Name:

Kennedy High School
Summer 2021 Math Packet
For Rising Geometry Students

This packet is a review of the skills that will help you be successful in Geometry (**Honors or Regular**) in the fall. By completing this packet over the summer, you will keep your brain mathematically active and you will also be able to identify skills that you need to strengthen. Complete the practice problems and then check your answers with the Answer Key. If you struggle with any of the exercises, please seek help from a friend, parent, sibling, book, or online resource. Enjoy your math review and we look forward to meeting you in September!

Formulas that you may find helpful are on the very last page of this packet.

Solving Equations I

Examples:

1. $4x - 6 = -14$

$$\begin{aligned} 4x - 6 &= -14 \\ +6 \quad &\quad +6 \\ \frac{4x}{4} &= \frac{-8}{4} \\ x &= -2 \end{aligned}$$

Solve: $4(-2) - 6 = -14$
$-8 - 6 = -14$
$-14 = -14$

2. $\dfrac{x}{-6} - 4 = -8$

$$\begin{aligned} \frac{x}{-6} - 4 &= -8 \\ +4 \quad &\quad +4 \\ -6 \cdot \frac{x}{-6} &= -4 \cdot -6 \\ x &= 24 \end{aligned}$$

Solve: $(24/-6) - 4 = -8$
$-4 - 4 = -8$
$-8 = -8$

Exercises: Solve the following equations using the rules listed on the previous pages:

1. $-4t + 3t - 8 = 24$
2. $\frac{m}{-5} + 6 = 4$
3. $-4r + 5 - 6r = -32$
4. $\frac{x}{-3} + (-7) = 6$
5. $6g + (-3) = -12$
6. $\frac{y}{-2} + (-4) = 8$
7. $9 - 5(4 - 3) = -16 + \frac{x}{3}$
8. $6t - 14 - 3t = 8\,(7 - (-2))$
9. $7(6 - (-8)) = \frac{t}{-4} + 2$
10. $7\,(3 - 6) = 6\,(4 + t)$
11. $4r + 5r - 8r = 13 + 6$
12. $3(7 + x) = 5(7 - (-4))$

Squares, Square Roots, and the Laws of Exponents

Hints/Guide:

Exponents are a way to represent repeated multiplication, so that 3^4 means 3 multiplied four times, or 3 · 3 · 3 · 3, which equals 81. In this example, 3 is the base and 4 is the power.

Roots are the base numbers that correspond to a given power, so the square (referring to the power of 2) root of 81 is 9 because 9 · 9 = 81 and the fourth root of 81 is 3 because 3 · 3 · 3 · 3 is 81.

$\sqrt[n]{x}$, where n is the root index and x is the radicand

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers	$a^m a^n = a^{m+n}$
Multiplying powers	$(a^m)^n = a^{mn}$
Subtracting powers	$\frac{a^m}{a^n} = a^{m-n}$
Negative powers	$a^{-n} = \frac{1}{a^n}$
To the zero power	$a^0 = 1$

Exercises: Evaluate:

1. $(8 - 4)^2 =$
2. $(4 - 2)^2\,(5 - 8)^3 =$
3. $5\,(8 - 3)^2 =$
4. $\sqrt{25 - 16} =$
5. $\sqrt{5(9 \bullet 125)} =$
6. $\sqrt{(8 - 4)(1 + 3)} =$

Simplify the following problems using exponents (Do not multiply out):

7. $5^2\ 5^4 =$

8. $(12^4)^3 =$

9. $5^9 \div 5^4 =$

10. $10^3 \div 10^{-5} =$

11. $7^{-3} =$

12. $3^{-4} =$

13. $(3^3 \bullet 3^2)^3 =$

14. $5^3 \cdot 5^4 \div 5^7 =$

Simplifying Radicals

Hints/Guide:

To simplify radicals, first factor the radicand as much as possible, then "pull out" square terms using the following rules:

$\sqrt{a^2} = a$ $\sqrt{ab} = \sqrt{a}\sqrt{b}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ as long as $b \neq 0$

A radical is in simplest form when:

- there is no integer under the radical sign with a perfect square factor,
- there are no fractions under the radical sign, and
- there are no radicals in the denominator.

