Albert Einstein High School

Summer Task Cover Sheet

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Course: AP Calculus AB and AP Calculus BC

- ✓ **Purpose of the Summer Assignment:** In Calculus, it's rarely the calculus that will get you; it's the algebra and pre-calculus. Students entering AP Calculus absolutely *must* have a strong foundation in algebra and pre-calculus. It's been said the Calculus is just Algebra wearing fancy pants.

- ✓ **Relationship between Summer Task and 1st Quarter Objectives:** Most questions in this packet were included because they concern skills and concepts that will be used extensively in AP Calculus. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and – more frequently – the ability to problem solve.

- ✓ **Description of the Task:** This packet includes a sampling of problems that students entering AP Calculus should be able to answer without much hesitation. Please note that there are separate packets for AB and BC. The questions are organized by topic; there are 15 topics for AB and 18 topics for BC. **When completing the problems each topic should be on its own page. Yes, AB students should have 15 or more separate sheets; BC students should have at least 18 sheets.**

- ✓ **Format for the Task:** AP graders must work quickly so it is important that you present your work in a neat, orderly format. I expect you to demonstrate this skill on this assignment and format will account for part of your grade. You will

  - Identify each page with your name clearly written on the upper right side of the page
  - Label each page by topic; write the topic name on the upper left side of the page
  - Identify each problem by number
  - Show all necessary work in a vertical format
  - Leave a blank line between problems
  - Box or circle your final answer

- ✓ **Supportive Resources:** Internet sites such as Khan Academy, PatrickJMT, & university websites; Ms. Griffin will respond to email but may not answer immediately.

- ✓ **Grading:** On the first day of school you will place each topic sheet in its own container. You will not pass in the packet listing the problems but I suggest that you annotate it so that you can ask questions regarding any problems that you found difficult to complete. Your grade will be based on meeting the due date, completing the problems, and correct answers on a randomly chosen subset of the assigned problems.

  In addition to the Summer Assignment grade described above, there will be four mini quizzes covering the material. I will distribute the calendar for the mini quizzes on the first day of school.

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- ✓ **DEADLINE:** First day of school
- ✓ **Grading Category:** Formative
- ✓
- ✓ **Points**: 12.5% of Formative as this category is 40% of your grade.
- ✓ **Extent to which the summer task counts towards the marking period grade:** As (.40)(.125) = .05, this will be 5% of your first quarter grade**.**

Please understand, the curriculum for AP Calculus (and your teacher) will expect you to approach problems with the mathematical *toolkit* needed to do the calculations and the mathematical *understanding* needed to make sense of unusual problems. This is not a class where every problem you see on tests and quizzes is identical to problems you've done dozens of times in class. This is because the AP test itself (and, truly, all "real" mathematics) requires you to take what you know and apply it, rather than to simply regurgitate a rote process. These are skills we will polish all year.

Now that I've said all of this, I encourage you to take a deep breath and start working. Do not wait until the last few days of summer to begin on this packet. If you spread it out, you will most likely retain the information much better. When you have the basics down and you put in the work needed, you'll see how amazing Calculus is! AP Calculus is challenging, demanding, rewarding, and – simply put – totally awesome!

AP Calculus AB Summer Assignment Name: _____

This packet contains the review problems you will submit on the scheduled first day of school. **You do not turn in this packet.** Follow the directions on the cover sheet for the work you will submit. I suggest you annotate this set of problems so that you can ask questions on the topics you find troubling.

I. Simplify. Show the work that leads to your answer.

1. $\dfrac{x-4}{x^2-3x-4}$ 2. $\dfrac{x^3-8}{x-2}$ 3. $\dfrac{5-x}{x^2-25}$ 4. $\dfrac{x^2-4x-32}{x^2-16}$

II. Trigonometric Identities.

1. Pythagorean = _____ 2. cos 2x = _____

3. sin 2x = _____

III. Simplify each expression.

1. $\dfrac{1}{x+h}-\dfrac{1}{x}$ 2. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$ 3. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}$ 4. $\dfrac{2x}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}$

IV. Solve for **z**:

1. $4x + 10yz = 0$ 2. $y^2 + 3yz - 8z - 4x = 0$

V. If: $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$ $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$
determine each of the following:

1. $(f + h)(1) =$ 2. $(k - g)(5) =$ 3. $(f \circ h)(3) =$ 4. $(g \circ k)(7) =$

5. $f^{-1}(x) =$ 6. $k^{-1}(x) =$ 7. $\dfrac{1}{f(x)} =$ 8. $(kg)(x) =$

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\dfrac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.
2. Expand $(x + y)^3$
3. Simplify: $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$
4. Eliminate the parameter and write a rectangular equation for $x = t^2 + 3$
$y = 2t$

VII. Expand and simplify

1. $\sum_{n=0}^{4} \frac{n^2}{2}$

2. $\sum_{n=1}^{3} \frac{1}{n^3}$

VIII. Simplify

1. $\frac{\sqrt{x}}{x}$

2. $e^{\ln 3}$

3. $e^{(1+\ln x)}$

4. ln 1

5. $\ln e^7$

6. $\log_3(1/3)$

7. $\log_{1/2} 8$

8. $\ln \frac{1}{2}$

9. $e^{3 \ln x}$

10. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

11. $27^{2/3}$

12. $(5a^{2/3})(4a^{3/2})$

13. $(4a^{5/3})^{3/2}$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

1. with slope –2, containing the point (3, 4) — 1. _____
2. containing the points (1, -3) and (-5, 2) — 2. _____
3. with slope 0, containing the point (4, 2) — 3. _____
4. parallel to $2x - 3y = 7$ and passes through (5 , 1) — 4. _____
5. perpendicular to the line in problem #1, containing the point (3, 4) — 5. _____

