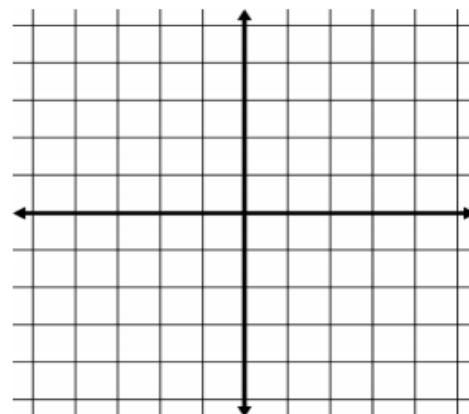
$$g(x) = \sqrt{x + 3} - 2$$



Describe the transformation of $g(x)$ from $h(x)$ from the parent function $f(x) = \sqrt{x}$

The graph is _____

$$h(x) = 2|x + 1| + 1$$



Describe the transformation of the parent function $f(x) = |x|$

The graph is _____

Factoring - follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

- a) if you have one (other than 1) factor it out front. Ex: $3x^2 - 6 = 3(x^2 - 2)$
b) if you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

- a) Is it a difference of two squares? $a^2 - b^2 = (a + b)(a - b)$ Ex: $x^2 - 25 = (x + 5)(x - 5)$
b) Is it a sum or difference of two cubes? $a^3 - b^3 = (a + b)(a^2 + ab + b^2)$
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

3 Terms: $ax^2 + bx + c$

- a) $a = 1$ --- Find 2 numbers that multiply to c and add to b Ex: $x^2 + 7x + 12 = (x + 3)(x + 4)$

- b) $a \neq 1$ --- Follow these Steps:

1) Multiply $a \cdot c$

Ex: $8x^2 + 14x + 3$
 $8 \cdot 3 = 24$

2) Find 2 numbers that multiply to this number and add to b

$12 \cdot 2 = 24$
 $12 + 2 = 14$

3) Divide each of these numbers by a and simplify

$\frac{12}{8} = \frac{3}{2}$; $\frac{2}{8} = \frac{1}{4}$

4) Write factors from the bottom up

$(2x + 3)(4x + 1)$

4 Terms: $ax^3 + bx^2 + cx + d$

- a) Pair up the first two terms and last two terms Ex: $x^3 + 3x^2 + 9x + 27$
 $(x^3 + 3x^2) + (9x + 27)$
b) Factor out front of the parentheses the terms have in common $x^2(x + 3) + 9(x + 3)$
c) Put the leftover terms in the parentheses $(x + 3)(x^2 + 9)$

Factor completely. You can check your answer by multiplying.

1. $3x^3 - 15x$ _____

2. $x^2 + 5x - 14$ _____

3. $x^2 - 16$ _____

4. $25x^2 - 9y^2$ _____

5. $3x^2 + 9x - 30$ _____

6. $4x^2 - 21x - 18$ _____

7. $a^2 - 14a + 49$ _____

8. $x^2 + 20x + 100$ _____

9. $25x^2 - 10x + 1$ _____

10. $2x^2 - 11x + 5$ _____

Solving Quadratics - To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the *quadratic formula*.

Ex:	$x^2 - 4x = 20$	set equal to zero FIRST
	$x^2 - 4x - 20 = 0$	factor (if possible)
	$(x + 3)(x - 7) = 0$	set each factor equal to zero
	$x + 3 = 0$ $x - 7 = 0$	solve each for x.
	$x = -3$ $x = 7$	

Quadratic Formula- allows you to solve any quadratic for all its real and imaginary roots. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex:	Solve $x^2 + 2x + 3 = 0$	
	$x^2 + 2x + 3 = 0$	cannot be factored
	a=1 b=2 c=3	determine the values for a, b, and c.
	$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$	substitute a, b, and c into the quadratic formula
	$x = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = -1 \pm i\sqrt{2}$	
	$x = -1 + i\sqrt{2}$ and $x = -1 - i\sqrt{2}$	

Solve.