Exercises: Simplify each expression.

1. $\sqrt{\frac{15}{81}} =$

2. $\sqrt{24} + 5\sqrt{6} =$

3. $\sqrt{75} + \sqrt{243} =$

4. $4 + 2\sqrt{10} =$

5. $\sqrt{28} + \sqrt{7} =$

6. $\sqrt{\frac{27}{49}} =$

7. $5\sqrt{3} - \sqrt{75} =$

8. $4\sqrt{3} \bullet \sqrt{18} =$

9. $\sqrt{128} - \sqrt{8} =$

10. $(5\sqrt{3})^2 =$

11. $\sqrt{128} + \sqrt{50} =$

12. $\sqrt{75} \bullet \sqrt{27} =$

13. Nina says that $16 + 4\sqrt{2}$ cannot be simplified. George says that is can be simplified to $20\sqrt{2}$. Who is correct? Explain how you know.

Solving Equations II

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{array}{rcl} 8x - 6 & = & 4x + 5 \\ \underline{-4x} & & \underline{-4x} \\ 4x - 6 & = & 5 \\ \underline{+6} & & \underline{+6} \\ \frac{4x}{4} & = & \frac{11}{4} \\ & & \\ x & = & 2\frac{3}{4} \end{array}$$

$$\begin{array}{rcl} 5 - 6t & = & 24 + 4t \\ \underline{+6t} & & \underline{+6t} \\ 5 & = & 24 + 10t \\ \underline{-24} & & \underline{-24} \\ \frac{-19}{10} & = & \frac{10t}{10} \\ & & \\ -1\frac{9}{10} & = & t \end{array}$$

Exercises: Solve the following problems:

<u>SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.</u>

1. $4r - 7 = 6r + 16 - 3r$
2. $13 + 3t = 5t - 9$
3. $-3x + 5 = 3x - 3$
4. $6y + 5 = 6y - 15$
5. $5x - 8 = 6 - 7x + 2x$
6. $7p - 8 = -6p + 8$

7. **Rowboat Rentals:** \$5.00 per hour plus a \$100.00 deposit. *Deposit will be refunded if the boat is returned undamaged.*

   Which equation represents the total cost for renting and returning a row-boat undamaged? Let c be the total cost in dollars and t be the time in hours.

   a. $c = 5t + 100$ b. $c = 500t$

   c. $c = 100t + 5$ d. $c = 5t$

8. Ted wants to buy a \$400.00 bike. He has two options for payment.

   Option One: Ted can borrow the \$400.00 from his father and repay him \$40.00 a month for a year.

   Option Two: The bike shop will finance the bike for one year at a 15% annual interest rate.

The formula for the total amount paid (a) is:

$a = p + prt$, where p in the amount borrowed, r is the rate of interest, and t is the time in years.

Which option would cost Ted the least amount of money?

Explain how you determined your answer. Use words, symbols, or both in your explanation.

Inequalities

Hints/Guide:

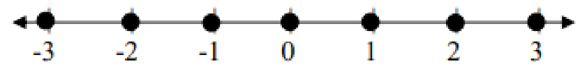
In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

<u>Inequality</u> - a statement containing one of the following symbols:

$<$ is less than $>$ is greater than $\leq$ is less than or equal to $\geq$ is greater than or equal to _ is not equal to

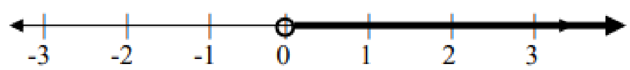
Examples:

1. Integers between -4 and 4.



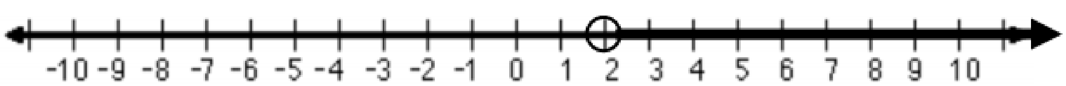
2. All numbers between -4 and 4.

