Precalculus: Unit 1 – Power, Polynomial, and Rational Functions

Topic	Instructional Foci
Topic 1: Piecewise-Defined Functions/Composition of Functions/*Limits*	In this topic, students learn that piecewise-defined functions can be used to model real-world situations. They explore some basic function types (i.e., absolute value) that can be expressed as piecewise-defined functions. Students extend their repertoire of function operations to include composition, and they learn how to verify that two functions are inverses using composition. Building on their knowledge of key features of functions, they learn that if a function is one-to-one, then its inverse is also a function. They classify functions as even, odd, or neither using the definitions, and they learn that the behavior of a function near a point can be used to classify a function as continuous or to identify the type of discontinuity. Honors students also learn that the concept of limits can be applied to describe the behavior of functions. ***Background:*** In Algebra 1, students did extensive work with linear, quadratic, and exponential functions and their transformations. In Algebra 2, they extended their knowledge of functions to include polynomial functions, simple rational functions (limited to transformations of $y=\frac{1}{x}$ and $y=\frac{1}{x^2}$), radical functions (limited to $y=\sqrt{x}$ and $y=\sqrt[3]{x}$) and logarithmic functions (limited to base 2, 10, and *e*). They identified key features of the graphs (domain, range, intervals of increasing/decreasing, end behavior, symmetry, intercepts, and extreme values, asymptotes). They performed operations on functions (addition, subtraction, multiplication, and long division). They found inverses of functions numerically, graphically, and algebraically. They recognized that $y=x^2$ needed a restricted domain to have an inverse that is a function. ***Concepts:*** 1. Explore continuity and types of discontinuity and boundedness, formalize definitions of even and odd, *and introduce limit notation for asymptotes and end behavior*. (Addison Wesley §1.2, Glencoe §3.1, §3.5) 2. Revisit piecewise-defined functions, extending them to include pieces that are polynomial, rational, or exponential. (Addison Wesley §1.3, Glencoe §1.7) 3. Compose and decompose functions, and find the domains of composed functions. (Addison Wesley §1.4, Glencoe §1.2) 4. Define one-to-one functions, and use the Inverse Composition Rule to verify that two functions are inverses. (Addison Wesley §1.5, Glencoe §3.4)

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Topic 2: Power Functions	In this topic, students learn that a function of the form $f(x) = k\,x^a$, where *k* and *a* are real numbers, represents a power function, whose properties are dependent on the value of *k* and *a*. Honors students also learn that the concept of limits can be applied to describe the behavior of power functions. ***Background:*** In their study of linear, quadratic, polynomial, radical, and rational functions in Algebra 1 and Algebra 2, students studied the parent functions $y = x^n$ and their transformations, for positive integer values of *n* and for $n = -2, -1, 0, \frac{1}{2}$, and $\frac{1}{3}$. They applied the terms direct and inverse variation and determined the constant of proportionality. In their study of exponential functions in Algebra 2, they learned that exponential functions are continuous and defined for all real number exponents. Honors Algebra 2 students explored how to numerically approximate values of expressions such as 2^{π} or $2^{\sqrt{2}}$. ***Concepts:*** 1. Define general power functions. (Addison Wesley §2.2, §A.1, Glencoe §11.2) 2. Analyze and categorize power functions and their key features, *including limits*. (Addison Wesley §2.2) 3. Transform power functions. (Addison Wesley §2.2) 4. Apply power functions to real-world situations. (Addison Wesley §2.2)

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Topic 3: Graphs of Rational Functions Extended	In this topic, students learn that the graph of a rational function, whose denominator is at most quadratic, can be analyzed in terms of its domain, range, intercepts, asymptotes (vertical, horizontal, oblique) and discontinuities. Honors students also learn that the concept of limits can be applied to describe the behavior of rational functions. ***Background:*** In Algebra 2, students studied transformations of $y=\frac{1}{x}$ and $y=\frac{1}{x^2}$ .They used long division of polynomials to investigate key features of the graphs. Honors students explored the difference between holes and vertical asymptotes. ***Concepts:*** 1. Graph rational functions with denominators of degree 2 that have holes. (Addison Wesley §2.6, Glencoe §3.7) 2. Graph rational functions with denominators of degree 2 that have oblique asymptotes. (Addison Wesley §2.6, Glencoe §3.7) 3. *Compute limits near holes and vertical asymptotes and as* $x \to \pm\infty$. (Addison Wesley §2.6) 4. *Explore the behavior of rational functions near horizontal asymptotes when given as partial fraction decompositions, where denominators have distinct linear factors only.* (Addison Wesley §7.4)

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Topic 4: The Algebra of Rational Expressions/Equations/Inequalities/*Partial Fractions*	In this topic, students learn that rational expressions can be combined using arithmetic operations. Honors students extend this learning to understand that some rational expressions can be expressed as the sum of two or more other rational expressions by partial fraction decomposition, although denominators are limited to products of linear and non-repeating factors. All students learn that polynomial inequalities and rational equations and inequalities can be used to solve real-world problems. *Note: Rational equations and inequalities are limited to denominators that are at most quadratic.* ***Background:*** In Algebra 2, students solved simple rational equations by multiplying both sides of the equation by the least common multiple of the denominators. Denominators contained at most two distinct linear factors. Students encountered extraneous roots due to division by 0. ***Concepts:*** 1. Add, subtract, multiply, and divide rational expressions (Addison Wesley §A.3) 2. *Find partial fraction decompositions of rational expressions, where the denominator is a product of distinct linear factors, and identify the impact on the graph.* (Addison Wesley §7.4) 3. Simplify complex fractions. (Addison Wesley §A.3) 4. Solve rational equations. (Addison Wesley §2.7) 5. Solve polynomial inequalities when suitable factorizations are available. (Addison Wesley §2.8) 6. Solving rational inequalities when suitable factorizations are available. (Addison Wesley §2.8)