1. $x^2 + 10 = 7x$

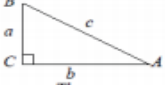
2. $x^2 - 10 = -3x$

3. $3x^2 - 7x + 8 = 0$

4. $4x^2 - 6x + 1 = 0$

Special Triangles

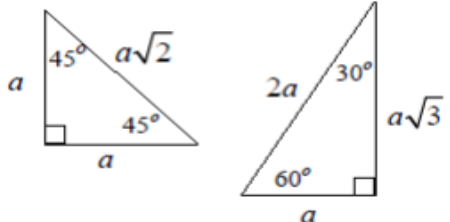
Formulas for Right Triangles



Pythagorean Theorem:
 $a^2 + b^2 = c^2$

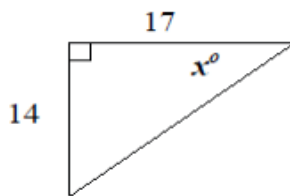
$\sin A = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\cos A = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$

Special Triangles

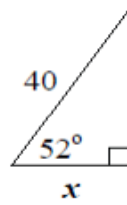


Determine the value of x and/or y in each figure below.

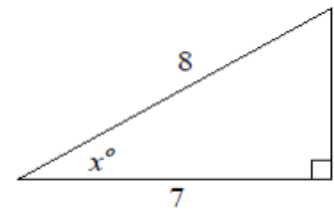
1.



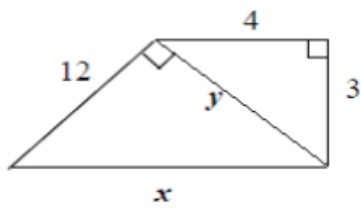
2.



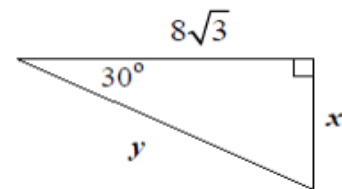
3.



4.



5.



Radicals:

To simplify means that 1) no radical has a perfect square factor and
 2) there is no radical in the denominator (rationalized).

Recall- the Product Property $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the Quotient Property $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24x^4} = \sqrt{4} \cdot \sqrt{6} \cdot \sqrt{x^4}$ find a perfect square factor
 $= 2x^2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both numerator and the denominator by
 $\sqrt{2}$

$$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{2}}{2} \quad \text{multiply straight across}$$

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60} \cdot \sqrt{105}$

7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

8. $\frac{1}{\sqrt{2}}$

9. $\frac{2}{\sqrt{3}}$

10. $\frac{3}{2-\sqrt{5}}$

Systems of Equations:

Substitution:

Solve 1 equation for 1 variable.

Rearrange.

Plug into 2nd equation.

Solve for the other variable.

Then plug answer back into an original equation to solve for the 2nd variable.

Ex: $3x + y = 6$
 $2x + 2y = 4$

$$\begin{array}{ll} 3x + y = 6 & \text{solve 1}^{\text{st}} \text{ equation for } y \\ 2x - 2(6 - 3x) = 4 & \text{plus into 2}^{\text{nd}} \text{ equation} \\ \text{opposite} & \\ 2x - 12 + 6x = 4 & \text{distribute} \\ 8x = 16 & \text{simplify} \\ x = 2 & \end{array}$$

$$\begin{array}{l} \text{Plug } x = 2 \text{ back into original } 3(2) + y = 6 \\ 6 + y = 6 \end{array}$$

$$y = 0$$

Solution: (2, 0)

Elimination:

Find opposite coefficients for 1 variable.

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable).

Solve for variable.

Ex: $3x + y = 6$
 $2x + 2y = 4$

$$\begin{array}{ll} 6x = 2y = 12 & \text{multiply 1}^{\text{st}} \text{ equation by 2} \\ 2x - 2y = 4 & \text{coefficients of } y \text{ are} \\ 8x = 16 & \text{add} \\ x = 2 & \text{simplify} \end{array}$$

Solve each system of equations. Use any method.

1. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

2. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

3. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$