3. The positive numbers.



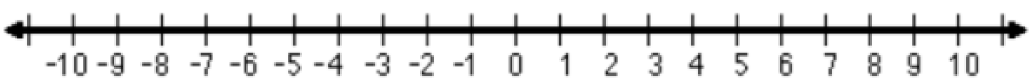
So, to solve the inequality -4x < -8 becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore x > 2 is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:

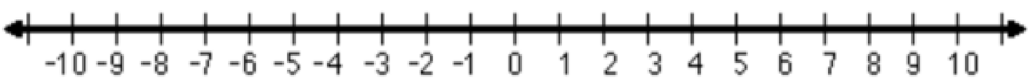


Exercises: Solve the following problems: No Calculators!

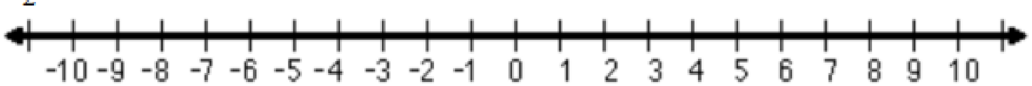
1. $4x > 9$



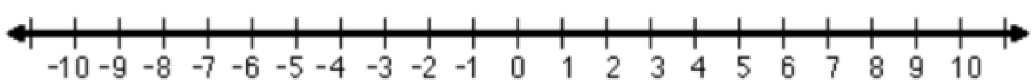
2. $-5t \geq -15$



3. $\frac{x}{2} \geq 3$



4. $\frac{x}{-4} > 2$

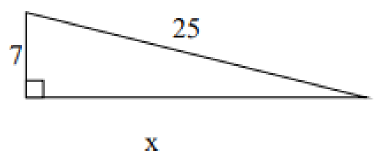


Pythagorean Theorem

Hints/Guide:

The Pythagorean Theorem states that in a right triangle, and only in a right triangle, the length of the longest side (the side opposite the right angle and called the hypotenuse, or c in the formula) squared is equal to the sum of the squares of the other two sides (the sides that meet to form the right angle called legs, or a and b in the formula). The formula is $a^2 + b^2 = c^2$.

Find the missing side.



$a^2 + b^2 = c^2$
$7^2 + x^2 = 25^2$
$49 + x^2 = 625$
$-49 \qquad -49$
$x^2 = 576$
$\sqrt{x^2} = \sqrt{576}$
$x = 24$

Exercises: Solve for the variable:

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

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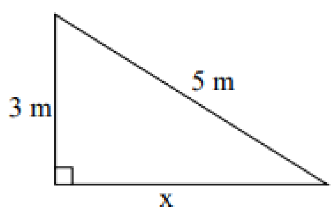

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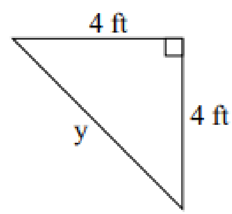

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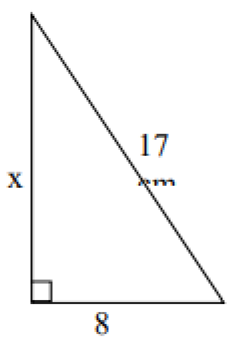

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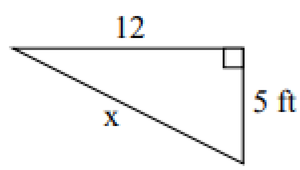