X. Without a calculator, determine the exact value of each expression.

1. sin 0
2. $\sin \frac{\pi}{2}$
3. $\sin \frac{3\pi}{4}$
4. $\cos \pi$
5. $\cos \frac{7\pi}{6}$
6. $\cos \frac{\pi}{3}$
7. $\tan \frac{7\pi}{4}$
8. $\tan \frac{\pi}{6}$
9. $\tan \frac{2\pi}{3}$
10. $\tan \frac{\pi}{2}$
11. $\cos(\mathrm{Sin}^{-1} \frac{1}{2})$
12. $\mathrm{Sin}^{-1}(\sin \frac{7\pi}{6})$

XI. For each function, determine its domain and range.

1. $y = \sqrt{x-4}$
2. $y = \sqrt{x^2-4}$
3. $y = \sqrt{4-x^2}$
4. $y = \sqrt{x^2+4}$

XII. Determine all points of intersection.

1. parabola $y = x^2 + 3x - 4$ and line $y = 5x + 11$
2. $y = \cos x$ and $y = \sin x$ in the first quadrant

XIII. Solve for x, where x is a real number. Show the work that leads to your solution.

1. $x^2 + 3x - 4 = 14$
2. $\dfrac{x^4 - 1}{x^3} = 0$
3. $(x - 5)^2 = 9$
4. $2x^2 + 5x = 8$
5. $(x + 3)(x - 3) > 0$
6. $x^2 - 2x - 15 \le 0$
7. $12x^2 = 3x$
8. $\sin 2x = \sin x$, $0 \le x \le 2\pi$
9. $|x - 3| < 7$
10. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$
11. $27^{2x} = 9^{x-3}$
12. $\log x + \log(x - 3) = 1$
13. $e^{3k} = 5$
14. $\ln y = 2t - 3$

XIV. Graph each function. Give its domain and range.

1. $y = \sin x$
2. $y = \cos x$
3. $y = \tan x$
4. $y = x^3 - 2x^2 - 3x$
5. $y = x^2 - 6x + 1$
6. $y = \dfrac{x + 4}{x - 1}$
7. $y = \dfrac{x^2 - 4}{x + 2}$
8. $y = e^x$
9. $y = \sqrt{x}$
10. $y = \sqrt[3]{x}$
11. $y = \ln x$
12. $y = |x + 3| - 2$
13. $y = \dfrac{1}{x}$
14. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \le x \le 3 \\ 4 & \text{if } x > 3 \end{cases}$

XV. Growth & Decay.

1. Write the equation for the exponential model that passes through….

a. (-2 , 12) and (0 , 3)

b. $\left(-5, \dfrac{243}{2}\right)$ *and* $\left(-1, \dfrac{3}{2}\right)$

3. Fill in the blanks for $f(x) = 2(5)^{-x}$.

Continuous (yes/no) _____

$\lim_{x \to \infty} f(x) =$ _____

$\lim_{x \to -\infty} f(x) =$ _____

H.A. _____

Domain: _____

Range: _____

Increasing: _____

Decreasing: _____

Upper bound: _____

Lower bound: _____

AP Calculus BC Summer Assignment Name: _____

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III. Simplify each expression.

1. $\dfrac{1}{x+h}-\dfrac{1}{x}$ 2. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$ 3. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}$ 4. $\dfrac{2x}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}$

IV. Solve for **z**:

1. $4x + 10yz = 0$ 2. $y^2 + 3yz - 8z - 4x = 0$

V. If: $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$ $h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$
determine each of the following:

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VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\dfrac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.
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11. $27^{2/3}$

12. $(5a^{2/3})(4a^{3/2})$

13. $(4a^{5/3})^{3/2}$

14. $\frac{3(n+1)!}{5n!}$

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

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X. Given the vectors **v** = -2**i** + 5**j** and **w** = 3**i** + 4**j**, determine

1. $\frac{1}{2}$**v**
2. **w** – **v**
3. length of **w**
4. the unit vector for **v**

XI. Without a calculator, determine the exact value of each expression.

1. sin 0
2. $\sin \frac{\pi}{2}$
3. $\sin \frac{3\pi}{4}$
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7. $y = \dfrac{x^2 - 4}{x + 2}$
8. $y = e^x$
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14. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 3 \\ 4 & \text{if } x > 3 \end{cases}$

XVI. **Graph** each of the following. Then, dentify, by name, each polar graph. Give at least one characteristic of each graph (e.g. radius, location, length of petal, point (other than the pole) on the graph, etc.)

1. $r = 2$ _____ _____
2. $r = 3 \sin \theta$ _____ _____
3. $r = 1 + \sin \theta$ _____ _____
4. $r = 2\cos 3\theta$ _____ _____

XVII. Sequences and series. Evaluate each of the following.

1. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ 2) $\sum_{n=1}^{5}(2n-4)$ 3) $\sum_{n=1}^{4} n^2 - n$ 4) $\sum_{n=1}^{50}(2n-4)$

5. Find the 131st term in the arithmetic sequence in which $t_3 = 7$ & $t_9 = 61$.

6. Find the first two terms of the geometric sequence for which $t_5 = 36\sqrt{2}$ & $r = \sqrt{2}$.

XVIII. Growth, decay, and logistics.

1. Write a logistic model with the following characteristics...

Limit to growth = 30 Initial Value = 12 Passes through (2 , 20)

2. Write the equation for the exponential model that passes through....

2a) (-2 , 12) and (0 , 3) 2b) $\left(-5, \frac{243}{2}\right)$ *and* $\left(-1, \frac{3}{2}\right)$

3. Fill in the blanks...

Fill in the blanks for $f(x) = 2(5)^{-x}$.

Continuous (yes/no) _____

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H.A. _____

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