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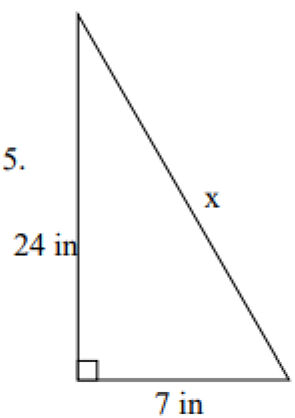

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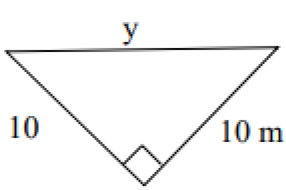

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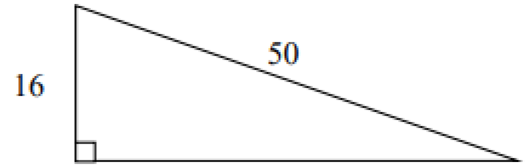


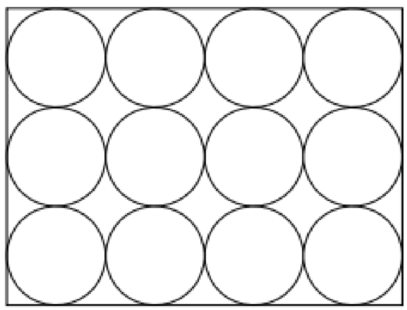
Irregular Area

Hints/Guide:

To solve problems involving irregular area, use either an additive or a subtractive method. In an additive area problem, break the object down into know shapes and then add the areas together. In a subtractive area problem, subtract the area of known shapes from a larger whole.

Exercises:

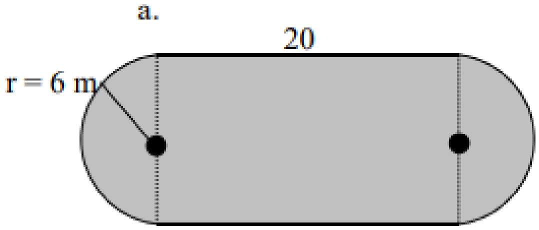
1. The baking sheet shown holds 12 cookies. Each cookie has a diameter of 3 inches.



What is the area of the unused part of the baking sheet? Round your answer to the nearest square inch.

2. Find the area of the shaded regions.

a.



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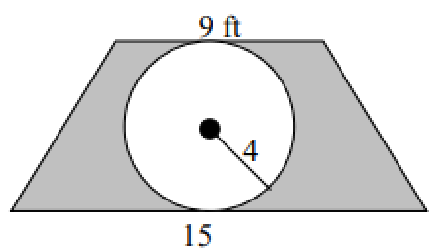

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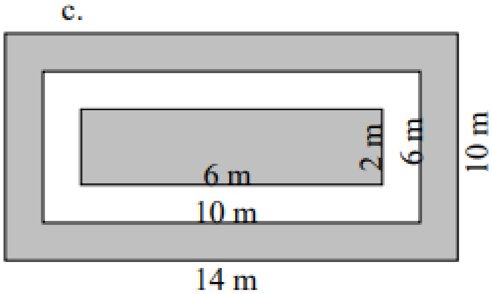

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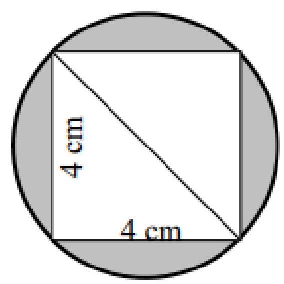

Volume and Surface Area

Hints/Guide:

To find the volume of prisms (a solid figure whose ends are parallel and the same size and shape and whose sides are parallelograms) and cylinders, we multiply the area of the base times the height of the figure. The formulas we need to know are:

The area of a circle is $A = \pi r^2$ The area of a rectangle is A = bh

The area of a triangle is $A = \frac{1}{2} b h$ The volume of a prism is V = Base Area • h

So, the volume of a rectangular prism can be determined if we can find the area of the base and the perpendicular height of the figure.

To determine the surface area of an object, we must find the areas of each surface and add them together. For a rectangular prism, we find the area of each rectangle and then add them together. For a cylinder, we find the area of each base and then add the area of the rectangle (the circumference of the circular base times the height) which wraps around to create the sides of the cylinder.

Exercises: Find the volume and surface area of the following figures: Note: Use $\pi = 3.14$

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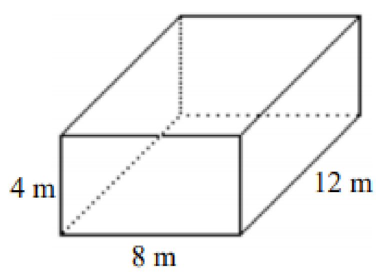

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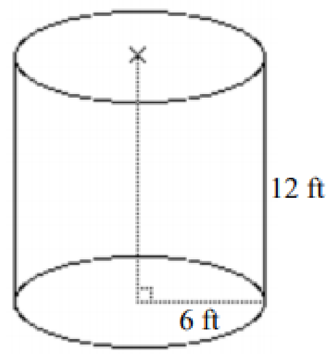

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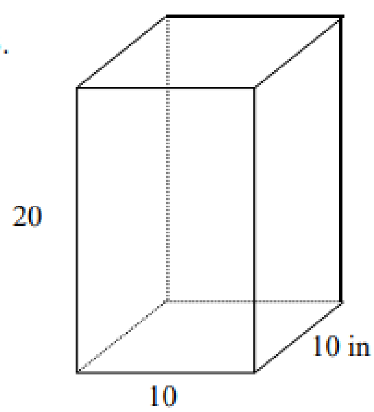

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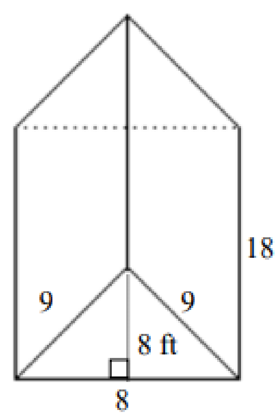
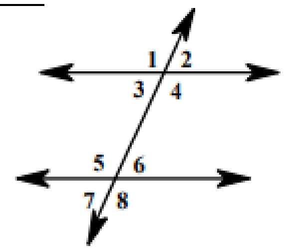


Angle Relationships

Hints/Guide:

To solve these problems, you will need to know some basic terms:

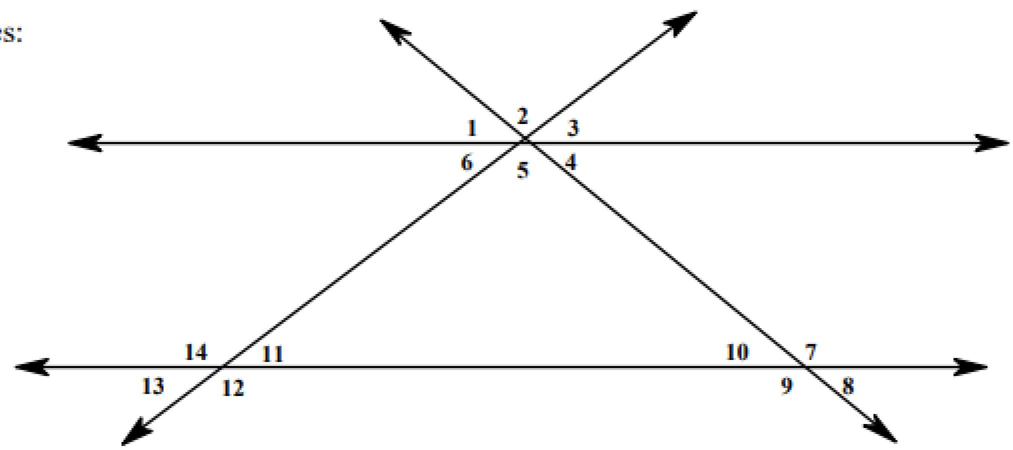
Two angles that sum to 180 degrees are called supplementary.
Two angles that sum to 90 degrees are called complementary.
Two angles that have the same angular measure are called congruent.
When a line (called a transversal) intersects a pair of parallel lines, it forms eight angles.



Angles 1 and 5 are corresponding.
Angles 1 and 8 are alternate exterior.
Angles 3 and 6 are alternate interior.
Angles 1 and 7 are same side exterior.
Angles 3 and 5 are same side interior.

Same side interior and same side exterior are supplementary angles.
Alternate interior and alternate exterior are congruent angles.
Corresponding angles are congruent angles.

Exercises:



If the measure of Angle 10 is 54º and Angle 11 is 46º, what is the measure of:

1. Angle 1 = ____________
2. Angle 2 = ____________
3. Angle 3 = ____________
4. Angle 4 = ____________
5. Angle 5 = ____________
6. Angle 6 = ____________
7. Angle 7 = ____________
8. Angle 8 = ____________
9. Angle 9 = ____________
10. Angle 12 = __________
11. Angle 13 = __________
12. Angle 14 = __________

Solving Problems Involving Shapes

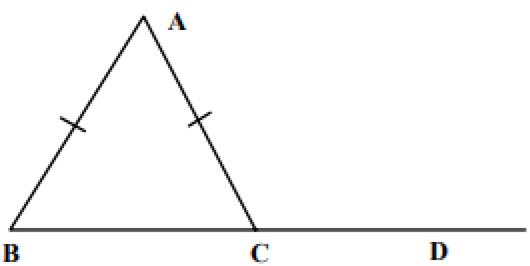
Hints/Guide:

To solve these problems, you will need to recall many facts, including:

There are 180 degrees in every triangle.
Isosceles triangles have two equal sides and two equal angles.
Equilateral triangles have three equal sides and three equal angles.
Quadrilaterals have an interior angle sum of 360 degrees.
Regular polygons have all sides equal and all angles equal.
The sum of the angles of a regular polygon can be found using $(n - 2) \cdot 180$.
The number of diagonals of any polygon can be found using $n(n - 3) \div 2$.
Concave polygon has one or more diagonals with points outside the polygon.
Convex polygon has all interior angles less than 180 degrees.

Exercises:

1. Triangle ABC is isosceles with the measure of $\angle A = 30^{\circ}$



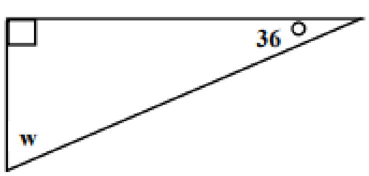
What is the measure of $\angle ACD$?

2. How many degrees in each angle of a regular hexagon? a regular octagon?

3. How many diagonals in a pentagon? an octagon? a decagon?

4. Solve for the missing angle (not drawn to scale)

a.



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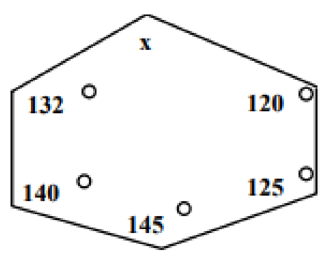

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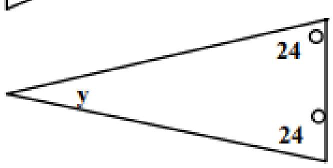

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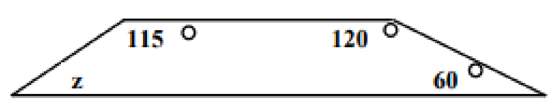
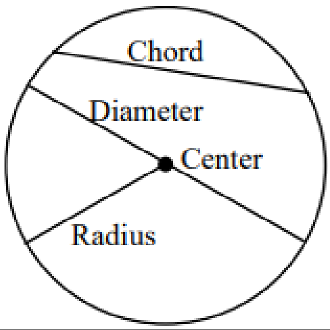


Circles

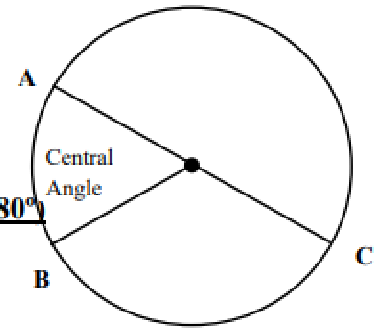
Hints/Guide:

There is some basic terminology that is needed for geometry. You need to know:



Circumference = $2\pi r = \pi d$

Area = πr^2



AB is a minor arc (less than 180º)

ACB is a major arc (greater than 180º)

<u>Exercises: Find the circumference and area of each circle. Use $\pi = 3.14$</u>

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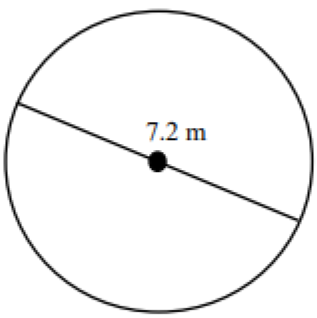

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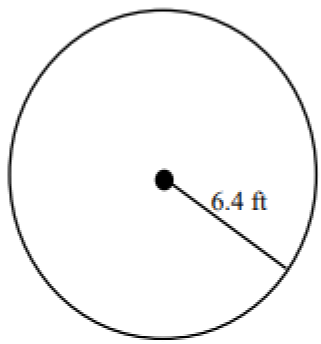

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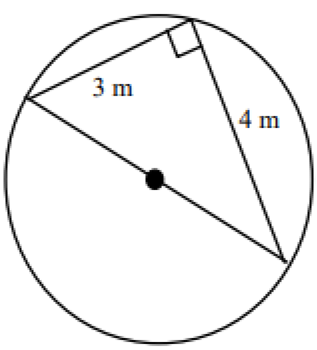

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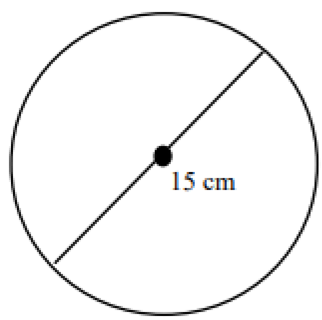
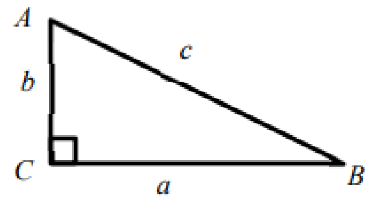


Geometry Formula Sheet

Pythagorean Theorem:
In right triangle ABC with right angle at point C:

$$a^2 + b^2 = c^2$$



Trigonometry:
In a right triangle with acute angle A:

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A}$$

Area/Circumference:

Triangle: $A = \frac{1}{2}bh$ Rectangle: $A = bh$ Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Parallelogram: $A = bh$ Regular Polygon: $A = \frac{1}{2} \times \text{apothem} \times \text{perimeter}$

Circle Area: $A = \pi r^2$ Circle Circumference: $C = 2\pi r = \pi d$

Volume:

Prism/Cylinder: $V = Bh =$ area of base $\times$ height Sphere: $V = \frac{4}{3}\pi r^3$

Pyramid/Cone: $V = \frac{1}{3}Bh = \frac{1}{3} \times \text{area of base} \times \text{height}$

Density = Mass $\div$ Volume

Coordinate Geometry:

Slope: $\frac{y_2 - y_1}{x_2 - x_1}$ Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Equations of Lines:

Slope-Intercept Form: $y = mx + b$
Point-Slope Form: $y - y_1 = m(x - x_1)$
Standard Form: $Ax + By = C$